Quick modal reanalysis for large modification of structural topology based on multiple condensation model

He Jian-jun, Chen Xiang-zi
(School of Automobile and Mechanical Engineering, Changsha University of Science and Technology, Changsha 410004, China)

Abstract: The paper focuses on the modal reanalysis for large topological modified structure based on the multiple dynamic condensation model. Firstly，basing on multiple dynamic condensation model of original structure and topological modified structures, with combining the independent mass orthogonalization and Rayleigh-Ritz analysis, the proposed method was generated. Finally, two numerical examples were used to show that the presented method can provide quite efficient and high quality approximation result for modal reanalysis even when large modifications of the structural topology are made.

Key words: modal reanalysis; large topological modified structure; multiple dynamic condensation; independent mass orthogonalization

1. Introduction

In many large-scale and complex structural systems, linear eigenvalue analysis is very important and useful for predicting modal response. However, in order to grasp the dynamic characteristic of huge structure, we usually directly calculate the Eigen-problem by LQ algorithm or subspace method. Therefore, the computational cost may be too time consuming. Modal reanalysis [1] provides efficient and quick numerical procedures for calculating the eigenvalues and eigenvectors due to modifying the multiple properties of the original structure, without having to repeat the dynamic equation problem several times. Thus, the aim of reanalysis is to reduce lots of computational cost in optimization design. For example, in structural topology optimization design with lots of iterations, the direct FEA analysis must be repeated for each modification of the design variables. Therefore, the computational cost may be too time consuming especially for large structure.

However, the modal reanalysis based on multiple dynamic condensation model are rarely studied in the corresponding reanalysis literature [2-8]. so in the paper, we focus on the structural modal reanalysis for large modifications by using multiple condensation model. The aim is to further reduce the global CPU time than the general modal reanalysis with the non-condensation model. Thus the proposed reanalysis method could be an high-efficiency method for large modifications in optimization design.

As for the large modifications, the computational effort is significantly reduced by the proposed approach. The reanalysis procedure is easy to implement and realize. Moreover, the approximate eigensolution of modified structure still has high accuracy even if the original structure and modified structure are reduced model. Section 2 briefly introduces the Eigen-equations for modal reanalysis problem based on reduced model. Section 3 describes the proposed method and its steps. Section 4 discusses the efficiency of the precision and the CPU time of the proposed method by two numerical examples. Section 5 summarizes our conclusions.

2. modal reanalysis formulations for multiple condensation model

2.1 Eigenproblem of original structure based on multiple condensation model

In finite element analysis, the natural vibration of undamped original structures with m DOF leads to a general algebraic eigenproblem

$$\boldsymbol{K}_m^0 \boldsymbol{\Psi}_{mi}^0 = \lambda_i^0 \boldsymbol{M}_m^0 \boldsymbol{\Psi}_{mi}^0 \tag{1}$$

where $\boldsymbol{K}_m^0$, $\boldsymbol{M}_m^0$, λ_i^0 and $\boldsymbol{\Psi}_{mi}^0$ are the stiffness matrix, the mass matrix, the *i*th eigenvalue and *i*th normalized eigenvector of the original structure, respectively.

By using the dynamic condensation method in literature [9], the three times condensation model for the original structure is obtained. Due to the limit of space, the reduced process is omitted here. The full iterative process is expressed as the literature [9].

Through the three time dynamic condensation process, finally, the eigenprobelm of reduced model for original structure can be expressed as

$$\boldsymbol{K}_{R_0}^{(i+1)} (\boldsymbol{\Psi}_{11}^0)^{(i+1)} = \boldsymbol{M}_{R_0}^{(i+1)} (\boldsymbol{\Psi}_{11}^0)^{(i+1)} (\boldsymbol{\Lambda}_{11}^0)^{(i+1)} \tag{2}$$

2.2 Eigenproblem of reduced model for topologically modified structure with added DOF

Similarly, the vibration eigenproblem of the complete model for topologically modified structure with added n DOF is expressed as

$$\boldsymbol{K}_{m+n} \boldsymbol{\Psi}_i = \lambda_i \boldsymbol{M}_{m+n} \boldsymbol{\Psi}_i \tag{3}$$

where $\boldsymbol{K}_{m+n}$ and $\boldsymbol{M}_{m+n}$ are the stiffness and mass matrix of the topologically modified structure, respectively; and the *i*th corresponding eigenpair are denoted as λ_i and $\boldsymbol{\Psi}_i$.

Basing the dynamic condensation method and its implementation for topologically modified structure, similarly, the stiffness matrix $\boldsymbol{K}_R^{(i)}$ and mass matrix $\boldsymbol{M}_R^{(i)}$ of topologically modified structure could be obtained as the original structure.

$$\boldsymbol{K}_R^{(i)} = \begin{bmatrix} \boldsymbol{K}_{R_0}^{(i)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \Delta \boldsymbol{K}_{R_{mm}} & \Delta \boldsymbol{K}_{R_{mn}} \\ \Delta \boldsymbol{K}_{R_{nm}} & \Delta \boldsymbol{K}_{R_{nn}} \end{bmatrix}, \quad \boldsymbol{M}_R^{(i)} = \begin{bmatrix} \boldsymbol{M}_{R_0}^{(i)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \Delta \boldsymbol{M}_{R_{mm}} & \Delta \boldsymbol{M}_{R_{mn}} \\ \Delta \boldsymbol{M}_{R_{nm}} & \Delta \boldsymbol{M}_{R_{nn}} \end{bmatrix} \tag{4}$$

3. Modal reanalysis of the multiple condensation model

The proposed modal reanalysis method is similar to the method in the reference [10]. Due to the limit of space, the complete modal reanalysis process is omitted here.

4. Numerical examples

For the purpose of validation verifications, the eigenvalues of the large topologically modified structure are computed using the direct method (LQ algorithm) and the proposed approximate reanalysis method, respectively. Define the relative error of the eigenvalues by using the criterion:

$$ree = (|\lambda_{ai} - \lambda_{ei}| / \lambda_{ei}) \times 100\% \tag{5}$$

where the exact eigenvalue λ_{ei} is directly calculated by using equation (3); basing on the multiple condensation model in equation (4), and the approximate eigenvalue λ_{ai} is computed by using the approximate modal reanalysis method from the literature [10].

Example 1 Considering a square spatial prismatic truss structure as shown in Fig.1, with its parameters given by: elasticity modulus is E=1.0×10^{11}Pa, cross-section area of each rod is A=0.0025m^2, mass density is ρ=8.90×10^3 kg/m^3. Considering large topological modification by adding 20 new nodes and 65 members as shown in Fig.2. The four lower-order eigenvalues, which resulted from those three methods, are shown in the Table 1.

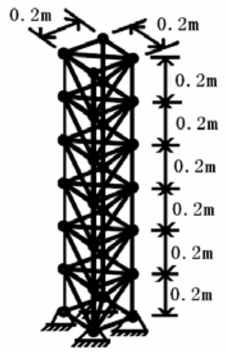

Figure.1. Initial design of spatial truss structure

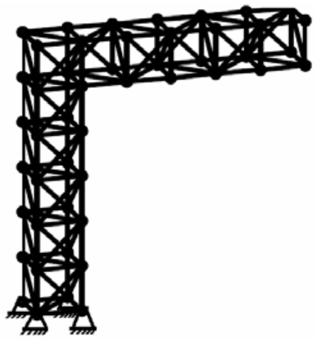
Figure. 2. Topologically modified structure

Table 1. The comparisons of the four lower-order eigenvalues for large modified structure

Mode	The direct method	Literature [11]'s method	Ree (%)	The proposed method	Ree (%)
1	1.15E+4	1.41E+4	22.82	1.15E+4	3.27E-3
2	2.27E+4	3.95E+4	73.86	2.27E+4	1.04E-1
3	9.57E+4	9.73E+4	1.72	9.64E+4	7.39E-1
4	1.28E+6	9.31E+5	625.70	1.29E+6	6.49E-1

Here, the comparisons of computational cost are as follow: the average computational time of exact computation by using the direct method is 0.1508 second; and the average computational time by using the proposed reanalysis method is 0.009514 second. So the saving of computational time is nearly 93.7 percentages.

Example 2: Considering an original rectangle bending plate structure, shown in Fig.3, with its material and structural parameters given by: elasticity modulus is E=2.1×10^{11}Pa, the thickness of plate is t=0.01m, mass density is ρ=7.80×10^3 kg/m^3, the Poisson's ratio is equal to 0.3, the length and width of plate are 1m and 0.3m, respectively. The original plate is fixed at left, right boundary and discretized into 44 nodes and 30 square plate elements. Considering large topological modifications as shown in Fig.4. The modified plate structure has 76 nodes and 54 elements, with its material parameters given as follow: elasticity modulus is E=1.0×10^{11}Pa, mass density is ρ=8.90×10^{3} kg/m^{3}, the Poisson's ratio is $\mu = 0.3$. The sizes of the modified structure are transformed to 1.2m×0.84m.

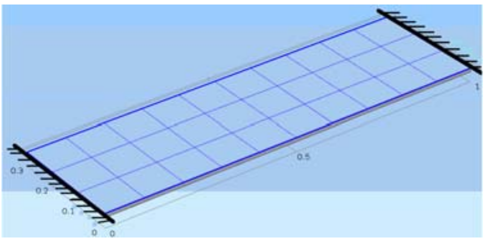

Figure.3. The original bending plate structure

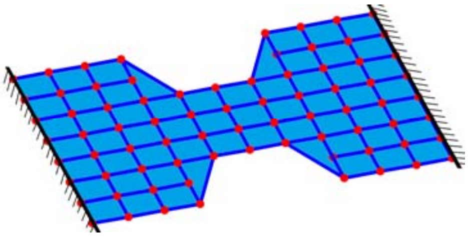

Figure.4. The modified structure

The six lower-order eigenvalues, which resulted from those three methods, are shown in the Table 2.

Table 2. The comparisons of the six lower-order eigenvalues for large modified structure

Mode	The direct method	Literature [11]'s method	Ree (%)	The proposed method	Ree (%)
1	2.86E+4	3.23E+4	12.8	2.86E+4	4.86E-2
2	8.35E+4	8.49E+4	1.64	8.40E+4	4.70E-1
3	1.20E+5	1.52E+5	25.89	1.23E+5	1.83
4	1.61E+5	2.05E+5	27.22	1.66E+5	3.33
5	3.00E+5	4.76E+5	59.21	3.10E+5	3.87
6	4.63E+5	9.08E+5	96.11	4.81E+5	3.94

Here, the comparisons of computational cost are as follow: the average computational time of exact computation by using the direct method is 0.1630 second; and the average computational time by using the proposed reanalysis method is 0.01357 second. So the saving of computational time is nearly 91.7 percentages.

5. Conclusion

For large changes in structural topological and parameter modifications with added DOF, an improved modal reanalysis method for the finite element system based on multiple condensation model of the original structure and the modified structure is proposed. By comparing to the method from literature [9], the presented method has higher accuracy and efficiency. From the approximate results of numerical examples and the comparisons of average computational cost, the proposed method not only has high approximation, but also can reduce vast amount of computational efforts than the direct calculation method, so it is much efficient than the direct method. In spite of the complete model or reduced model, the proposed modal method can be used with a general reanalysis for multiple condensation model when large changes of structure topology and parameters are made, and it may be one of the highest efficient and exact modal reanalysis methods for large topologically modified structure so far.

Acknowledgements

The authors would like to thank for the supports by Natural Science Foundation of China under grant 51305048, and Scientific Research Fund of Hunan Provincial Education Department under grant 11C0045.

Reference

[1] U. Kirsch, Reanalysis of Structures—A Unified Approach for Linear, Nonlinear, Static and Dynamic Systems, Springer, Dordrecht, 2008

[2] Oded Amir a, Ole Sigmund, Boyan S. Lazarov, et al. Efficient reanalysis techniques for robust topology optimization [J]. Comput. Methods Appl. Mech. Engrg., 2012, 245-246: 217-231

[3] Liang Ma, Su Huan Chen , Guang Wei Meng. Combined approximation for reanalysis of complex eigenvalues [J]. Computers and Structures, 2009, 87: 502-506

[4] P. Cacciola, G. Muscolino. Reanalysis techniques in stochastic analysis of linear structures under stationary multi-correlated input [J]. Probabilistic Engineering Mechanics, 2011, 26: 92-100

[5] F. Massa, T. Tison, B. Lallemand. Structural modal reanalysis methods using homotopy perturbation and projection techniques [J]. Comput. Methods Appl. Mech. Engrg., 2011, 200: 2971-2982

[6] Mitsuhiro Kashiwagi. A numerical method for eigensolution of locally modified systems based on the inverse power method [J]. Finite Elements in Analysis and Design, 2009, 45: 113-120

[7] F. Massa, B. Lallemand, T. Tison. Multi-level homotopy perturbation and projection techniques for the reanalysis of quadratic eigenvalue problems: The application of stability analysis [J]. Mechanical Systems and Signal Processing, 2015, 52-53: 88-104

[8] Guoqiang Gao, Hu Wang, Guangyao Li. An adaptive time-based global method for dynamic reanalysis [J]. Struct Multidisc Optim, 2013, 48: 355-365

[9] Zu-Qing Qu, Wenjun Chang. Dynamic condensation method for viscously damped vibration systems in engineering [J]. Engineering Structures, 2000, 22(11): 1426-1432

[10] He Jianjun, Jiang Jiesheng, Kang Xingwu. New method of dynamical reanalysis for large topologically modified structures [J]. Zhendong Gongcheng Xuebao, 2007, 20(4): 407-411

[11] Huang C, Chen S H, Liu Z S. Structural modal reanalysis for topological modifications of finite element systems [J]. Engineering Structures, 2000, 22(4): 304-310