

Application of the quadrilateral area coordinates: A 4-node quadrilateral membrane element beyond MacNeal's theorem

*Pei-Lei Zhou¹, †Song Cen^{1,2,3}

¹Department of Engineering Mechanics, School of Aerospace Engineering, Tsinghua University, Beijing 100084, China

²High Performance Computing Center, School of Aerospace Engineering, Tsinghua University, Beijing 100084, China

³Key Laboratory of Applied Mechanics, School of Aerospace Engineering, Tsinghua University, Beijing 100084, China

*Presenting author: sea_sky_love@yeah.net

†Corresponding author: censong@tsinghua.edu.cn

Abstract

In this paper, the second form of quadrilateral area coordinate method (QACM-II) was employed to develop a novel 4-node quadrilateral membrane element insensitive to mesh distortion. First, two stress functions in terms of the QACM-II (S , T) are given to represent exact solutions of pure bending state in x and y directions. Their linear combination can reflect exact solutions for pure bending in all directions because the QACM-II is a local natural coordinate system. Thus, the basic analytical solutions of stresses, strains and displacements corresponding to the pure bending state for the isotropic and anisotropic plane problems can be obtained. Then, the unsymmetric element technique was adopted to formulate an unsymmetric 4-node, 8-DOF (2DOFs per node) quadrilateral membrane element. This approach is derived from the virtual work principle, and uses different test and trial functions. The test function employs the isoparametric coordinate interpolation to satisfy inter-element compatibility, while the trial function adopts a novel composite coordinate interpolation of the Cartesian coordinates and QACM-II to satisfy the completeness requirements in physical space. Due to the linear relationship between quadrilateral area and Cartesian coordinates, the order of trial displacement fields will not degrade when the meshes are distorted. Since the test displacement fields satisfy the compatibility requirements and the trial displacement fields contain analytical solutions of pure bending in all directions, the new element can produce exact solutions in both constant stress/strain patch test and pure bending problem in any directions, which means the well-known contradiction declared by MacNeal's theorem: any 4-node, 8-DOF plane membrane element will either lock in in-plane bending or fail to pass a C_0 patch test when the element's shape is an isosceles trapezoid, has been perfectly broken through.

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