Construction of convex models for non-probabilistic correlation quantification and uncertainty analysis

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Abstract

Non-probabilistic convex models need to be provided only the variation interval bounds of parameters rather than their exact probability distributions; thus, such models can be applied to uncertainty analysis of complex structures when experimental information is inadequate for probability modelling. Convex model utilizes a convex set to quantify the uncertainty domain of the uncertain-but-bounded parameters. It is not a single mathematical model; it means a series of models. The ellipsoid model and the parallelepiped models are convex models which could take correlation between uncertain parameters into consideration. In this work, a unified method for construction of these convex models—one ellipsoid model and four kinds of parallelepiped models, is proposed. The analytic mathematical expression of each convex model can be formulated once the correlation matrix of the uncertain-but-bounded parameters is obtained. With the convex uncertainty domain, such as a multidimensional ellipsoid or parallelepiped obtained, subsequent uncertainty analysis such as reliability analysis, optimization design, etc. can be carried out.

Keywords: Non-probabilistic, Convex model, Correlation quantification, Uncertainty analysis

Introduction

Uncertainties associated with manufacturing imperfection, usage variation and imprecise knowledge widely exist in practical engineering problems, and quantifying and controlling their effects on the structural performance is of great importance for current product design. Since entering the 1990s, the non-probabilistic convex model [Ben-Haim and Elishakoff (1990); Elishakoff et al. (1994)] has been developed for uncertainty analysis and reliability design, in which only the variation bounds of the uncertain parameters are required rather than their precise probability distributions. Thus, it provides a potential and promising way to conduct the reliability analysis and design for many complex problems with limited samples. Traditionally, the minimum volume method is used to identify the best convex set. However, the “minimum volume method” will bring about some severe difficulties; especially the optimization problems, which can be effectively solved only for problems with very small number of parameters. For this end, a mathematical foundation for correlation analysis of the non-probabilistic convex model was created, and whereby an effective method was proposed to construct the multidimensional ellipsoids on the uncertainty [Jiang et al. (2011)].

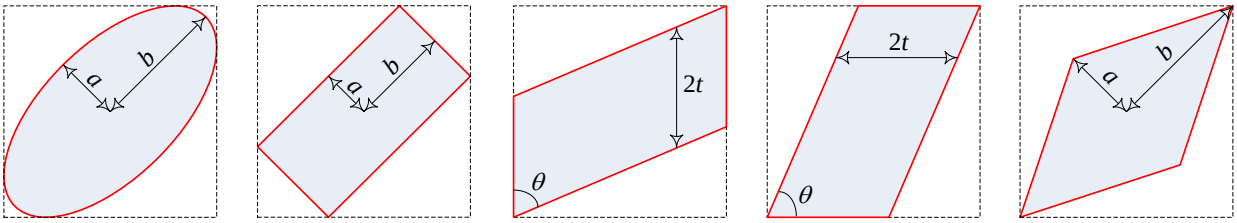

Figure 1. Ellipsoid model and parallelepiped models (2 dimensional)

Based on the correlation analysis method, this work aims to provide a unified framework for construction of ellipsoid model and parallelepiped models, including one ellipsoid model and four kinds of parallelepiped models, as shown in Fig. 1. Two kinds of correlation coefficient between

uncertain-but-bounded parameters are utilized to construct a convex model, and the comparisons are made. A concept of "unbiasedness" is proposed to evaluate the adaptability of each convex model to a certain group of samples. The indexes such as "fitness" and "ratio of uncertainty volume" are presented as a reference for decision making among the various kinds of convex models. Through illustration, some important characteristics such as definitions of correlation coefficients and UnBiasedness of different methods for construction of the convex models are presented in Table 1. For some certain group of uncertain-but-bounded parameters and corresponding sample points, the construction procedure of a convex model quantifying the uncertainty is discussed in detail by several numerical examples.

Table 1. Some important characteristics of different convex models

Shape	Convex Models	Convex Correlation Coefficient	Mathematical Expression	UnBiasedness CCC	UnBiasedness SCC
	Ellipsoid Model	$\rho = \frac{b^2 - a^2}{b^2 + a^2}$	$\mathbf{x}^{\mathrm{T}}\mathbf{G}\mathbf{x} \le 1$	UnBiased	UnBiased
	Cuboid Model	$\rho = \frac{b^2 - a^2}{b^2 + a^2}$	$\left\|\mathbf{C}^{-1}\mathbf{x}\right\| \le \mathbf{e}$	Biased	UnBiased
	LTRi-Parallelepiped Model	$\rho = \frac{\left\|1-t\right\|}{\sqrt{(1-t)^2 + t^2}}\mathrm{sgn}(\cos\theta)$	$\left\|\mathbf{C}^{-1}\mathbf{x}\right\| \le \mathbf{e}$	Biased	UnBiased
	UTRi-Parallelepiped Model	$\rho = \frac{\left\|1-t\right\|}{\sqrt{(1-t)^2 + t^2}}\mathrm{sgn}(\cos\theta)$	$\left\|\mathbf{C}^{-1}\mathbf{x}\right\| \le \mathbf{e}$	Biased	UnBiased
	Parallelepiped Model	$\rho = \frac{b^2 - a^2}{b^2 + a^2}$	$\left\|\mathbf{C}^{-1}\mathbf{x}\right\| \le \mathbf{e}$	Biased	UnBiased

Results and discussions

A numerical example with 3 uncertain-but-bounded parameters considered. The variation bounds of the parameters are all set as [-1, 1]. A certain group of sample points are observed for correlation approximation. Figure 2 shows a three dimensional ellipsoid model and a parallelepiped model that enclose all the sample points of the uncertain parameters. The ellipsoid model and the parallelepiped models are efficiently constructed by correlation information between uncertain parameters, rather than the traditional minimum volume method. The analytic expressions of the ellipsoid and the parallelepiped models are formulated.

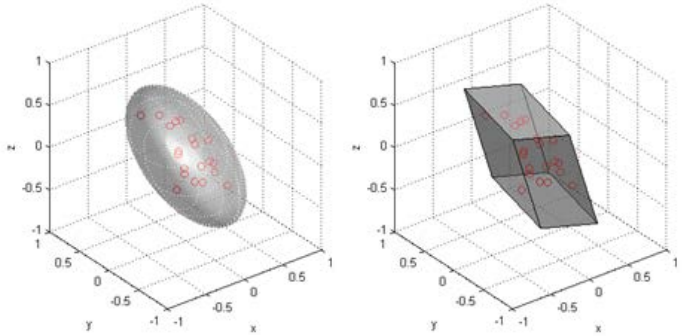

Figure 2. Ellipsoid and parallelepiped models constructed based on sample points

Conclusions

This work mainly discusses the construction of the uncertainty domain for uncertain parameters when experimental information is inadequate for the probability modelling. It provides engineers with 5 kinds of convex models to choose for uncertainty quantification. From both theoretical analysis and numerical examples, two major phenomena are observed. Firstly, the SCC (Sample Correlation Coefficient)-based construction method shows better qualities, and the shape of uncertainty domain depicted in this way is more reasonable; secondly, among all of the five convex

models, the Ellipsoid model and the Parallelepiped model present better adaptability. With the uncertainty domain of the uncertain parameters constructed in the form of a convex set, subsequently analyses such as non-probabilistic reliability analysis, uncertainty propagation, optimization etc. can be performed conveniently.

References

Y. Ben-Haim, I. Elishakoff. (1990) *Convex Models of Uncertainties in Applied Mechanics*, Elsevier Science Publisher, Amsterdam.

I. Elishakoff, P. Elisseeff, S.A.L. Glegg. (1994) Nonprobabilistic, convex-theoretic modeling of scatter in material properties, *AIAA Journal*, **32**, 843-849.

C. Jiang, X. Han, G.Y. Lu, J. Liu, Z. Zhang, Y.C. Bai. (2011) Correlation analysis of non-probabilistic convex model and corresponding structural reliability technique, *Computer Methods in Applied Mechanics and Engineering*, **200**, 2528-2546.