A Kansa-RBF method for elliptic boundary value problems in annular domains

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Abstract

In this presentation, we couple the Kansa method [Kansa (1990)] and a matrix decomposition technique for solving problems using a large number of interpolation points. Since global RBFs are used as basis functions, we can also achieve high accuracy in numerical results. To be more specific, we consider the discretization of elliptic boundary value problems in annular domains using the Kansa method. For any choice of RBF, such appropriate discretizations lead to linear systems in which the coefficient matrices possess block circulant structures and which are solved efficiently using matrix decomposition algorithms (MDAs). An MDA [Bialecki et al (2011)] is a direct method which reduces the solution of an algebraic problem to the solution of a set of independent systems of lower dimension with, often, the use of fast Fourier Transforms (FFTs). MDAs lead to considerable savings in computational cost and storage. This decomposition technique not only allows us to handle large-scale matrices but also makes it possible to implement the LOOCV technique to find the sub-optimal shape parameter of the RBFs used. It should be noted that the LOOCV technique is not suitable when the size of the matrix is too large. Such MDAs have been used in the past in various applications of the method of fundamental solutions (MFS) to boundary value problems in geometries possessing radial symmetry, see e.g., [Karageorghis (2010; 2011)]. A similar MDA was also applied for the approximation of functions and their derivatives using RBFs in circular domains in [Karageorghis et al. (2014)], see also, [Heryudono et al. (2010)].

Keywords: radial basis functions, Poisson equation, biharmonic equation, Cauchy-Navier equations of elasticity, Fast Fourier transforms, Kansa method.

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