Simulation of the acoustic filed of a structure vibrating in a subsonic uniform flow using the fast BEM

Q. Zhou[1], C. Z. Li[1], and †Z. Y. Yan *[1]

[1]Department of Aerodynamics, College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics , PRC.

*Presenting author: jutsjtu@nuaa.edu.cn
†Corresponding author: jutsjtu@nuaa.edu.cn

Abstract

In this paper, the acoustic radiation of a structure vibrating in an uniform subsonic flow is investigated using the pre-corrected fast Fourier transform accelerated BEM (pFFT-BEM). The governing equation together with the boundary conditions are transformed into the classical Helmholtz wave equation and the corresponding boundary conditions. The pFFT-BEM is then applied to simulate the acoustic field. A vibrating spheroid in a subsonic uniform flow is simulated to validate the algorithm.

Keywords: pFFT, BEM, Acoustics, Subsonic flow

Governing Equation

The velocity potential of the acoustic filed of a structure vibrating in a subsonic uniform flow along x direction satisfies the equation

$$\nabla^2\phi + k^2\phi - 2ikM\frac{\partial\phi}{\partial x} - M\frac{\partial^2\phi}{\partial x^2} = 0 \tag{1}$$

Where M is the Mach number of the flow field.
Using the Prandtl-Glauert transformation

$$\tilde{x} = \frac{x}{\beta},\ \tilde{y} = y, \tilde{z} = z, \beta = \sqrt{1-M^2} \tag{2}$$

and assuming that

$$\tilde{\phi} = \phi e^{-i\kappa\tilde{x}} \tag{3}$$

Then Eq.1 can be transformed into the classical Helmholtz wave equation

$$\tilde{\nabla}^2\tilde{\phi} + \tilde{k}^2\tilde{\phi} = 0 \tag{4}$$

The boundary condition can be correspondingly transformed as

$$\frac{\partial\tilde{\phi}}{\partial\tilde{n}} = e^{-i\kappa\tilde{x}}\left[\frac{\partial\phi}{\partial\tilde{n}} - i\kappa\phi\tilde{n}_1\right] \tag{5}$$

Eq.(4) with the boundary condition (5) can be simulated using the pre-corrected fast Fourier transform accelerated boundary element method.

Numerical Example

To validate the algorithm, the acoustic field of a spheroid $x^2+\beta^2y^2+\beta^2z^2=\beta^2$ vibrating in a subsonic uniform flow with $M=0.7$ as shown in Figure 1 is simulated. After the Prandtl-Glauert transformation (2), this spheroid is transformed into a sphere as shown in Figure 2.

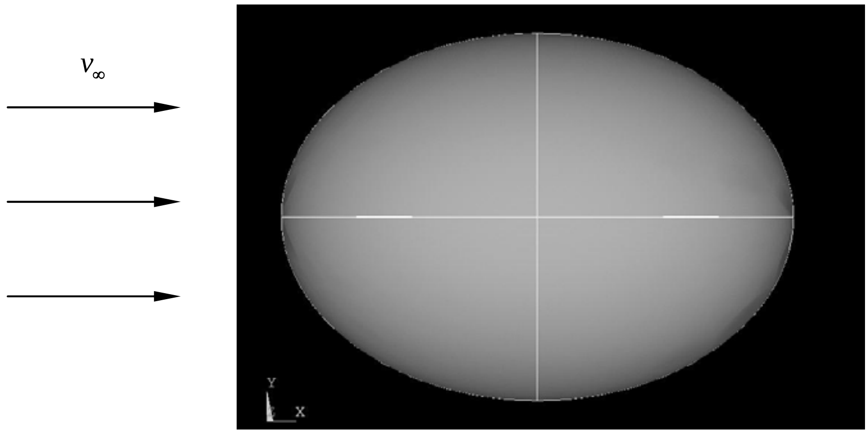

Figure 1 A spheroid vibrating in a subsonic uniform flow

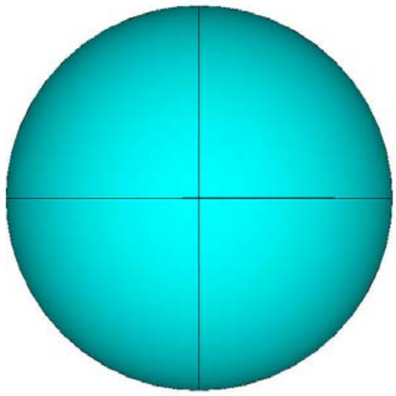

Figure 2 The shape of the spheroid shown in figure1 in the transformed domain

To define the boundary condition, we assume there is a point sound source at the center of the spheroid in the subsonic uniform flow. The normal velocity described by Eq. 5 is generated by this point sound source. The velocity potential of the acoustic field is

$$\phi = \frac{\beta}{4\pi}\frac{e^{-ik\frac{\sqrt{x^2+\beta^2\left(y^2+z^2\right)}-Mx}{\beta^2}}}{\sqrt{x^2+\beta^2\left(y^2+z^2\right)}} \tag{6}$$

After the Prandtl-Glauert transformation (2), Eq. 6 became as

$$\tilde{\phi} = \frac{1}{4\pi}\frac{e^{-i\tilde{k}\tilde{r}}}{\tilde{r}} \tag{7}$$

Therefore the boundary condition in the simulation satisfies

$$\frac{\partial\tilde{\phi}}{\partial\tilde{n}} = -\frac{1}{4\pi}\left(1+i\tilde{k}\right)e^{-i\tilde{k}} \tag{8}$$

The model shown in Figure 2 is discretized using 19200 constant triangular elements. The velocity potential of the acoustic field as a function of the wave number is shown in Figure 3. In this figure, both the numerical results obtained by the pFFT-BEM based on the Helmholtz integral equation and the composite Helmholtz integral equation are presented. From this figure, it is clear that the numerical results obtained by the composite Helmholtz integral equation agree very well with the corresponding analytical solutions given by Eq. 6. While, there exist some characteristic frequencies where the numerical results obtained by the Helmholtz integral equation are nonunique.

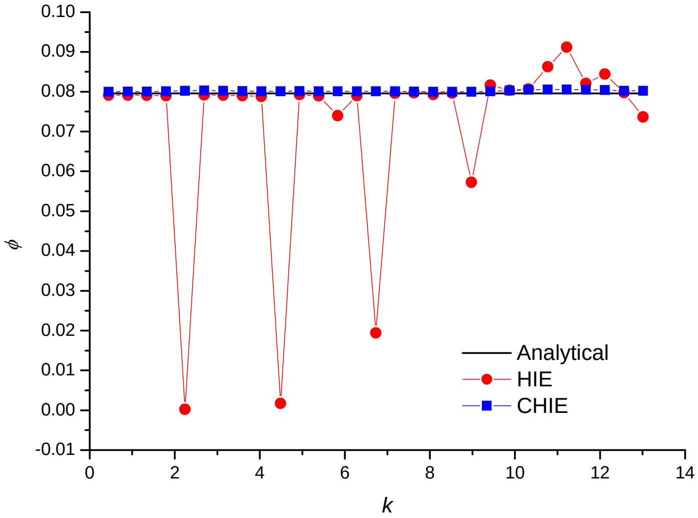

Figure 3 The shape of the spheroid shown in Figure1 in the transformed domain

Acknowledgements

This project was supported by NSFC (11172132); the Priority Academic Program Development of Jiangsu Higher Education Institutions.

References

Wu TW, Lee L. (1994) A direct boundary integral formulation for acoustic radiation in a subsonic uniform flow. *Journal of Sound and Vibration*, **175(1)**, 51-63.

Yan Z.Y., Gao X.W. (2013) The development of the pFFT accelerated BEM for 3-D acoustic scattering problems based on the Burton and Miller's integral formulation. *Engineering Analysis with Boundary Elements*, **37**, 409-418.