Effect of distribution between stress and couple stress on Saint-Venant's decay rates for micropolar elastic solids in pure bending

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Abstract

Saint Venant's principle in heterogeneous solids with microstructure plays an important role from a viewpoint of structural integrity. The micropolar elasticity which is characterized by additional degrees of freedom associated with rotation of microstructure is one of a promising theory to predict such heterogeneous material in a framework of generalized continuum theory. The mathematical formulation and proof of Saint Venant's principle have been studied on micropolar elasticity [Berglund (1977)]. In the literature, the decay of strain energy based on the locality of effects of self-equilibrated loads has been studied with micropolar material parameters. Numerical analysis for Saint Venant's principle on micropolar theory has also performed, e.g. [Nakamura and Lakes (1995)]. In the linear micropolar elasticity, strain energy density involves a term of micro-curvatures which is spatial gradient of micro rotations, and couple stress is associated with the micro-curvature. In this study, two dimensional finite element code which takes into account the rotational degrees of freedom is developed and it is used to solve a problem of a beam in pure bending to clarify the Saint Venant's end effect of linear micropolar elasticity. Since micro-rotations is independent of the translational displacements, contribution of couple stress and that of stress to the traction force on a boundary can be assumed independently under a resultant force. The different cases of distribution between stress and couple stress are studied parametrically. The results generally show that non-zero shear stress components yield near loaded boundaries when the distribution on the boundary does not satisfy the self-equilibrium condition between stress and couple stress. However the ratio of the stress and the couple stress decays rapidly to the appropriate value that is determined by the characteristic length. The decay of stress distribution corresponds to a generalization of Saint Venant's principle.

Keywords: Micropolar elasticity, Couple stress, Saint Venant's principle, Finite element method

References

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