Topology optimization of anisotropic constrained damping structures based on ESO method

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Abstract

The topology optimization design of structures with anisotropic constrained damping layers (ACDL) is investigated in present paper. The evolutionary structural optimization (ESO) method is employed to find the optimal layout of ACDL with aim to maximum the modal damping loss factor of the sandwich structure. Modal strain energy (MSE) method is used to calculate the damping loss factor of whole sandwich structure, and the sensitivity is analyzed. Optimization topology configuration of ACDL is obtained under the constraint of total amount of ACLD materials in terms of percentage added weight to the base structure. The merit of constrained damping treatment is evaluated quantitatively by damping efficiency formula introduced.

Keywords: Evolution Structural Optimization, Modal Strain Energy, Anisotropic, Topology optimization

Introduction

Passive constrained damping layer (CDL) treatment is one of the common and effective means to reduce vibration and noise of thin-walled structures. However, aero structures require strictly for additional mass quality. On one hand, optimizing the layout of the CDL parts is needed to reduce the amount of material, and obtain a better suppression damping effect; on the other hand, we suppose to employ anisotropic constrained damping layer (ACDL), whose density is smaller and damping loss factor may not be neglected.

Lall[1,2]used analytical methods to study the structural characteristics of fragmentation damping structure for beams and plates respectively. Zheng[3]has studied the structural characteristics of fragmentation damping structure for cylindrical shell by same methods and explored the fragmentation arrangement of constraint damping layout optimization problem. From their work, we learn that better vibration control effect with less structural mass increment could be achieved by arranging damping structure appropriately without full coverage, which leads to the topology optimization problem of constrained damping structure.

Several topology optimization methods are commonly used, such as homogenization method (Bendsøe and Kikuchi[4]), variable density method (Bendsøe[5]; Sigmund[6]), evolutionary structural optimization method (Xie and Steven[7]), level set method (Wang et al.[8]; Allaire et al.[9]) and so on. Guo[10]optimized a square plate with CDL based on ESO method, and obtained a clear topological structure by remove the inefficient damping materials gradually; Zheng[11] used SIMP interpolation model and MMA solution algorithm to optimized the layout of constraint damping structure with a target to maximize the modal loss factor. The research of optimization for anisotropic constrained damping layers (ACDL) is rare.

This paper introduces damping efficiency formula to evaluate the merit of constrained damping treatment results quantitatively and then anisotropic material is employed as constraint sheet and the topology optimization problem of structures with ACDL is investigated based on ESO method. Finally, a numerical example is presented to show the superiority of ACDL in lightweight design concept.

Damping efficiency

The damping loss factor of structure will increase after an additional constrained damping layer treatment. We always want to attach additional mass as little as possible to get a bigger damping loss factor. In other word, a larger ‘damping efficiency’ after damping treatment is expected. To compare the results of damping treatments quantitatively, formula (1) is introduced to calculate the damping efficiency:

$$\eta_{eff} = \frac{\Delta\eta}{\Delta m} \tag{1}$$

In this formula, $\Delta\eta$ denotes the increment of the structure’s damping loss factor, and Δm denotes the mass of the damping layers, which is also additional mass.

Topology optimization model

The topology optimization problem of given thickness of both constraint layer and damping core, aiming at the maximum of damping loss factor, under the constraint of additional mass quantity, using element existence state as design variables, is formulated as:

$$\begin{aligned} & Max \quad \eta \\ & find \quad x = [x_1, x_2, \cdots, x_n] \\ & s.t. \quad \mathrm{W} = \mathrm{W_v} + \mathrm{W_c} \le \mathrm{W}^* \\ & \qquad x_i = \{0,1\} \end{aligned} \tag{2}$$

Where η is the damping loss factor of the sandwich structure; n denotes the number of viscoelastic damping core elements; x_i is design variable with 0 denoting element deleted and 1 denoting element reserved; $\mathrm{W_v}$ and $\mathrm{W_c}$ denote the mass quantity of the viscoelastic damping core and the constraint sheet respectively. W^* is the constraint mass quantity.

Finite element model

In a thin-walled structure with anisotropic constraint damping layer, energy consumption comes from both damping layer and constraint layer. The damping layer dissipates energy by shear deformation and the constraint layer dissipates energy by tensile/compressive and bending deformation. Thus, a finite element model reference to Johnson’s [12] method is employed using the commercial program package ABAQUS, as shown in figure 1. The constraint sheet and bottom sheet are modeled with quadrilateral shell elements called S4R producing stiffness at two rotational and three translational degrees of freedom per node. The viscoelastic damping core is modeled with solid elements called C3D8R producing stiffness at three translational degrees of freedom per node. All nodes are at element corners. A key feature of this kind of element in the present application is its ability to account for coupling between stretching and bending deformations. This feature allows the plate nodes to be offset to one surface of the plate, coincident with the corner nodes of the adjoining solid elements. In this way, a three-layer plate can be modeled with only two layers of nodes, and finite element analysis will not take too much time.

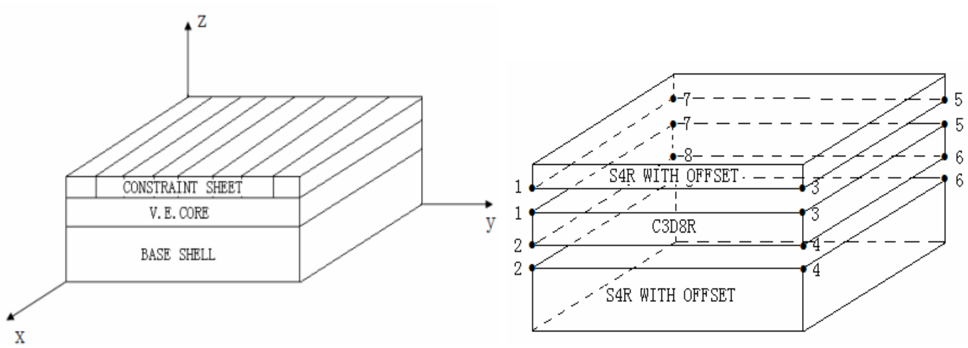

Figure 1. Finite element modeling of structure with ACDL

Sensitivity analysis of modal damping loss factor

According to modal strain energy (MSE) method, the *k*-th modal damping loss factor of the sandwich structure is calculated as follows:

$$\eta_k = \frac{\eta_v E_{vk} + \eta_c E_{ck} + \eta_b E_{bk}}{E_{tk}} \tag{3}$$

where η_v, η_c and η_b are damping loss factors of damping material, constraint sheet material and bass structure material respectively; E_{vk}, E_{ck}, E_{bk} denote the k-th modal strain energy of damping core, constraint sheet and bass structure respectively, and $E_{tk} = E_{vk} + E_{ck} + E_{bk}$.

In a difference method way, when the damping core element i and the corresponding constraint layer element are deleted, small change of the whole sandwich structure's damping loss factor can be approximately expressed as:

$$\Delta\eta_k = \frac{\eta_v \Delta E_{vk} + \eta_c \Delta E_{ck}}{E_{tk}} - \frac{(\eta_v E_{vk} + \eta_c E_{ck} + \eta_b E_{bk})\Delta E_{tk}}{E_{tk}^2} \tag{4}$$

In each iteration of the ESO method, the whole structure changes very little when an element i is deleted, so the following approximation can be adopted:

$$\begin{aligned} \Delta E_{vk} &\approx -E_{vki} \\ \Delta E_{ck} &\approx -E_{cki} \\ \Delta E_{tki} &\approx -E_{vki} - E_{cki} \end{aligned} \tag{5}$$

Where E always denotes model strain energy. From (3), (4) and (5), $\Delta\eta_k$ can be calculated as

$$\Delta\eta_k = \frac{(\eta_v E_{vk} + \eta_c E_{ck} + \eta_b E_{bk})(E_{cki} + E_{vki})}{E_{tk}^2} - \frac{\eta_v E_{vki} + \eta_c E_{cki}}{E_{tk}} \tag{6}$$

Formula (6) denotes the change amount of the $k-th$ modal damping loss factor after the damping core's element i and its corresponding constraint element are deleted. Thus formula (7) can be defined as the design sensitivity of element i.

$$S_{ki} = \frac{(\eta_v E_{vk} + \eta_c E_{ck} + \eta_b E_{bk})(E_{cki} + E_{vki})}{E_{tk}^2} - \frac{\eta_v E_{vki} + \eta_c E_{cki}}{E_{tk}} \tag{7}$$

If maximization of the former $m-th$ model damping loss factors is required, the design sensitivity of element i is formulated as

$$S = \sum_{k=1}^{m} \omega_k S_{ki} \tag{8}$$

where ω_k denotes the weight coefficients of each model.

Mesh-independent filtering

Discrete mesh-independent filtering method is employed during the optimization process to avoid checkerboard phenomenon. Referring to Sigmund's work, the new sensitivity of element *i* after filtering is:

$$S_i^{new} = \frac{1}{x_i \sum_{i=1}^{N} H_i} \sum_{j=1}^{N} H_j x_j S_j \tag{9}$$

Where x_j is design variable (0 or 1). The weight factor H_i is written as

$$H_i = \max(0, r - dist(i, j)) \tag{10}$$

In formula (10) the operator $dist(i, j)$ is defined as the distance between center of element i and center of element j and operator $\max(\ )$ is defined to take larger one of two values; r denotes filtering radius.

Optimization process

The initial idea of ESO topology optimization method is the ‘survival of the fittest’, which will not change when the traditional ESO method is applied to ACDL topology optimization, but its form varies. Design sensitivities of elements could be negative or positive, or they could all be positive. In the former case, elements with maximum positive sensitivities should be removed in each iteration so that the loss factor of whole structure may increase, and in the latter case, elements with minimum absolute sensitivity value should be deleted so that the loss factor may decrease slowly.

The optimization process aiming at the maximization of the modal damping loss factor of constrained damping structure with assigned material properties, boundary conditions, and the single iteration delete ratio R is shown in figure 2.The specific steps are as follows:

1. Generate the finite element model of the sandwich structure.
2. Execute structural modal analysis and calculate model strain energy.
3. Calculate the sensitivities of elements and filter them.
4. Find the maximum sensitivity.
5. If the maximum value is negative, remove $N_i = N * R$ (N denotes the total number of original damping core elements) damping core elements with maximum sensitivities and corresponding constraint sheet elements; If the maximum value is positive, remove damping core elements with maximum value and corresponding constraint layer elements.
6. Judge whether the amount of material constraint conditions is reached, if not, repeat steps 1-5, otherwise finish the result.

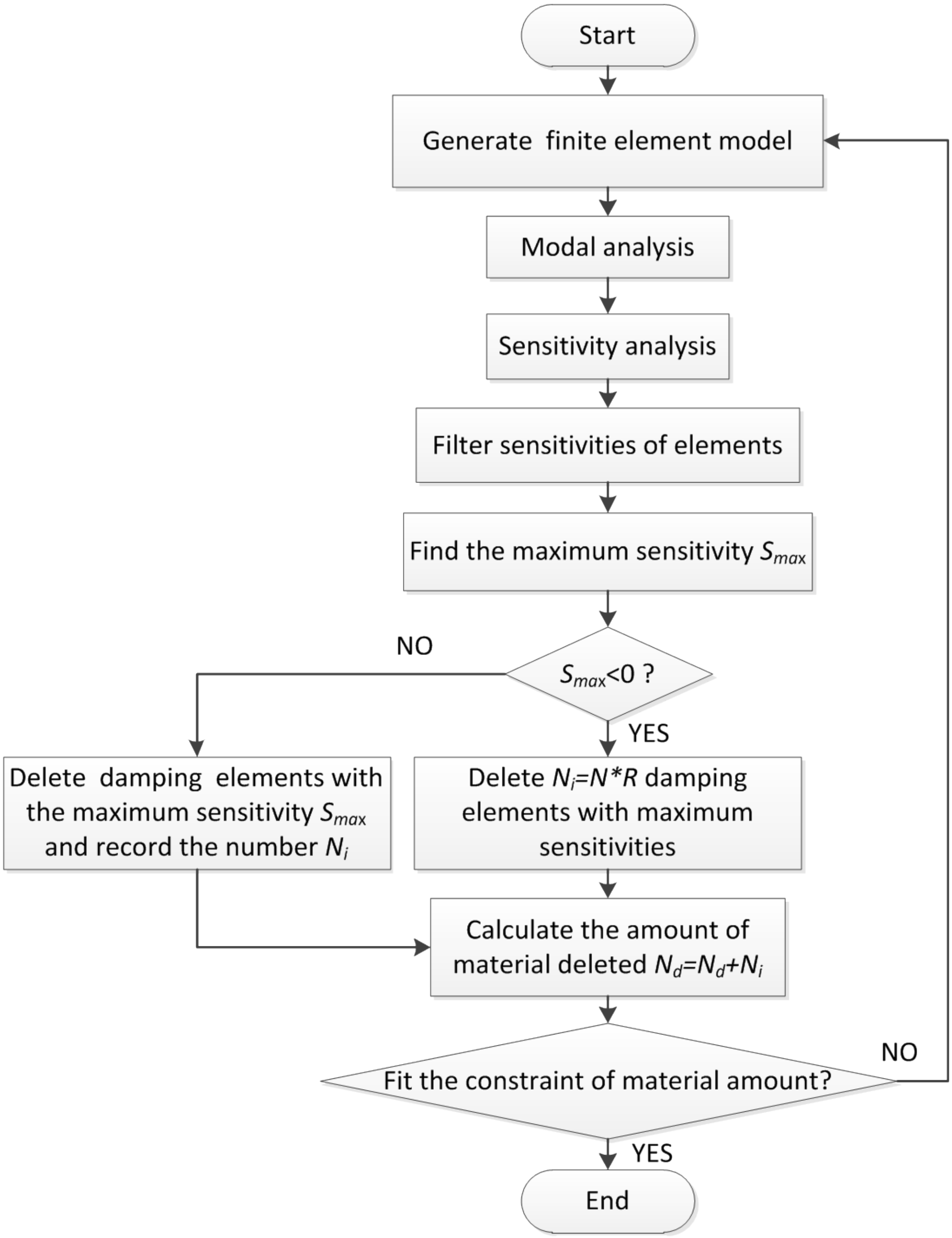

Figure 2. Topology optimization process based on ESO method

However, it should be noted that, if the original structure is symmetrical, the number of elements removed will not necessarily equal to $N*R$ in step 6 to obtain a reasonable symmetrical topology result. The reason may be that the elements in symmetrical positions share equal sensitivities. The deleted method now is to obtain a sensitivity threshold using $N*R$ and then remove the elements with sensitivities below (or above) the threshold value.

Numerical examples

Consider a square aluminum plate fully covered with ACDL and four edges clamped, which finite element model is shown in figure 3. Resin-based carbon fiber composite unidirectional laminates is adopted as the constrained sheet and C fiber direction parallels to the x-axis. The viscoelastic material 3M112D is employed as the damping core. Properties of each material are shown in table 1 and table 2.

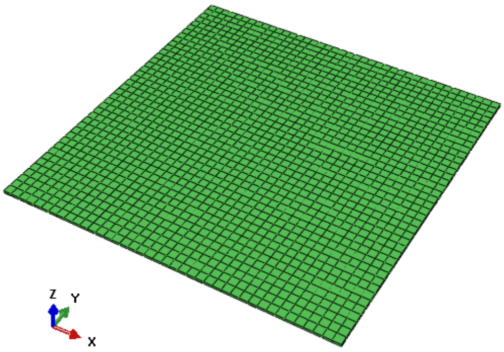

Figure 3. Initial design model

Table 1. Material properties

Properties	Bass shell	Damping core	Constraint sheet
Elastic modulus, E(GPa)	70	—	—
Density, ρ (kg/m^3)	2.7×10^3	0.98×10^3	1.64×10^3
Thickness, h (m)	0.002	0.003	0.001
Shear modulus, G (MPa)	—	10	—
Poisson ratio, μ	0.33	0.49	—
Loss factor, η	0.0001	1	0.01

Table 2. Modulus of the constraint sheet

E_1/GPa	E_2/GPa	μ_{12}	G_{12}/GPa	G_{13}/GPa	G_{23}/GPa
125	10.3	0.29	4.3	4.3	4.3

Taking 50% amount of ACDL material as constraint and aiming at the maximization of the first modal damping loss factor, optimization result is shown in figure 4:

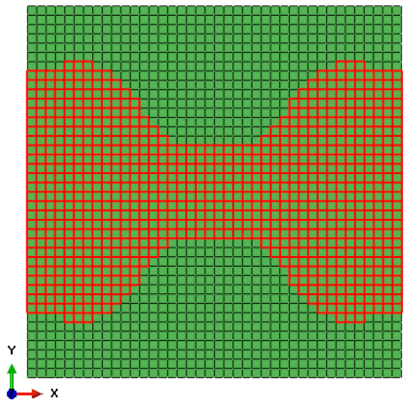

Figure 4. Topology optimization design of ACDL

Isotropic material can be treated as a special anisotropic material. The method used here are equally applicable to optimize structure with isotropic constrained damping layers (ICDL). Replace constraint layer material with aluminum in the same finite element model, we can get an optimal result of square plate with traditional ICDL, which is shown in figure 5.

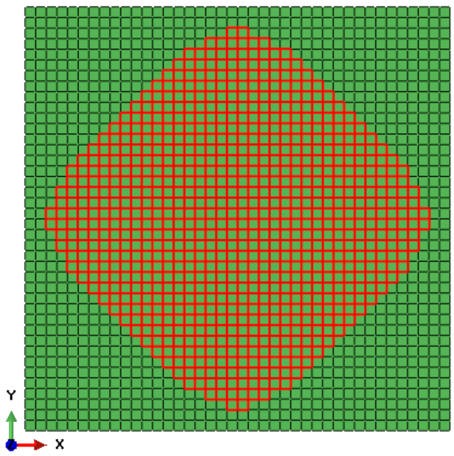

Figure 5. Topology optimization design of ICDL

Figure 6 shows damping loss factor's variation history with delete ratio. Loss factors of both model decline slowly when more and more damping elements are deleted. After eliminating 50% constraint damping layer material, structure with ACLD dropped from an initial 0.41 to 0.36, decreased by 12.2%; structure with ICLD dropped from an initial 0.43 to 0.37, decreased by 14.0%.The optimization results are satisfactory.

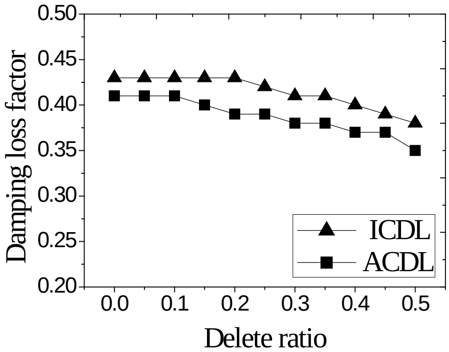

Figure 6. History of damping loss factor

Calculate damping efficiency using formula (1), and damping efficiency was increased greatly after optimization, as shown in table 3. We can also find that ACDL has higher damping efficiency than traditional ICDL.

Table 3. Damping efficiency of the plates with CDL（kg^{-1}）

	Before optimization	After optimization
ACDL	0.56	0.98
ICDL	0.48	0.82

Conclusions

The topology optimization problem of thin-walled structure attached ACDL with given thickness is investigated based on ESO method in present paper. The numerical example shows that the damping efficiency increases after optimization, and damping efficiency of ACDL with assigned materials is higher than traditional ICDL due to the non-negligible damping loss factor of composites. The advantage of ACDL is shown in lightweight design of vibration suppression and the approach presented has a strong engineering practicability.

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