Stochastic prediction and update for macroscopic property of heterogeneous material considering microstructural uncertainties

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For heterogenic structure, multiscale analysis is a powerful tool for microstructual design to improve the property of composite material. Nowadays after 3D printing machine is used, the optimal microstructure design by numerical simulation can be fabricated into reality, which makes computer aided engineering more appeals. However, there always exists a gap between numerical simulation and fabricated ones. The most probable reason lies in uncertainties in the microstructure. The introduction of ASME V&V10-2006 standard improved the awareness among the analysts about the importance of appropriate choice of quality of interest and uncertainty quantification in contributing to the credibility or trustworthiness of the computational mechanics model [1]. The stochastic simulation for macroscopic property by microstructure design seems accessible and update by fabricated products or feedback of experiments could associate with a variety of underlying uncertainties.

The uncertainties of macroscopic elastic property $\boldsymbol{D}^H$ are originated from the microstructure architecture $\boldsymbol{X}$ and constituent material properties, expressed by $\boldsymbol{D}^H \equiv \boldsymbol{D}^H\left(\boldsymbol{X}, \boldsymbol{D}(\alpha)\right)$. Here $\boldsymbol{D}(\alpha)$ is not only applicable for isotropic elastic matrix, but also for orthotropic elastic matrix $\boldsymbol{D}(\alpha_1, \alpha_2)$ and multiphase constituent elastic matrixes $\boldsymbol{D}(\alpha_1, \alpha_2, \alpha_3)$. Uncertainties in microstructure architecture $\boldsymbol{X}$ are hard to parametrize in statistic distribution. In the standpoint of modeling, a mathematical equation is written for coated particulate embedded composites as Eq.(1). The geometry $\boldsymbol{X}$ contains random variables of radius of particles R_p, coating thickness h_c, particulate volume fraction V_p, coating volume fraction V_c, disordering arrangement A_p and A_c. The disordering as uncertainty factors in morphology is a non-parametric factor described by q and a dummy parameter m. In order to simplify the problem, microstructures for spherical porous material as Eq. (2) are firstly determined as demonstrative examples. Here there are two algorithms to make the architecture as slight disordering and clustering geometry.

$$\boldsymbol{X} \equiv \boldsymbol{X}\left(R_p, h_c, V_p, V_c, A_p(m), A_c(q)\right) \tag{1}$$

$$\boldsymbol{X} \equiv \boldsymbol{X}\left(R, V, A(m, c)\right) \tag{2}$$

$$\boldsymbol{D}^H \equiv \sum_{j=1}^{n} \Pr\left(Y_j\right) \boldsymbol{D}^H_{Y_j}\left(\boldsymbol{D}\left(\alpha_i\right)\right) \tag{3}$$

The first-order perturbation based homogenization method is used to analysis computational model with random variables in constituent material, which is assumed as Gaussian probability density function. However, the non-parametric uncertainties introduced by morphology variation are neglected by perturbation based method. A new computational method is proposed by collecting statistic data in a mixture probabilistic way shown as Eq. (3). $\boldsymbol{D}_{Y_j}{}^H$ is the calculated results by perturbation based homogenization analysis. $\Pr(Y_j)$ is a probability of computational model Y_j. Additionally, a regression curve for property of interest and design variable can be determined iteratively by accumulation of calculation samples. Providing that the experimental results are available, the update of the simulation is shown to feedback in accordance with various uncertainties in reality by changing the probability of computational model $\Pr(Y_j)$.

Two demonstrative examples are shown in detail. The obtained properties are expressed in the form of probabilistic way. The first example is spherical porous material with slight disordering. As mentioned above, the regression curve and 95% reliability regression curve are obtained. The second one is porous material with slight clusters. This results has applied update scheme by adding virtual experimental results. Through this way, it can report probabilistic sensitivity of random variables to result in cost-effective changing leading to improved reliability.

References

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