Bulk and surface balance of osteocytes during bone tissue formation and remodelling: Continuous and stochastic models

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Abstract

At the microscopic level, bone tissues sustain changes both in their bulk properties and geometry. Slowly evolving properties of bone such as mineral content and maturity of matrix-embedded cells (osteocytes) may suddenly and locally be erased and overwritten with immature content (newly formed bone) during bone remodelling. As a result, bone is "patchy" and its state depends on the history of formation and resorption processes.

In this contribution, the equations governing the evolution of osteocytes and other properties of inclusions in bone matrix are developed. These equations express bulk and surface material balance due to maturation processes and creation/dissolution at the bone surface during bone formation and remodelling. Numerical simulations of osteocyte density and mineral content exemplify how these equations make it possible to model patchy bone states and their evolution without explicitly maintaining the record of when/where resorption and formation processes occurred. This formalism also enables the systematic derivation of equations governing the temporal evolution of spatial averages of bone properties. Traces that formation and resorption processes are localised at the surface are retained in the resulting ordinary differential equations, which prevent these spatially-averaged equations from being written in closed-form.

Finally, a stochastic simulation algorithm generating discrete inclusions in bone matrix is developed. The stochastic generation of discrete inclusions is important when the inclusion density or the inclusion specific locations have important consequences. This is the case of osteocytes, which regulate local mineral content and signal cells at the bone surface. This stochastic model confirms that bone substrate curvature and bone-forming cell density do not explicitly influence the density of inclusions generated, in accordance with the governing surface balance equations [Buenzli (2015)].

Keywords: tissue growth, osteocyte, mineralisation, surface mass balance, curvature, stochastic simulation algorithm

References

Buenzli, P. R. (2015) Osteocytes as a record of bone formation dynamics: A mathematical model of osteocyte generation in bone matrix, *Journal of Theoretical Biology* **364**, 418–427.