Lower Bound of Eigenvalue Solution Using the NS-FEM

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Abstract

The smoothed finite element method (S-FEM) has been recently developed as an effective solver for solid mechanics problems. The S-FEM uses different smoothed domains and can have various models. The smoothing domains can be cell-based (CS-FEM), edge-based (ES-FEM) and node-based (NS-FEM), and each of these S-FEM models has its own useful properties. We represents here a unique approach to compute the lower bounds of vibration modes or eigenvalues of elasto-dynamic problems, by making use of the important softening effects of the node-based smoothed finite element method (NS-FEM). We first use the NS-FEM, standard FEM and the analytic approach to compute the eigenvalues of transverse free vibration in strings and membranes. It is found that eigenvalues by NS-FEM is always smaller than those by FEM and the analytic method. However, NS-FEM produces spurious unphysical modes simultaneously, due to its overly soft behavior. A technique is then proposed to remove these spurious modes by analyzing the shape vibration modes. The technique is established based on the observation that they these spurious modes have excessively large wave numbers that are not related to the physical deflection shape of the vibration modes, but related to the discretization density. Therefore these modes can be easily removed by computing their wave numbers. The final results of NS-FEM become the lower bound of the vibration modes, and the accuracy of the solution can be improved via mesh refinement due to the proven stability of the NS-FEM. It offers, for the first time, a valuable practical computational means to effectively compute the lower bounds of eigenvalues problems.

Keywords: NS-FEM, Lower bound, Soft behavior, Spurious unphysical mode, Eigenvalue, Eigenvector, Vibration mode, Stationary rectangular membrane.

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