A probabilistic approach to inverse material parameter identification

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Abstract

In this paper, a probabilistic approach is adopted to determine material parameters. The proposed algorithm is tested to estimate material parameters of tubular materials subjected to a spherical punch test. We assume the measured Force-Deflection (F-D) data, Major Strain-Deflection (MS-D) and anisotropy parameters are corrupted with a Gausian noise σ=1% and σ=4% and zero mean at the all data points. Then we employ a Markov chain Monte Carlo (MCMC) method in a Bayesian framework to solve the inverse material parameter identification problem. The results show how uncertainty of the measurement values influence the uncertainty of the estimated material parameters. This approach can be adopted to study stability of the inverse problem in the presence of the experimental noise.

Keywords: Material parameter identification, Bayesian inference, Markov chain Monte Carlo, Inverse identification, Probabilistic inverse method.

Introduction

The accuracy of Finite Element Analysis (FEA) depends heavily on the use of well-evaluated material parameters during the simulation process. To identify the mechanical parameters of the materials, experiments are conducted that impose loads on an instrumented specimen, and subsequently the measured data, such as load-displacement data, is converted to the material parameters, e.g. stress-strain relationship.

One popular approach to identify the material parameters from the experimental measurements is to solve an inverse problem using either a forward or backward inverse identification method. In the forward approach, the material parameters are identified by solving an optimization problem, where the material parameters in the FEA are adjusted to minimize discrepancies between the measured experimental data and the simulation results [Zribi et al. (2013)]. An alternative is a backward inverse identification method, or inverse map, which is based on mapping the experimentally measured response to the parameters of a material model directly [Asaadi et al. (2014)]. These approaches for material parameter identification are mostly deterministic; however uncertainty of the obtained parameters must be evaluated against the uncertainty of the experimental measurements. To address this limitation, this study demonstrates material parameter identification using a MCMC method for Bayesian inference. We intend to identify how uncertainty in the measured force, strain and anisotropy parameters influences the obtained material parameters. We present a Bayesian inference approach, like the work done by [Karandikar et al. (2014)], for the solution of the inverse material parameter identification problem. The system is defined as a constrained half tube subjected to the spherical punch test, Fig. 1.

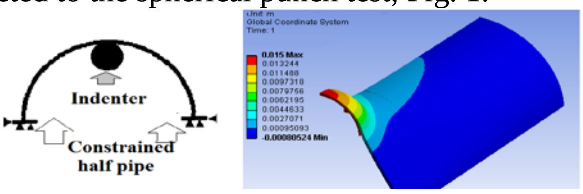

Fig. 1. Schematic representation of the FE model. 1/4 of the pipe is simulated

The system parameters are the material hardening parameters, and the coefficient of friction between the specimen and the indenter. The system responses are F-D data, MS-D and anisotropy coefficients, obtained in uniaxial tensile tests. We conduct our study in a virtual environment where FEA simulations of the tube punch test replace the experimental data. We opt for the virtual experiments since experimental data for the large number of tube punch test of different materials are not available. In addition, we can control the errors present in the virtual experiment to obtain a proper understanding of how these errors influence the identified material parameters. For our numerical experiments we use FEA of the tube punch test for a known set of the material parameters and the MS-D on the dome and F-D responses are then computed. Afterwards, Bayesian inference is adopted to predict the material parameters used for the punch test simulation, while we added artificial 1%-4% normally distributed stochastic noise to the responses of the FEA. In contrast with the deterministic inverse parameter identification problems, the estimated parameters are not single values and the range of their probable values is determined by the uncertainty in the measured experimental (virtual in this case) information.

Bayesian Inference

Bayesian inference is a statistical tool for updating the probability of a hypothesis as new evidence becomes available. Bayesian inference is based on Bayes formula which is given by

$$P(H|E) \propto P(E|H)P(H) \tag{1}$$

in which H represents the hypothesis, e.g material parameters, E is the observation related to this hypothesis, e.g experimental measurements of the test such as F-D, MS-D and anisotropy responses, P(H|E) is the posterior probability density function, P(E|H) is the likelihood function and P(H) is the prior probability density function. We use the posterior probability density function for estimating the material parameters when it is explored by Metropolis–Hastings algorithm.

Metropolis–Hastings (MH) algorithm

MH algorithm is one of the most popular MCMC methods for obtaining samples from a probability distribution for which direct sampling is difficult. In the algorithm, a candidate sample is selected given current value of the chain, e.g current estimated material parameter. It can be rejected or accepted according to the acceptance ratio, obtained from the posterior probability density function. The chain moves to the candidate point if it is accepted or remains at the current value if it is rejected. Calculating the posterior in each iteration needs FEA, which is computationally expensive. Therefore, we suggest using a cheap-to-evaluate surrogate model of the FEA.

In order to construct the surrogate model of the FEM, we trained two separate Artificial Neural Networks (ANNs) to map material parameter sets to the F-D and MS-D responses of the FEA. The training set is constructed based on FEA of 100 material parameter sets. The Voce hardening law with three parameters, k, n and y, and Hill48 yield model with two parameters, R1 and R2, determine the material behavior. Ranges of the parameters are represented in Tab. 1.

Tab. 1 Selected range of the material parameters for constructing the surrogate model of the FEM

Material parameter	k(MPa)	n	y(MPa)	R1	R2	f
Parameter range	1200-1800	1-2.5	290-360	0.95-1.05	1-1.2	0.03-.07
Parameters of the virtual Exp.	1650	2.5	290	1.02	1.1	0.03

Results and discussion

In this section, the MCMC method for Bayesian updating of material parameters is demonstrated using a virtual experiment. The parameters selected in the Tab.1 are used to simulate the F-D and MS-D responses of the punch test simulation. Then, results of the simulation are used as the target for constructing the likelihood density function, while we assume normal stochastic noise with the standard deviation of 1% and 4% for the mean value of FEA results at each data point. The prior distributions of the parameters are assumed to be joint uniform distribution in the selected range. The MCMC converged after 500 iterations. Results of the chain for estimating the parameters are shown in Fig. 2(a-b). Fig.2 (a) indicates that the estimated parameters for 1% noisy responses tend to be very close to the mean of the parameters, which is also very close to the nominal values represented in the Tab.1. Fig 2(b) also demonstrates that increasing the assumed noise in the responses of the system from 1% to 4% (300% increase) , causes 380%, 1000%, 800% and 500% increases in the standard deviations of the estimated k, n ,y and coefficient of friction. This example illustrates how Bayesian inference can be employed for tackling a probabilistic inverse problem to study how the quality of the experimental measurements affects the estimated parameters in an inverse problem. This approach can be employed as a tool in the decision-making process where comparison between different measurements of the responses of the system is necessary to build a robust inverse problem.

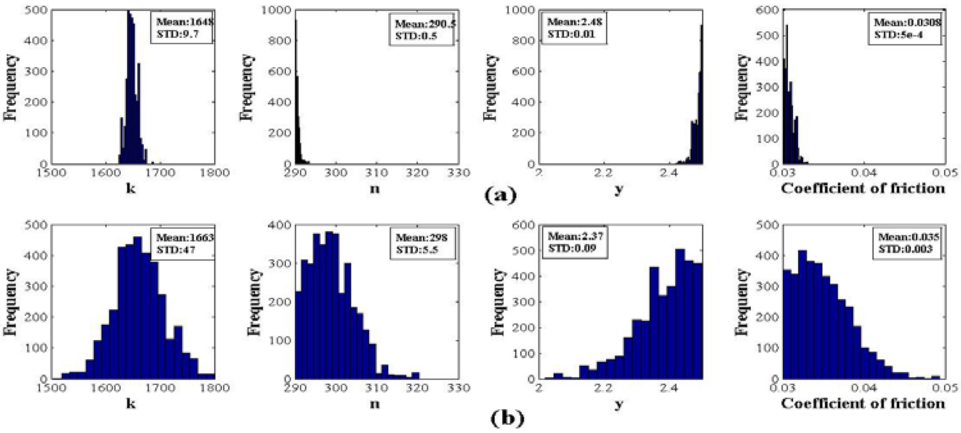

Fig. 2. Variation of the estimated parameters when assuming 1% normally distributed stochastic noise for the measured responses, (a), and 4% normally distributed stochastic noise for the measured responses of the virtual experiment, (b).

Conclusion

In this study we employed Bayesian inference for solving a probabilistic inverse material parameter identification problem. The proposed approach can be used to determine how uncertainty in the measurement data influences the identified material parameters. Therefore, the proposed algorithm can be used as a tool for studying ill-posedness of an inverse problem, stability type, and comparing the effect of quality of different measurements for identifying material parameters.

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