

PROBABILISTIC STRUCTURAL ANALYSIS OF THE COMPOSITE CREW MODULE- SUBSTRUCTURING WITH HIGH RESOLUTION GRID

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ABSTRACT

A probabilistic structural analysis of an experimental Composite Crew Module (CCM) has been performed using the most current Orion Crew Module design loads. The purpose of the analysis was to quantify the risk of failure for different factors of safety in response to uncertainties in the design variables.

The CCM was found to be very safe with ample margin of safety and a low probability of failure. As a further analysis activity, the loads were scaled up and the composite material's strength was scaled down to assess a bounding scenario; still the design was found to be safe.

This analysis shows that for given factor of safety and uncertainty in the design space, the probability of failure associated with the design can be significantly different. Further, designing structures based on factor of safety without quantifying the influence of uncertainties in the design variables can lead to structures with unknown risk.

KEY WORDS: Composite Analysis, Probability of Failure, Composite Crew Module, Space Vehicle, Composite Failure Theories Applications

INTRODUCTION

NASA is evaluating the use of composite materials for building its next generation crew exploration vehicle (CEV) [Kirsch 2011]. In addition, NASA is interested in assessing the applicability of probabilistic methods for developing reliability based designs of components, structures and systems made from composite materials. For these two reasons, NASA initiated a task to probabilistically analyze the composite material that was developed for the new crew module. This paper describes a procedure for estimating the reliability of the CCM structure given various missions.

Most design practices use deterministic analysis combined with factors of safety (FOS) to provide a margin between the assumed peak load and a mean failure point. Unfortunately, this approach does not provide any information on the potential failure rate or reliability of the system. Factors of safety do not provide any information which relates the loading range relative to the structural allowable range. In reality, there is statistical scatter in both the loading variables and the structure's ability to resist those loads. Hence designs using deterministic analysis combined with FOS can lead to an overly conservative design with excess weight and cost or under design with unknown magnitude of risk.

Probabilistic analyses quantify the effect that uncertainties in the input variables have on the response variables. As a result, the probabilistic analyses provide quantitative information to enable designers to either calculate a realistic risk or the actual FOS in an existing design. Thus the designer can improve the design by either selecting a predetermined risk or adjusting the FOS to meet the predetermined risk. The authors [Nagpal et al 2010 and Pai 2008] have used probabilistic approaches in quantifying probability of

failure and performing sensitivity analyses of different structural problems and furthermore demonstrated its application to improving design practices.

The probability of failure, a measure of reliability, is critical for determining the structural integrity of the vehicle under the various flight and ground loads encountered during operation. Performing an accurate estimation for the probability of failure is more critical now than before because of recent changes in NASA's requirements. The objective is to reduce the loss-of-crew/loss-of-vehicle rate by a factor of 2 to 3. In addition, the cost/pound of payload is also to be reduced by the same factor. These requirements present a formidable challenge given the rigors and uncertainties of space flight coupled with possible future budget constraints.

The use of Polymer Matrix Composite (PMC) materials for structural components is steadily increasing particularly for aerospace applications [Harris et al, 2008]. The key advantages of using composite materials are that they are light weight and can be manufactured with desired properties [NASA SP-8108]. However, the fabrication process of a component made from composite materials is quite complex and uniquely developed specifically for a given structural component [NASA PD-ED-1217]. This complexity in the fabrication process is due to a large number of variables such as fiber volume ratio, ply orientation, density, type and strength of fibers and other variables.

Designing, components made from composite materials using probabilistic methods requires the characterization of uncertainties in the material properties, fabrication processes and loading variables. In addition, there are further uncertainties associated with the degradation of composite materials in a space environment.

Probabilistic methods using physics-of-failure approaches provide valuable quantitative information which is not only used to develop a risk-based design, but also provides the probability of failure for a design which has been developed using a factor of safety, Figure 1. This figure displays the relationship between this CCM model's factor of safety and the probability of failure. There is an inverse relationship between the probability of failure and the factor of safety. That being, demanding a larger factor of safety leads to a decrease in the probability of failure. For human rated systems NASA HQ requires that the probability of failure be no greater than $1.0E-04$. Due to the safe nature of this design, this study estimates that one could relax the safety of factor to just below 1.4 and still satisfy the risk requirement. Figure 1 assumes that the uncertainties in the design variables are known to some extent. Later it will be shown how to deal with the situation when the design variables have a large amount of variance.

Inherent uncertainties in material properties, loads and geometry complicate the design process [Andrzej, Nowak, and Collins, 2000] and affect safety, performance and cost. Traditionally, factors of safety (FOS) have been developed empirically based on years of engineering experience and are assumed to account for these uncertainties [Melchers 1999]. Any design based on the FOS lacks quantitative information on risk of failure, consequently it remains unknown whether the design is within the acceptable risk limits or not.

In the absence of expensive-to-acquire experimental data, a probabilistic approach provides extremely valuable quantitative information for generating cost effective risk-based design and additionally provides precise direction for further resource investments. Uncertainties in the design variables are defined through probability distributions which are used in conjunction with a probabilistic methodology to propagate these uncertainties up to a probabilistic system response. Figure 2 shows a family of curves that relate the probability of failure with an assumed uncertainty in the design variables. As the uncertainty in the design variables increase, for a given factor of safety, the probability of failure also increases. The probabilistic analysis was first conducted assuming that all the material properties had 5% uncertainty, for which a probability of failure is then produced. The full probabilistic analysis is again conducted for each: 10%, 15%, 18%, 20%, and 40% uncertainty in the design variables. The 5% and 10% cases are considered to represent realistic scatter in the material properties, due to fabrication issues, whereas the other cases represent an attempt to envelope the design space to find cost savings based on less stringent manufacturing quality controls. All six test cases were used to generate the curve named "NO Cov", where all the design variables were assumed to be independent random variables. The entire procedure

described above is repeated. This time assuming that the design variables are not fully independent but have 10% and 30% mutual correlation.

Technical Approach

The following procedure was used to quantify the probability of failure. First to be discussed is the deterministic finite element model; its material properties and the loads that were analyzed in this study. Next, several probabilistic analyses of the finite element model is presented, each case assuming different levels of uncertainty in the design variables. Each probabilistic analysis generates a distribution for the stresses or strains in a region of interest to the designer. These stress/strain distributions are input into the reliability calculation, which is used to determine a probability of failure. The probability of failure is estimated by considering the area under a material level limit state equation such as $(1 - \frac{\text{stress}}{\text{strength}})$. This type of the limit state equation is more realistic when the stresses and the material properties' allowables are not statistically independent, as can be explicitly seen in the Hoffman's failure criterion.

Finite element model

The finite element model analyzed was provided by the NASA Langley Research Center, Figure 3. The ANSYS workbench translator was used to convert the model from a NASTRAN format. The translated model was then verified for its accuracy of translation by comparing results of the original model obtained using ANSYS and NASTRAN. This comparison showed that the values of the maximum stress from the two analyses were within 0.25%.

The model was of the CCM pressure vessel's outer structure and included the mass of the secondary structure. This ensured that the loads were accurate for the mass and center of gravity of the CCM with a payload. A preliminary global analysis was performed using the material properties provided by NASA. From the results of this analysis, regions of high stresses, also called critical regions, in the structure were identified. The finite element meshes in these regions were further refined to perform high resolution analysis

Composite shell elements with orthotropic material properties were used to model the structure. The properties indicating the thickness of each ply, fiber angle and materials in the refined mesh area are given in Table 1. The CCM wall is made of 10 composite material plies and one aluminum honeycomb layer ("Shapegrid" TradeMark). The honeycomb layer, numbered as 6, is sandwiched between layers of composite plies numbered 1-5 and 7-11, as listed in Table 1. The composite plies are made of two different materials; plies 3 and 9 are made of one type of material, labeled as 1, and plies 1, 2, 4, 5, 7, 8, 10 and 11 are made of another type of material, labeled as 2. The composite plies are designed using high strength carbon fiber, IM7 and Cytec's 977-2 epoxy. Properties for both types of materials 1 and 2 including their elastic properties and allowable strengths are listed in Table 5. From these properties, it is obvious that material 1 is considerably stiffer and stronger than material 2. The honeycomb material, labeled as material 6, is a filler material and it doesn't have appreciable stiffness and strength properties.

Loading and boundary conditions

The CCM is subjected to three different loading conditions during its flight and landing. The loadings were provided by the NASA Johnson Space Flight Center as adaptations of the Orion Crew Vehicle loadings. In all cases the CCM is subjected to 15.2 psi internal pressure. This analysis considers only the first two of the three loading conditions, see Table 2. Since the original FE model was designed for the launch abort and docking load conditions. For these two loading conditions, the analysis includes 5 sub-loading conditions for abort loads and 8 sub-loading conditions for docking loads. The sub-loadings represent different variations in the two loadings. The two loading conditions considered are:

1. High altitude abort loads with five (5) sub-load conditions.
2. On-orbit docking loads with eight (8) sub-load conditions.

Both the abort and docking load conditions include acceleration loads. Where A_x , A_y and A_z are accelerations in x (along the height), y (perpendicular to the paper) and z (horizontal in the plane of the paper) directions.

Deterministic analyses

Deterministic structural analyses were performed to identify the most critical regions with high stresses in the crew module, see Table 3. The high stress region is in the cylinder near its interface with the cone shape structure near one of the door openings, Figure 4.

Of the thirteen different load cases analyzed only two loads, high altitude abort loads and on-orbit docking loads were found to be the most critical loads. The most critical regions were identified using these two loads. A high resolution analysis at the ply level was conducted to determine peak stresses and identify potential failures.

Overall, the high altitude abort sub-load conditions resulted in higher stresses than the on-orbit docking sub-loading conditions. Since all high altitude abort sub-loading conditions are very similar in loading, variations in stresses among these sub-loading conditions are insignificant. Out of all the thirteen sub-loading conditions, the maximum stresses occurred in ply 3 under the fourth case, which was a high altitude abort sub-loading. Therefore this loading case was selected to perform the subsequent probabilistic analyses. The peak stress and type of ply failure were used in performing probabilistic structural analyses for several different levels of assumed uncertainties in the design variables. Ply 3 is the stiffest of all plies found in the critical stress region. Figure 4 illustrates stress distribution in ply 3, for the fourth high altitude abort load, without external and internal connections with the aero-shell and payloads.

Failure criteria

The probability of failure was estimated from the area which is both under a given failure criteria's PDF and to the left of the failure criteria limit of 0.0. A distribution for each of the following three ply level failure criteria was constructed from the probabilistic analysis: (1) inter-ply delamination criterion, (DELFC), (2) Hoffman's failure criterion (HFC) and (3) modified distortion energy criterion (MDE) [Robert Aiello, 1989], see Appendix A.

The MDE and HFC criteria have been expressed in terms of actual and allowable stresses and the DELFC has been expressed by actual and allowable strains. Allowable strain for DELFC is the shear strain between two plies. Plies 2, 3 and 5 are identified to have the most probable failures based on a deterministic analysis and thus selected for the probabilistic analyses, Table 4. The three ply level failure criteria were used to define the limit state equation in a "stress-strength formulation".

All three failure criteria have been calculated independently to estimate a probability of failure in the critical regions based on uncertainties in ten input design variables; three material properties and strengths (limiting stress) for each of materials 1 and 2 and the pressure and acceleration loads. There are five limiting stresses (σ_{xt} , σ_{xc} , σ_{yt} , σ_{yc} and σ_{xy}) for both material 1 and material 2. σ_x , σ_y and σ_{xy} are the allowable stresses in both the x and y directions respectively. Further, the subscript (t) represents the materials' allowable in tension and (c) is the material's allowable in compression. The allowable with respect to shear stresses is denoted by σ_{xy} .

The DELFC is based on strains; ϵ_x , in the x direction, and ϵ_y , in the y direction, and shear strain, ϵ_{xy} and shear strength of the epoxy which holds the plies together. DELFC will occur only if the actual shear strain in epoxy is more than allowable shear strain. The allowable shear strain is calculated from allowable shear stress, ($S_{xy} = 7,000$ psi), Elastic Modulus ($E_x = 500,000$ psi) and Poisson's ratio ($\nu = 0.41$) using the relations $G_{xy} = E_x / 2(1+\nu)$ and $\Phi(\text{Allowable}) = S_{xy}/G_{xy}$. This allowable shear deformation is calculated as 0.0395.

Table 4 shows the deterministic results for all three failure criteria for each ply in the critical region. For the HFC and MDE criteria, plies 2 and 3 have the lowest of the Failure Criteria (FC) values for both first

and second cases. On a scale from zero to one the MDE for ply 2 is 0.663. For the DELFC criterion, plies 2 and 5 have the lowest FC values. Since plies 3 and 9 have a zero fiber angles and are sandwiched between plies which also have a zero fiber angle, they have no response for DELFC. Ply 2 is next to ply 1 which has fiber angle of 45, and ply 5 has fiber angle of 45, they show the lowest FC values for DELFC response. Again low FC is more likely to fail than the plies with higher FC values.

Note at this stage, Table 4 gives point values for the three failure criteria. In the next section the probabilistic analysis will fit distributions to these criteria to quantify the expected scatter in the failure of the plies.

Probabilistic analyses

All design variables including material and geometric properties, boundary conditions and applied loading variables were selected as input variables for the probabilistic analysis. The input variables' inherent uncertainties are represented by a standard deviation. The design variables' uncertainties are then propagated through the finite element model of the CCM. The resulting response function, such as elastic stress, is then fit to a distribution. The distribution of the uncertainties in the peak stresses given systematic perturbations to all the relevant design variables are then inputted into the reliability calculations. A distribution for each failure criteria was generated based on the stress distribution in the critical region.

The analyses began by introducing uncertainty into the relevant state variables. A probabilistic discretization of the design space is created by assigning a probability distribution function (PDF) to each design variable, see Table 5. On successive sample runs of the finite element model, one uses these PDFs to determine the numerical values that the design variables take on. PDF and cumulative distribution function (CDF) for the response function are then constructed. This is accomplished by fitting a response surface, via a least squares methodology, through both the perturbed basic variables and the subsequent structural response.

This technique permits the entire spectrum of system response for the structure under uncertain loading conditions to be discovered. To arrive at this information traditional deterministic solution techniques were used, only the basic variables were modified to permit uncertainty in the model's design variables [Miller 2006].

Results of probabilistic analyses

The deterministic stress analysis results together with mean strengths and uncertainties of input variables were used to perform probabilistic structural analyses using FPI (Fast Probability Integrator) module of the NESSUS code. FPI determines the PDF of the response variable, stress, based on the uncertainties of the input variables and subsequently conducts the reliability analyses using the limit state function for all the failure criteria.

For the sake of the probabilistic analysis the material's components of strength (σ_{xt} , σ_{xc} , σ_{yt} , σ_{yc} and σ_{xy}) were considered to be one variable for each constituent ply material; they were perturbed concomitantly. All three components of the acceleration vector also increase or decrease simultaneously by the same proportion.

For the first probabilistic analysis, Table 5, all input variables were assumed to have uncertainties with standard deviations of 5%; the NASA experts concurred with this level of uncertainties. These standard deviations are considered realistic and represent a high quality fabrication of composites. The results for this level show that the PDF of the failure criteria does not overlap a limit state value of zero, see Figures 5-10. This means that there is practically no risk of failure. Hence, the design is found to be completely safe with respect to all three criteria of failure.

In the second probabilistic analysis, Table 5, the uncertainties standard deviations are doubled to 10% for all variables, twice the magnitude of uncertainties of those in the first scenario. This case represents a lower quality fabrication of composites. The uncertainties in this scenario are exaggerated intentionally in order to demonstrate the reserved safety of the current design. The HFC failure criterion PDF for ply 2

approaches the limiting value but does not cross it; once again implying a completely safe design. Figures for this scenario are not shown because of the page limitations.

Based on several scores of stress analyses for both the 5% and 10% uncertainty scenarios, normalized CDF and PDF using all three failure criteria for all the plies have been generated, more than 30 plots were created. These plots show the CDFs and PDFs of the three failure criteria (HFC, MDE and DELFC) along with their limit states for ply 2 for 5% uncertainties in the input variables. The limiting value of 0.0 for all the failure criteria is also shown in the PDF plots only. However, only three of these plots are shown here in Figures 5-10. These Figures are representative of the other 30 results which are not shown here to avoid repetitiveness of discussion. The results for all cases are shown in an unpublished report [N&R Engineering 2011] submitted to NASA.

Trade study for reliability and cost effectiveness

Since the analyses for both scenarios yield safe designs, the next natural step is to conduct a trade study with the objective of finding a lower cost composite material. In the trade study uncertainties in the design were systematically increased in set bounding scenarios: 15%, 18%, 20%, and 40%. This was done to provide guidelines for key managers on how they might be able to reallocate their resources more cost effectively. The uncertainties in these analyses have been highly exaggerated to show their quantitative impact on reliability and cost of a design. These analyses illustrate how probabilistic methods can assist the designer in selecting a lower cost composite i.e. composite fabricated using lenient tolerances while maintaining the desired level of reliability. Figure 2, discussed earlier shows a summary of results from the first two scenarios and this trade study.

Simulating a fabrication processes for composites

It is important to quantify the impact that the quality of a fabrication process can have. The next example demonstrates that for a given reliability a designer can select the appropriate tolerance level and corresponding cost for the design.

In this scenario, consider the case of three composites with same mean strength but fabricated with different quality standards; high, medium and low. A high quality composite implies tight-tolerances fabrication process, poor quality composite implies lenient-tolerances fabrication process and medium quality composite implies composite fabrication process with tolerances in between the two.

A graphical comparison of the composites with the same mean values of and hypothetically large uncertainties in all the input variables is shown in Figure 11. In this figure, there are 3 PDFs with the same mean value, a normalized mean of 0.588. The one with tightest scatter, green color, represents the case of tight and most expensive fabrication tolerances, and the one with widest scatter, blue color, represents the case of most lenient and inexpensive of the three cases fabrication tolerances. The third PDF, purple color, represents the case of tolerances in between the other two.

The three tolerances cases are represented by three standard deviations (SD); SD of 0.207 represents the case of tightest tolerance and SD of 0.287 the most lenient tolerance and SD of 0.239 the in-between tolerance. The quantified risk is area under the PDF to the left of the red vertical line. For the tightest tolerance case the risk is 0.0225%, most lenient tolerance case is 2.204% and for the in-between tolerance case is 0.696%.

This figure shows that for the same factor of safety, the risk in the design is significantly different and it illustrates that designing based on factor of safety without quantifying the influence of uncertainties could lead to a design with unknown risk. The factor-of-safety for all three tolerance cases is the same, 2.427. It is obvious from this demonstration that the level of risk would have been unknown if the probabilistic method was not used to quantify it.

Figure 12 illustrates a graphical comparison of the three PDFs representing three composite fabricated using the same tolerances. This example demonstrates how probabilistic structural analysis is used for accurately selecting an appropriate material for design. The analysis was performed using the same

magnitudes of uncertainties but with different mean values for the input variables. All three PDFs overlap the failure-criteria limit of zero, thereby allowing one to find the lower bound on acceptable materials. Such a study provides the relationship between the probability of failure and a factor of safety. Thus an analyst has more control over the amount of risk involved with the choice of a given factor of safety.

Also, this scenario demonstrates how probabilistic methods can be used in making key decisions such as setting cost-saving fabrication tolerances in addition to providing a quantitative estimate of the risk for a given factor of safety.

Summary

A probabilistic structural analysis of an experimental Composite Crew Module (CCM) of the future spacecrafts has been performed using the most current Orion Crew Module design loads obtained from the NASA Johnson Space Center. The purpose was to quantify the probability of failure for different factors of safety in response to uncertainties in the design variables. The results of these reliability calculations provide a quantitative means for selecting appropriate composite materials. An additional purpose was to demonstrate how probabilistic methods provide cost savings by allowing a reliability manager the ability to select proper factors of safety for predetermined risk.

Structural analyses were performed for 13 different load cases and the most critical load condition and the corresponding critical regions of high stresses were identified. A high resolution analysis at ply level was conducted in that region to determine peak stresses and identify potential failures. The peak stress and type of failure were then used in performing the probabilistic structural analyses.

Based on probabilistic structural analysis, the CCM is found to be very safe with ample margin of safety and a low probability of failure. As a further analysis activity, the loads were scaled up and the composite material's strength was scaled down to assess a bounding scenario. Still the design is found to be safe.

Probabilistic methods demonstrate how to select materials for the structure based on setting tolerances (cost) and factor of safety for predetermined risk. Without using probabilistic methods, the reliability of the design remains unknown; in contrast to using a Factor of Safety method alone.

Finally probabilistic methods provide one more important piece of information for the key managers when optimizing the use of the critical resources. This information consists of sensitivities of the input variables such as geometry, material properties, etc., on the response variables such as stress. For improving the quality of the end products, managers need to know which resources have high sensitivities, and thus are controlling the structures' performance or safety.

Table 1: Section Properties and Materials

Numbers of Layers = 11				
Total thickness = 1.0754 in.				
Ply (Layer)	Ply thickness in.	Material ID	Fiber angle	Ply Location
1	0.0081	2	45	
2	0.0081	2	0	
3	0.0053	1	0	
4	0.0081	2	0	
5	0.0081	2	45	
6	1	6	0	
7	0.0081	2	45	
8	0.0081	2	0	
9	0.0053	1	0	
10	0.0081	2	0	
11	0.0081	2	45	Outside

Table 2: Composite Crew Module Loading Conditions

Load Cases		Internal Pressure	Gravity Acceleration	Vehicle Accelerations					
		(psi)	X (in/s ²)	X (in/s ²)	Y (in/s ²)	Z (in/s ²)	Rx (rad/s ²)	Ry (rad/s ²)	Rz (rad/s ²)
High Altitude Abort	1	15.2	-386.04	5408.401	11.54460	-354.3054			
	2	15.2	-386.04	5413.442	228.0736	-138.1016			
	3	15.2	-386.04	5385.366	226.8812	-568.2328			
	4	15.2	-386.04	5386.233	-203.9546	-568.3412			
	5	15.2	-386.04	5414.363	-205.0386	-138.1558			
Water Landing			G's						
	6		-386.04	348.836	279.488	305.435	The water landing load cases are not considered in this analyses.		
	7		-386.04	222.436	-192.225	307.354			
	8		-386.04	-110.446	-112.088	-103.202			
On-Orbit Docking							0.08	0.137	0.041
	9	15.2		11.1952	3.8604	4.24644	0.50265	0.86080	0.25761
	10	15.2		11.1952	-3.8604	4.24644	0.50265	0.86080	0.25761
	11	15.2		11.1952	3.8604	-4.24644	0.50265	0.86080	0.25761
	12	15.2		-11.1952	-3.8604	-4.24644	0.50265	0.86080	0.25761
	13	15.2		-11.1952	3.8604	4.24644	0.50265	0.86080	0.25761
	14	15.2		-11.1952	3.8604	-4.24644	0.50265	0.86080	0.25761
	15	15.2		11.1952	-3.8604	-4.24644	0.50265	0.86080	0.25761
	16	15.2		-11.1952	-3.8604	4.24644	0.50265	0.86080	0.25761

Load Case	Ply 1	Ply2	Ply3
High Altitude Abort	Maximum Principal Stress (psi)		
1	28,848	32,478	55,355
2	28,269	32,432	54,947
3	30,218	33,109	55,542
4	29,362	32,491	56,377
5	27,409	31,811	55,751
On-Orbit Docking			
9	21,449	26,272	44,254
10	22,228	26,154	44,371
11	21,206	26,227	44,024
12	22,497	26,270	44,537
13	22,140	26,433	44,650
14	21,717	26,388	44,420
15	21,805	26,109	44,141
16	22,920	26,315	44,767
Pressure Only		25,079	419,767

Table 3: Typical Stress Distribution 11 plies

Table 4: Failure Criteria, calculated stresses and strains at the ply level for mean design values (5% uncertainty case)

Material	Ply	HFC	MDE	DELFC	SX	SY	SXY	eX	eY	eXY
2	1	0.980	0.913	1.000	6895.17	12319.16	-2938.25	6.13E-04	1.15E-03	-3.50E-03
2	2	0.694	0.663	0.963	27155.71	-8183.74	-448.21	2.61E-03	-8.70E-04	-5.34E-04
1	3	0.886	0.772	1.000	59352.79	-127.83	-388.22	2.99E-03	-8.70E-04	-5.34E-04
2	4	0.701	0.670	1.000	26770.13	-8199.09	-448.33	2.57E-03	-8.70E-04	-5.34E-04
2	5	0.951	0.918	0.950	6450.67	11878.02	-2870.68	5.72E-04	1.11E-03	-3.42E-03
3	6	1.000	1.000	0.981	0.00	0.00	0.00	1.18E-03	-8.68E-04	-5.39E-04
2	7	0.878	0.955	0.985	-8587.95	-3146.32	-569.47	-8.14E-04	-2.66E-04	-6.78E-04
2	8	0.872	0.949	0.999	-2703.88	-9372.75	-457.93	-2.25E-04	-8.81E-04	-5.45E-04
1	9	0.975	0.995	1.000	-5966.80	-1303.79	-376.21	-2.43E-04	-8.81E-04	-5.45E-04
2	10	0.869	0.949	1.000	-3089.46	-9388.10	-458.05	-2.51E-04	-8.81E-04	-5.45E-04
2	11	0.869	0.951	0.986	-9132.45	-3587.47	-501.91	-8.55E-04	-3.08E-04	-5.98E-04

Table 5: Material Properties, Loads and Standard Deviations used in the Probabilistic study

Variables with (%) Uncertainties and Standard Deviation. All values are in (psi) except accelerations Ax, Ay, Az in lb/in^2								
	Material	Variable	Mean Value	Case 1		Case 2		Direction
				Std. Dev.	%	Std. Dev.	%	
Elastic Modulus	1	Ex	22,950,000	1,147,500	5%	2,295,000	10%	X
		Ey	1,360,000	68,000	5%	136,000	10%	Y
		Gxy	690,000	34,500	5%	69,000	10%	XY
	2	Ex	10,533,000	526,650	5%	1,053,300	10%	X
		Ey	10,533,000	526,650	5%	1,053,300	10%	Y
		Gxy	840,000	42,000	5%	84,000	10%	XY
Allowable Stress	1	Sxt	124,779.00	12,477.90	10%	24,955.80	20%	Tension
		Sxc	85,787.10	8,578.71	10%	17,157.42	20%	Compression
		Syt	100,000.00	10,000.00	10%	20,000.00	20%	Tension
		Syc	100,000.00	10,000.00	10%	20,000.00	20%	Compression
		Sxy	12,870.00	1,287.00	10%	2,574.00	20%	Shear
	2	Sxt	57,265.20	5,726.52	10%	11,453.04	20%	Tension
		Sxc	39,370.49	3,937.05	10%	7,874.10	20%	Compression
		Syt	57,265.20	5,726.52	10%	11,453.04	20%	Tension
		Syc	39,370.49	3,937.05	10%	7,874.10	20%	Compression
		Sxy	12,870.00	1,287.00	10%	2,574.00	20%	Shear
Pressure		Pr	15.20	0.76	5%	0.76	5%	
Accelerations	Ax		-5,000	-1,000	20%	-2,000	40%	
	Ay		203.95	40.79	20%	81.58	40%	
	Az		568.34	113.67	20%	227.34	40%	

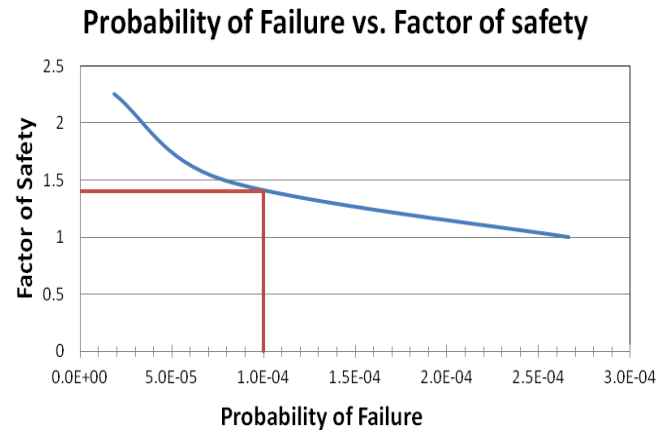


Figure 1: Probability of Failure vs. Factor of Safety

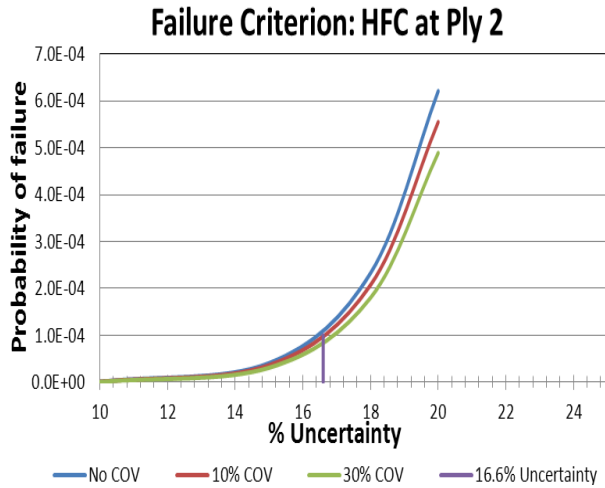


Figure 2: Probability of Failure as a function of the amount of uncertainty in the design space

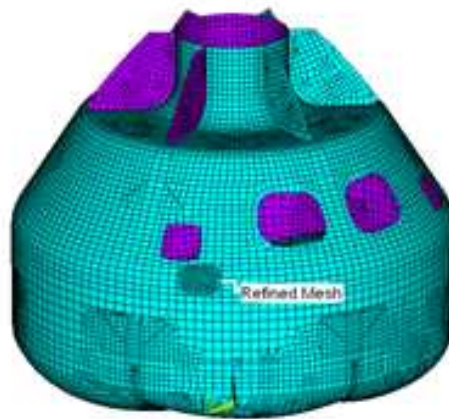


Figure 3: Refined Mesh for High Stress Location

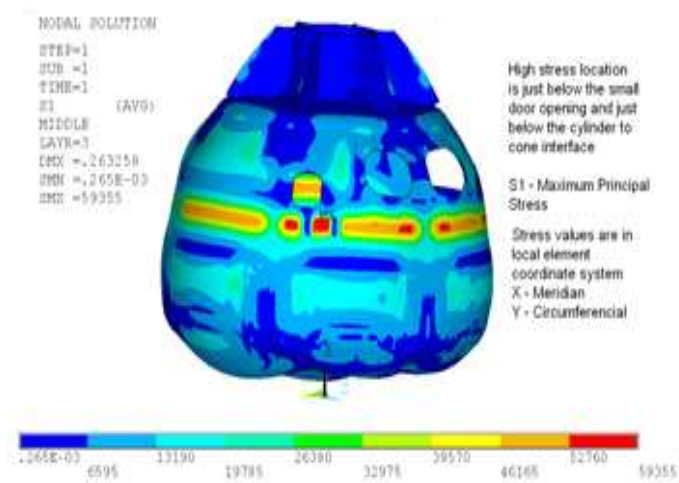


Figure 4: High Stress Plot for Maximum Principal Stress in Ply 3

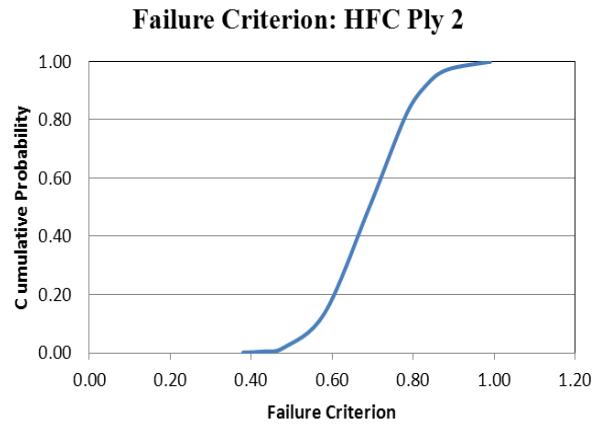


Figure 5: Cumulative Density Function (CDF) of Failure Criterion, HCF Ply 2

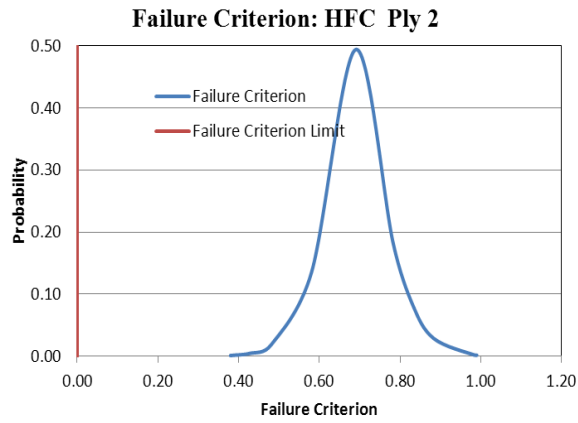


Figure 6: HCF Failure Criterion with Failure Criterion Limit for Ply 2 (5% uncertainties in the input variables)

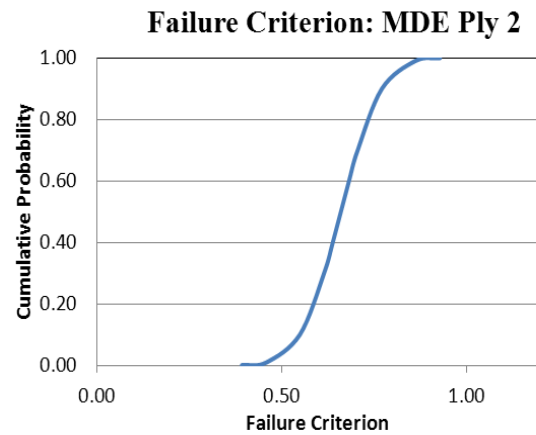


Figure 7: CDF of Failure Criterion, MDE Ply 2

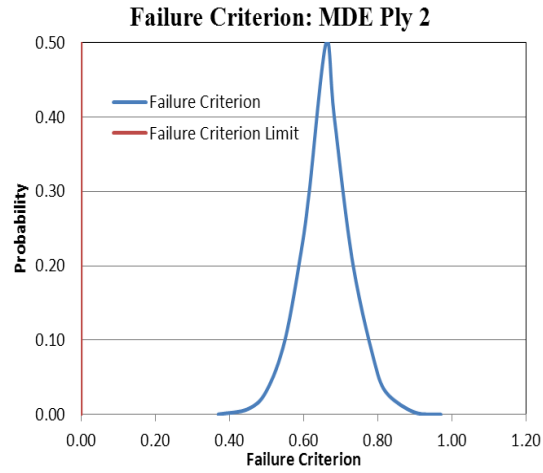


Figure 8: MDE Failure Criterion with Failure Criterion Limit for Ply 2 (5% uncertainties in the input variables)

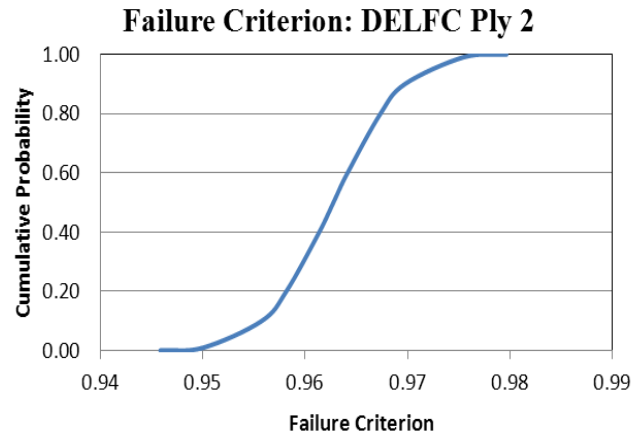


Figure 9: CDF of Failure Criterion: DELFC Ply 2

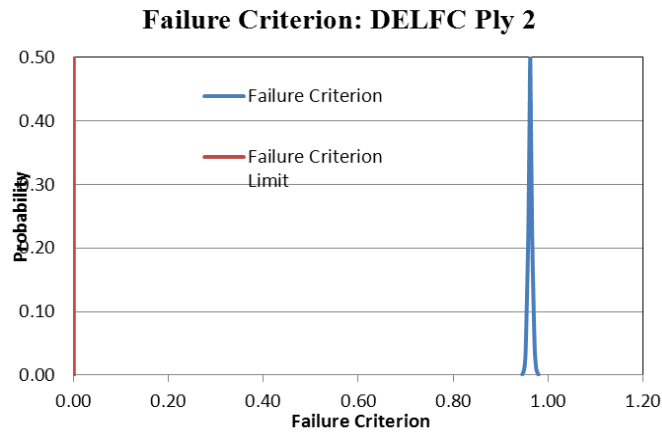


Figure 10: DELFC Failure Criterion with Failure Criterion Limit for Ply 2 (5% uncertainties in the input variables)

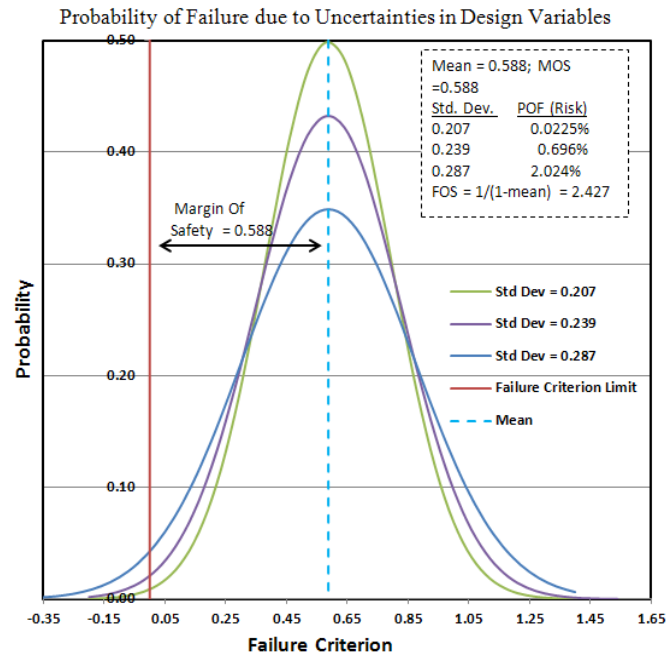


Figure 11: Probability of Failure due to Uncertainties in Design Variables. Factor of safety alone is not enough to properly assess risk.

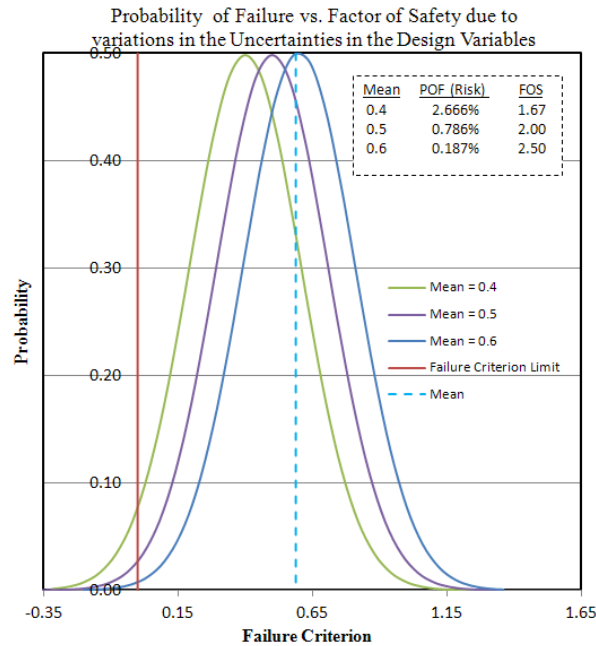


Figure 12: Probability of Failure (POF) vs. Factor of Safety (FOS) due to variation in the Uncertainties in the Design Variables

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ANNEX A

Failure Criteria equations and nomenclature

Delamination criterion:

$$FC = 1 - \left(\frac{|\Delta\phi|}{\Delta\phi_{del}} \right)$$

$$\Delta\phi_{del} = \frac{S_{12}}{G_{12}}$$

$$\Delta\phi = \frac{1}{2}(\varepsilon_{c22} - \varepsilon_{c11})(\sin 2\theta_i - \sin 2\theta_{i-1}) + \frac{1}{2}(\varepsilon_{c12})(\cos 2\theta_i - \cos 2\theta_{i-1})$$

Hoffman's criterion:

$$FC = 1 - \left[\left(\frac{\sigma_{P11\alpha}}{S_{P11\alpha}} \right)^2 + \left(\frac{\sigma_{P11\beta}}{S_{P11\beta}} \right)^2 - K_{P12\alpha\beta} \frac{\sigma_{P11\alpha}}{|S_{P11\alpha}|} \frac{\sigma_{P22\beta}}{|S_{P22\beta}|} + \left(\frac{\sigma_{P12}}{S_{P12}} \right)^2 \right]$$

Modified distortion energy criterion:

$$FC = 1 - \left[\left(\frac{\sigma_{P11\alpha}}{S_{P11\alpha}} \right)^2 + \left(\frac{\sigma_{P11\beta}}{S_{P11\beta}} \right)^2 - K_{P12\alpha\beta} \frac{\sigma_{P11\alpha}}{|S_{P11\alpha}|} \frac{\sigma_{P22\beta}}{|S_{P22\beta}|} + \left(\frac{\sigma_{P12}}{S_{P12}} \right)^2 \right]$$

$$K_{P12\alpha\beta} = K' \frac{(1 + 4\nu_{P12} - \nu_{P13})E_{P22} + (1 - \nu_{P23})E_{P11}}{\sqrt{E_{P11}E_{P22}(2 + \nu_{P12} + \nu_{P13})(2 + \nu_{P21} + \nu_{P23})}}$$

where K' , the correlation factor, is assumed to be 1.

$$\alpha = \begin{cases} T, & \sigma_{P11} \geq 0 \\ C, & \sigma_{P11} < 0 \end{cases}$$

$$\beta = \begin{cases} T, & \sigma_{P22} \geq 0 \\ C, & \sigma_{P22} < 0 \end{cases}$$

Symbol	Description
ε_{11}	Strain in the 11 direction
ε_{22}	Strain in 22 direction
ε_{12}	Shear strain in plane 12
θ_i	Fiber angle in the ith ply
θ_{i-1}	Fiber angle in the $i - 1$ th ply
S_{12}	Shear strength in plane 12
G_{12}	Shear strain in plane 12
σ_{P11}	Calculated stress in the 11 direction of ply p
σ_{P22}	Calculated stress in the 22 direction of ply p
σ_{P12}	Calculated stress in the 12 plane of ply p
S_{P11T}	Tensile strength in the 11 direction of ply p
S_{P11C}	Compressive strength in the 11 direction of ply p
S_{P22T}	Tensile strength in the 22 direction of ply p
S_{P22C}	Compressive strength in the 22 direction of ply p
S_{P12}	Strength in the 12 plane of ply p
ν_{12}	Poisson's ratio
ν_{23}	Poisson's ratio
T	Tension
C	Compression

$$S_{P11\alpha} = \begin{cases} S_{P11T}, & \alpha = T \\ S_{P11C}, & \alpha = C \end{cases}$$