

F-bar aided edge-based smoothed finite element method with tetrahedral elements for large deformation analysis of nearly incompressible materials

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Abstract

A new smoothed finite element method (S-FEM) with tetrahedral elements for finite strain analysis of nearly incompressible solids is proposed. The proposed method is basically a combination of the F-bar method and edge-based S-FEM with tetrahedral elements and is named “F-barES-FEM-T4”. The isovolumetric part of the deformation gradient ($\mathbf{F}_{iso}$) is derived from the $\mathbf{F}$ of ES-FEM, whereas the volumetric part ($\mathbf{F}_{vol}$) is derived from the cyclic smoothing of $J (= \det(\mathbf{F}))$ between elements and nodes. F-barES-FEM suppresses the pressure oscillation in nearly incompressible materials successfully with no increase in DOF.

Keywords: F-bar method, Smoothed finite element method, Tetrahedral element, Finite strain, Nearly incompressible, Pressure oscillation.

Introduction

In the practical numerical simulation for solids, the use of tetrahedral elements are indispensable due to the complexity of body shapes. The edge-based smoothed finite element method with first-order tetrahedral elements (ES-FEM-T4) [Liu et al. (2010)] is one of the most accurate FE formulations that resolve the shear locking issue of the standard tetrahedral (i.e., constant strain) FE formulation. However, ES-FEM-T4 raises volumetric locking in the analysis of nearly incompressible materials such as rubber. Therefore, development of accurate FE formulations with T4 elements is still in a research stage.

There are already several methods proposed for locking-free analysis with tetrahedral meshes, but they all have some sort of serious drawbacks as follows.

- Hybrid (or mixed) second-order tetrahedral elements:
significant increase in DOF; inevitable Lagrange multipliers; convergence problems in contact analysis; accuracy loss in severely large strain analysis.
- F-bar-Patch method [Neto et al. (2005)]:
difficulties in pre-definition of good-quality patches; shear locking.
- Selective reduced integration hexahedral elements as the subdivision of tetrahedral elements (4 hexahedrons in a tetrahedron):
significant increase in DOF; severe element distortion of initial mesh; pressure oscillation.
- Selective edge/node-based S-FEM-T4 (ES/NS-FEM-T4) [Onishi and Amaya (2014)]:
pressure oscillation; locking at corners.
- Bubble-enhanced ES-FEM (bES-FEM-T4) [Ong et al. (2015)]:
significant increase in DOF; quick pop out of bubble nodes; pressure oscillation.

In addition, formulations with the selective integration have a drawback that they cannot treat materials with deviatoric/volumetric coupling terms. For these reasons, a new formulation without intermediate nodes, mixed variational principle, pre-defined patches, element subdivision, selective integration, nor bubble nodes is awaited.

In this paper, we propose a new ES-FEM-T4 formulation combined with F-bar method [Neto et al. (1996)] named “F-barES-FEM-T4”. The smoothed deformation gradient and stress in the proposed method are defined at each edge, and the stress integration is performed over the smoothing domains of ES-FEM. The isovolumetric part of the deformation gradient ($\mathbf{F}_{iso}$) is derived from $\mathbf{F}$ of ES-FEM, whereas the volumetric part ($\mathbf{F}_{vol}$) is derived from the cyclic smoothing of $J (= \det(\mathbf{F}))$ between elements and nodes. Therefore, the proposed method can suppress the pressure oscillation without any unfavorable approaches listed above.

Method

Mutli-Smoothed Deformation Gradient

As is well known today, NS-FEM-T4 can avoid volumetric locking and gives solutions that have less pressure oscillation in nearly incompressible cases. It implies that the node-based smoothing of J ($\equiv \det(\mathbf{F})$) can be regarded as an operation to suppress pressure oscillation to some extent. Here we define the cyclic smoothing of J as the following operation:

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- 1) Calculate J at each element, J^{elem} , in the same way as the standard FEM-T4.
 - 2) Calculate smoothed J at each node, $\tilde{J}^{\text{node}}$, in the same way as NS-FEM-T4:

$$\tilde{J}^{\text{node}} = \frac{\sum_{i \in \mathbb{I}} J_i^{\text{elem}} V_i^{\text{elem}}}{\sum_{i \in \mathbb{I}} V_i^{\text{elem}}}, \quad (1)$$

where $\mathbb{I}$ is the set of elements attached to the each node and V denotes the volume.

- 3) Calculate smoothed J at each element, $\tilde{J}^{\text{elem}}$, as the average of $\tilde{J}^{\text{node}}$:

$$\tilde{J}^{\text{elem}} = \frac{\sum_{j \in \mathbb{J}} \tilde{J}_j^{\text{node}}}{4}, \quad (2)$$

where $\mathbb{J}$ is the set of four nodes comprising the each element.

- 4) Repeat 2) and 3) as necessary to calculate multi-smoothed J at each element, $\bar{J}^{\text{elem}}$. ($\tilde{J}^{\text{elem}}$ is considered as J^{elem} in Eq. (1), and the last $\tilde{J}^{\text{elem}}$ in Eq. (2) is considered as $\bar{J}^{\text{elem}}$.)
- 5) Calculate multi-smoothed J at each edge, $\bar{J}^{\text{edge}}$, in the same way as ES-FEM-T4:

$$\bar{J}^{\text{edge}} = \frac{\sum_{k \in \mathbb{K}} \bar{J}_k^{\text{elem}} V_k^{\text{elem}}}{\sum_{k \in \mathbb{K}} V_k^{\text{elem}}}, \quad (3)$$

where $\mathbb{K}$ is the set of elements attached to the each edge.

In F-barES-FEM-T4, $\bar{J}^{\text{edge}}$ is used to define the volumetric part of the deformation gradient as $\bar{\mathbf{F}}_{\text{vol}}^{\text{edge}} = (\bar{J}^{\text{edge}})^{1/3} \mathbf{I}$.

Meanwhile, ES-FEM-T4 can avoid shear locking and gives accurate solutions in the analysis with compressible materials. The smoothed deformation gradient of ES-FEM-T4 is calculated as

$$\tilde{\mathbf{F}}^{\text{edge}} = \frac{\sum_{k \in \mathbb{K}} \tilde{\mathbf{F}}_k^{\text{elem}} V_k^{\text{elem}}}{\sum_{k \in \mathbb{K}} V_k^{\text{elem}}}. \quad (4)$$

In F-barES-FEM-T4, $\tilde{\mathbf{F}}^{\text{edge}}$ is used to define the isovolumetric part of the deformation gradient as $\bar{\mathbf{F}}_{\text{iso}}^{\text{edge}} = (\bar{J}^{\text{edge}})^{-1/3} \tilde{\mathbf{F}}^{\text{edge}}$, where $\bar{J}^{\text{edge}} \equiv \det(\tilde{\mathbf{F}}^{\text{edge}})$.

Finally, the multi-smoothed deformation gradient at each edge, $\bar{\mathbf{F}}^{\text{edge}}$, is defined in manner of the F-bar method as

$$\bar{\mathbf{F}}^{\text{edge}} = \left(\frac{\bar{J}^{\text{edge}}}{\bar{J}^{\text{edge}}} \right)^{1/3} \tilde{\mathbf{F}}^{\text{edge}}. \quad (5)$$

Nodal Internal Force and Stiffness

The Cauchy stress at each edge $\mathbf{T}^{\text{edge}}$ is calculated in the standard way with $\bar{\mathbf{F}}^{\text{edge}}$. In contrast to the methods based on the selective integration, F-barES-FEM-T4 has no difficulty in treating dev/vol coupled materials.

The contribution of each edge to the nodal internal force is then calculated in manner of the F-bar method as

$$\{f_{\text{int}}^{\text{edge}}\} = [\tilde{B}^{\text{edge}}]\{T^{\text{edge}}\}V^{\text{edge}}, \quad (6)$$

where $[\tilde{B}^{\text{edge}}]$ is the smoothed B matrix of ES-FEM-T4 at each edge. In contrast to the methods based on mixed variational formulation, F-barES-FEM-T4 is purely a displacement-based formulation and thus eliminates the use of pressure or volumetric strain unknowns.

The contribution of each edge to the tangent stiffness matrix, $[K^{\text{edge}}]$, necessitates a little complicated way of derivation due to the cyclic smoothing, thereby omitting in this paper. As the number of the smoothing cycles is increased, the bandwidth of $[K]$ is also increased; however, there is no increase in DOF.

Results

Bending of Cantilever

Figure 1 shows the pressure distributions of a small deformation cantilever bending analyses in 2D plane-strain condition. The analysis domain is a $10 \text{ m} \times 1 \text{ m}$ rectangle discretized with a structural mesh of 500 triangular elements. Its left-hand side is perfectly constrained, and a concentrated bending load is assigned to the top-right node. The material is the neo-Hookean hyperelastic material with an initial Poisson's ratio of 0.499. It is clearly shown that NS-FEM-T3 raises severe pressure oscillation (Fig. 1 (a)), whereas our method with k times cyclic smoothing, F-barES-FEM-T3(k), suppresses pressure oscillation to some extent (Fig. 1 (b)-(d)). Our heuristic diagnostics suggest that $v_{\text{ini}} < 0.48 \Rightarrow k = 0$ (i.e., ES-FEM is recommended), $0.48 \leq v_{\text{ini}} < 0.49 \Rightarrow k = 1$, $0.49 \leq v_{\text{ini}} < 0.499 \Rightarrow k = 2$, $0.499 \leq v_{\text{ini}} < 0.4999 \Rightarrow k = 3$ and so on.

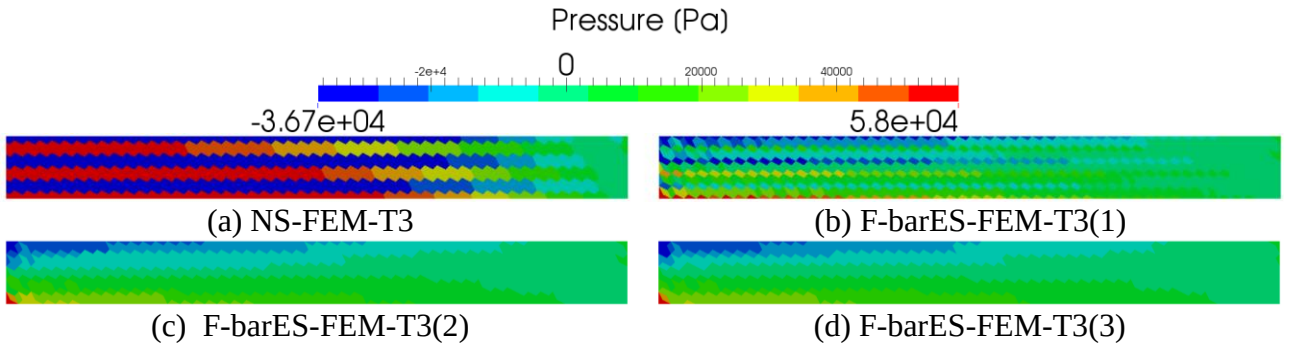


Figure 1. Pressure distributions of the plane-strain small deformation cantilever bending analysis. The contour range is adjusted to the result of (d). Increase in the number of cyclic smoothing helps suppressing the pressure oscillation in the nearly incompressible material.

Partial Compression of Block

Figure 2 shows the pressure distributions of a finite strain block compression analyses in 3D. The analysis domain is a 1 m cube discretized with an unstructured mesh of 55821 tetrahedral elements. Its $-x$, $-y$ and $-z$ faces are constrained in the normal direction of each face, its $+z$ face is constrained in x and y directions, and a pressure is applied to $1/4$ of $+z$ face. The material is the Arruda-Boyce hyperelastic material with an initial Poisson's ratio of 0.499. As noted in Introduction, ES/NS-FEM-T4 suffers from pressure oscillation (Fig. 2 (a)). In contrast, our F-barES-FEM-T3(2) and F-barES-FEM-T3(3) suppress pressure oscillation to some extent (Fig. 2 (b), (c)).

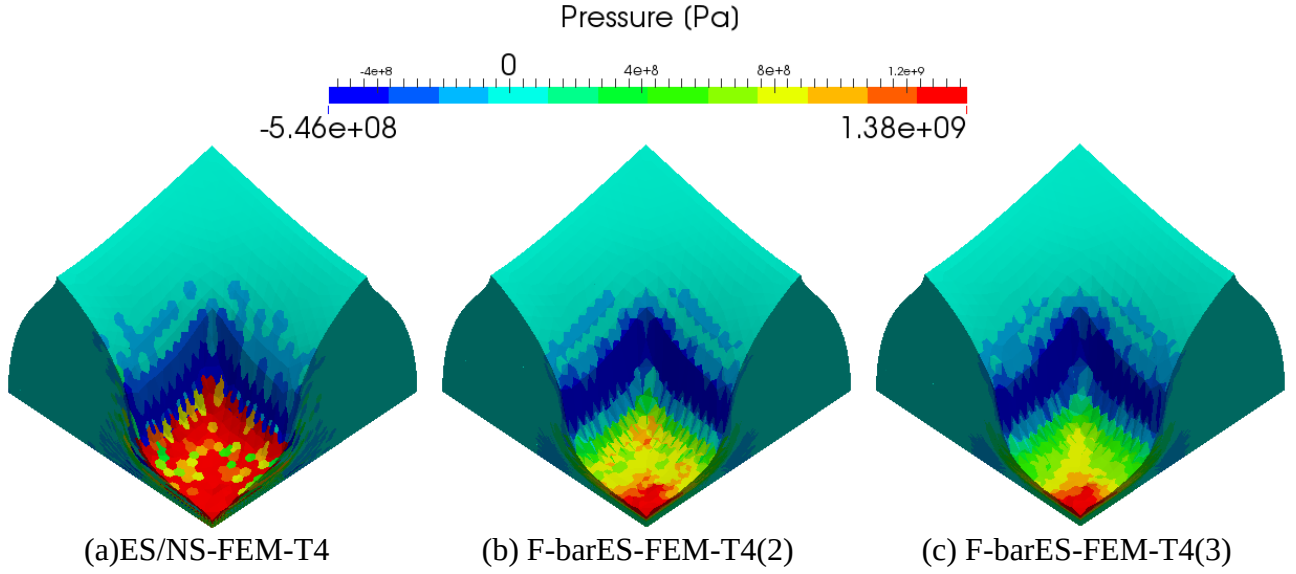


Figure 2. Pressure distributions of the finite strain block compression analysis. The contour range is adjusted to the result of (c). Our method with three-time cyclic smoothing, F-barES-FEM-T4(3), successfully suppresses the pressure oscillation in finite strain analysis of nearly incompressible materials.

Conclusions

A novel S-FEM called F-barES-FEM-T4 was proposed. The cyclic smoothing of $J (\equiv \det(\mathbf{F}))$ between elements and nodes was introduced to combine the idea of F-bar method and ES-FEM-T4 efficiently. Accordingly, F-barES-FEM-T4 suppressed the pressure oscillation in nearly incompressible materials with no increase in DOF.

Meanwhile, the relations among the Poisson's ratio, the number of smoothing cycles, underestimation of stress concentrations, etc. are still unclear. The impact of the increase in the matrix bandwidth on the computational efficiency is also unclear. Further investigation about them is our future work.

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