A better way for managing all of the physical sciences under a single unified theory of analytical integration.

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Abstract

If a new system of computational logic would be entirely based on the application of a true unified *computational-based* analytical theory of integration then what better way of validating such a system of mathematical logic then through the complete development of a unified theory of physics. The outcome of having successfully arrived at such a monumental theory in physics would represent a much greater expansion of our knowledge in terms of engineering science. This would be the direct consequence of having analytically resolved under one unified theory of analytical integration the vast majority of PDEs some of which would prove very similar in appearance to those encountered in theoretical physics.

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Content

0. Introduction

The new development for the physical science begins with the introduction of what appears to be a *computational-based* unified theory of *analytical* integration. This would eventually lead to the formation of some type of a new physical ideology by which a unified theory of physics could eventually be constructed over time.

Even *Albert Einstein* who has always claimed that God never plays with dices would have conceded if he were alive today, that somewhere out there in the vast realm of mathematics there has to exist some form of an algebraic system capable of addressing all of physics under one unified theory of ***"computation"*.**

And I would like to quote from the ending of his book *The meaning of Relativity* [Einstein (1974)]:

" This does not seem to be in accordance with continuum theory, and must lead to an attempt to find a purely algebraic theory for the description of reality. But nobody knows how to obtain the basis of such a theory ".

With a unified theory of integration now possibly well within our grasp, this would represent a very important step towards becoming much less dependent on experimental scientific and engineering methods of analysis. As a result of this, we would be adopting a far more theoretical approach towards every aspect of the physical sciences on a much wider universal scale than ever thought possible under existing traditional methods of analysis.

" The greatest problem in having to rely on traditional methods of mathematical analysis is mainly due to a severe lack of universality as a direct consequence of not having uncovered a unified theory of analytical integration in the past. "

Throughout the remaining of this article, I will first and foremost attempt to briefly summarize in more *layman's term* an entirely new mathematical ideology. Next, I will proceed to demonstrate how such a very powerful new ideology in mathematics can be mutated into a whole new branch of physics that I would like to introduce everyone as an "***idealistic physics***".

As a direct consequence of what appears to be a unified theory of analytical integration, I would like to address the importance of arriving at some unified theory of physics for a much greater expansion of our knowledge in terms of engineering science.

1. First and foremost, the new mathematical ideology

What gives the new ideology its own and very unique flavor in mathematics is that all analysis pertaining to anti-differentiation whether in the form of a DE or an integral is always performed at the differential level. The main reason for this is to insure that the concept of continuity be preserved throughout the entire anti-differentiation process. Because the laws of algebra apply equally well to finite quantities as they do to differential quantities without regards to any limiting process near zero, there is never a risk of violating any known mathematical principles. The type of continuity I am referring to can ideally be described by a DE or a system of DEs.

Under the new proposed mathematical ideology when it comes to solving for any type of DE or system of DEs, rather than working with complete mathematical equations, we instead only become interested in working with complete *differential* form representations.

This is where the new mathematical ideology now begins to deviate from the old traditional thinking of Calculus.

If we were to have complete access to every imaginable type of mathematical equations just from the computed values alone that would originate from the application of some very unique mathematical ideology then this would certainly represent a very significant discovery in mathematics. This would no doubt represent an extremely valuable tool for completely eliminating our most fundamental problem of not being able to select the most suitable type of mathematical equation for handling *all* aspects of the physical and biological sciences under one ***"unified theory of computation"*.** Over time, this would inevitably lead towards the development of some form of a unified theory of physics by which some type of a ***"theory of everything"*** would be constructed from.

The very first place we might want to look for the possible existence of such a potentially formidable mathematical theory of integration is in the following very simple integral equation:

$$t \;=\; \int \frac{dy}{ay^2 + by \;+ c} \tag{1.001}$$

Everyone would certainly agree that only because "y(t)" was initially presented in its complete differential form, this has provided us with the capability of defining a wide range of mathematical expressions just by varying the numerical values present inside this integral. From this very simple observation in Calculus, we can immediately deduce that differential forms could at least potentially represent a very powerful link between *numerical computation* and *complete mathematical expressions*.

So our primary objective now is to determine what possible variations in terms of differential form representations can we expect for including *"all"* types of mathematical equations regardless of the degree of complexity involved. Such mathematical equations would be constructed from the use of algebraic and elementary basis functions that would involve the presence of composite functions with no limit whatsoever as to each of their degree of composition. Furthermore, there would be no restriction whatsoever on the number of dependent and independent variables involved and finally, the entire mathematical equation may be expressible not only in explicit form but also in implicit form as well.

Such an ideal universal differential expansion form can only be described mathematically in terms of <u>*two*</u> fundamental parts that would involve the use of *multivariate polynomials* as well as complete *differentials* of *multivariate polynomials*.

For a general system of "k" number of implicitly defined *multivariate* mathematical equations in the form of "$f_k(z_m, x_n) = 0$" that consist of "m" number of dependent variables and "n" number of independent variables this may be described as follow:

(1). <u>*Primary Expansion:*</u>

$$F_i(W_j) \;=\; 0 \;=\; \sum_r a_{i,r}\left(\prod_j^p W_j^{E_{i,s}}\right) \qquad (1 \le i \;\le k) \tag{1.002}$$

where "W_j" are auxiliary variables, "p" is the total number of such auxiliary variables each of which are raised to some floating point value and "r" is the total number of terms present in each of the "k" number of implicitly defined multivariate polynomial equations.

(2). <u>*Secondary Differential Expansion:*</u>

$$dz_i \;=\; dW_i \qquad (1 \le i \le m) \tag{1.003}$$

$$dx_i \;=\; dW_{m+i} \qquad (1 \le i \le n) \tag{1.004}$$

$$\sum_{t=1}^{m} N_{i(m+n+1)-m-n-1+t} dz_t \;+\; \sum_{t=1}^{n} N_{i(m+n+1)-n-1+t} dx_t \;=$$

$$= \; N_{i(m+n+1)} dW_j \qquad [1 \leq i \leq p-m-n] \;\; [m+n+1 \leq j \leq p] \qquad (1.005)$$

$$N_c\left(W_j\right) \;=\; \sum_{t=(c-1)r+1}^{cr} b_{c,t} \left(\prod_{j}^{p} W_j^{E'_{c,s}} \right) \qquad (1.006)$$

$$[1 \leq c \; \leq i(m+n+1)] \;\; [1 \leq i \leq p-m-n]$$

There is at present no other possible differential form capable of representing *all* mathematical equations with such a high degree of universality then the one suggested above.

In complete expanded form we would write this as follow:

(1). <u>*Primary Expansion:*</u>

$$F_1 \;=\; 0 \;=\; a_{1,1} W_1^{m_{11}} W_2^{m_{12}} \cdots W_p^{m_{1p}} \;+\; a_{1,2} W_1^{m_{1,p+1}} W_2^{m_{1,p+2}} \cdots W_p^{m_{1,2p}} \;+\; \ldots \;+$$

$$+ \; \ldots \; + \; a_{1,r} W_1^{m_{1,p(r-1)+1}} W_2^{m_{1,p(r-1)+2}} \cdots W_p^{m_{1,rp}} \qquad (1.007)$$

$$F_2 \;=\; 0 \;=\; a_{2,1} W_1^{m_{2,1}} W_2^{m_{2,2}} \cdots W_p^{m_{2,p}} \;+\; a_{2,2} W_1^{m_{2,p+1}} W_2^{m_{2,p+2}} \cdots W_p^{m_{2,2p}} \;+\; \ldots \;+$$

$$+ \; \ldots \; + \; a_{2,r} W_1^{m_{2,p(r-1)+1}} W_2^{m_{2,p(r-1)+2}} \cdots W_p^{m_{2,rp}} \qquad (1.008)$$

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$$F_k \;=\; 0 \;=\; a_{k,1} W_1^{m_{k,1}} W_2^{m_{k,2}} \cdots W_p^{m_{k,p}} \;+\; a_{k,2} W_1^{m_{k,p+1}} W_2^{m_{k,p+2}} \cdots W_p^{m_{k,2p}} \;+\; \ldots \;+$$

$$+ \; \ldots \; + \; a_{k,r} W_1^{m_{k,p(r-1)+1}} W_2^{m_{k,p(r-1)+2}} \cdots W_p^{m_{k,rp}} \qquad (1.009)$$

(2). *Secondary Differential Expansion:*

$$dz_i = dW_i \qquad (1 \le i \le m) \tag{1.010}$$

$$dx_i = dW_{m+i} \qquad (1 \le i \le n) \tag{1.011}$$

$$[N_1 dz_1 + N_2 dz_2 + \dots + N_m dz_m] + [N_{m+1} dx_1 + N_{m+2} dx_2 + \dots +$$

$$+ \dots + N_{m+n} dx_n] = N_{m+n+1} dW_{m+n+1} \tag{1.012}$$

$$[N_{m+n+2} dz_1 + N_{m+n+3} dz_2 + \dots + N_{2m+n+1} dz_m] + [N_{2m+n+2} dx_1 +$$

$$+ N_{2m+n+3} dx_2 + \dots + N_{2(m+n+1)-1} dx_n] = N_{2(m+n+1)} dW_{m+n+2} \tag{1.013}$$

$$\begin{array}{ccc} . & . & . \\ . & . & . \\ . & . & . \\ . & . & . \end{array}$$

$$\Big[N_{(p-1)(m+n+1)+1} dz_1 + N_{(p-1)(m+n+1)+2} dz_2 + \dots + N_{(p-1)(m+n+1)+m} dz_m\Big] +$$

$$+ [N_{(p-1)(m+n+1)+m+1} dx_1 + N_{(p-1)(m+n+1)+m+2} dx_2 + \dots + N_{p(m+n+1)-1} dx_n] =$$

$$= N_{p(m+n+1)} dW_p \tag{1.014}$$

Assuming a system of implicitly defined mathematical equations consisting of *3* dependent variables and *5* independent variables with a total number of *12* auxiliary variables.

We will determine the correct index value for each of the multivariate polynomials present inside the *Secondary Differential Expansion* that would be responsible for defining the complete expression for the *10th* auxiliary variable:

Starting with equation (1.005):

$$\sum_{t=1}^{m} N_{i(m+n+1)-m-n-1+t} dz_t \;+\; \sum_{t=1}^{n} N_{i(m+n+1)-n-1+t} dx_t \;=$$

$$= \; N_{i(m+n+1)} dW_j \qquad [1 \le i \le p-m-n] \;\; [m+n+1 \le j \le p] \qquad (1.015)$$

The results are for "p=12", "m=3", "n=5" we have "i = 12 - 10 = 2" and "j = 10":

$$\sum_{t=1}^{3} N_{2(3+5+1)-3-5-1+t} dz_i \;+\; \sum_{t=1}^{5} N_{2(3+5+1)-5-1+t} dx_i \;=\; N_{2(m+n+1)} dW_{10} \qquad (1.016)$$

$$[\, N_{10} dz_1 + N_{11} dz_2 + N_{12} dz_3 \,] + [\, N_{13} dx_1 + N_{14} dx_2 + N_{15} dx_3 +$$

$$+ \; N_{16} dx_4 + N_{17} dx_5 \,] \;=\; N_{18} dW_{10} \qquad (1.017)$$

Before proceeding any further, a few simple mathematical definitions need to be in order.

The first one, the actual process of transforming a complete mathematical equation in terms of the above universal differential form representation is referred to as taking its *Multivariate Polynomial Transform.*

Next, the complete *reverse* process of going from a differential form representation back to its original complete mathematical equation would be referred to as taking the *inverse* of a *Multivariate Polynomial Transform*. This would require following a very unique integration process to be described later for determining the complete analytical expression corresponding to each auxiliary variable "W_j". They each in turn would be substituting back into the *Primary Expansion* for arriving at the complete original expression that we started with being in the form of "$f_k(z_m, x_n) = 0$".

As we are dealing mainly with multivariate polynomials and complete differentials of multivariate polynomials, new types of coefficients are being introduced along the way. During the process of inverting from a differential form back to the original complete mathematical equation, some of these coefficients would be entirely responsible for defining the basis functions by which the complete mathematical equation was originally constructed from. These particular types of coefficients are present only in the *Secondary Differential Expansion* of a *Multivariate Polynomial Transform*. The remaining types of coefficients will be described in more detail later on.

Example (1.1). Let us consider the simplest *two* dimensional case which would correspond to the case for "$k = m = n = 1$" and by replacing the dependent variable "z" with "y", we arrive at the following corresponding general *Multivariate Polynomial Transform* for "$y(x)$" :

(1). *Primary Expansion:*

$$F(W_j) = 0 \qquad [1 \le j \le p] \tag{1.018}$$

(2). *Secondary Differential Expansion:*

$$dx = dW_1 \tag{1.019}$$

$$dy = dW_2 \tag{1.020}$$

$$N_{3i-2}dx + N_{3i-1}dy = N_{3i}dW_j \qquad [1 \le i \le p-2] \; [3 \le j \le p] \tag{1.021}$$

For this general univariate two dimensional case, the *Secondary Differential Expansion* may be written in the following more general format upon replacing each auxiliary variable on the left hand side with the dependent and independent variables:

$$M(x,y)dy + N(x,y)dx = P(W_j)dW_j \tag{1.022}$$

The left hand side of this equation appears in exactly the same format by which all first order ODEs are written prior to applying Euler's method for specifically targeting those that are considered as exact differentials.

This test is well known in Calculus and is defined by:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{1.023}$$

When this condition is met than Euler's general integral formula can then be applied and the result is a vastly simplified integration process.

The formula has two equivalent form.

The first:

$$\int Mdx + \int \left(N - \frac{\partial}{\partial y}\int Mdx\right)dy = C \tag{1.024}$$

and the second one:

$$\int Ndy + \int \left(M - \frac{\partial}{\partial x}\int Ndy\right)dx = C \tag{1.025}$$

Example (1.2). We will apply the concept of an exact differential for demonstrating in detail the exact process involved for inverting the following *Multivariate Polynomial Transform* corresponding to a univariate implicitly defined equation in two dimension.

(1). <u>*Primary Expansion:*</u>

$$F(W_1, W_2, W_3\ \ W_4\ ) \ = \ 0 \ = \ W_4 \ + \ 2W_2 \tag{1.026}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$dx \ + \ 0 \cdot dy \ = \ dW_1 \tag{1.027}$$
$$0 \cdot dx \ + \ dy \ = \ dW_2 \tag{1.028}$$
$$-2W_1 dx \ + \ 0 \cdot dy \ = \ W_3 dW_3 \tag{1.029}$$
$$2W_1 dx \ - \ W_3 dy \ = \ W_3(W_2 \ + \ W_3) dW_4 \tag{1.030}$$

The first step is to naturally begin by integrating in ascending order of complexity each first order ODE that is present in the *Secondary Differential Expansion* for the expression of each auxiliary variable.

We begin first by defining "$W_1(x) \ = \ x$" and "$W_2(y) \ = \ y$".

For "W_3(x)", we integrate equation (1.029) by parts to arrive at:

$$W_3(x) \ = \ \pm\sqrt{C_3 - 2x^2} \tag{1.031}$$

For "W_4(x, y)", the corresponding first order ODE to integrate is obtained by substituting the expression for "$W_1(x)$" and "$W_2(y)$" into (1.030) to afterwards rearrange the resultant equation in the form of:

$$\frac{2x\ dx}{W_3(y + W_3)} \ - \ \frac{dy}{y + W_3} \ = \ dW_4 \tag{1.032}$$

Let:

$$M(x, y) \ = \ \frac{2x}{W_3(y \ + \ W_3)} \tag{1.033}$$

so that since "$W_3 \ = \ W_3(x)$" :

$$\frac{\partial M}{\partial y} \ = \ \frac{-2x}{W_3(y \ + \ W_3)^2} \tag{1.034}$$

Next, define:

$$N(x, y) \ = \ \frac{-1}{(y \ + \ W_3)} \tag{1.035}$$

so that:

$$\frac{\partial N}{\partial x} = \frac{1}{(y + W_3)^2} \frac{dW_3}{dx} \tag{1.036}$$

From equation (1.029):

$$\frac{dW_3}{dx} = \frac{-2W_1}{W_3} = \frac{-2x}{W_3} \tag{1.037}$$

Substituting this equation into equation (1.036), we get:

$$\frac{\partial N}{\partial x} = \frac{-2x}{W_3(y + W_3)^2} \tag{1.038}$$

Since:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = \frac{-2x}{W_3(y + W_3)^2} \tag{1.039}$$

it follows that equation (1.030) is an exact differential whose solution may be obtained using any one of Euler's integral formula mentioned earlier in equation (1.024) and (1.025).

The following general form will be used:

$$\int N dy + \int \left(M - \frac{\partial}{\partial x}\int N dy\right) dx = C \tag{1.040}$$

For the first integral:

$$\int N dy = \int \frac{-dy}{y + W_3} = -\ln(y + W_3) \tag{1.041}$$

For the second integral:

$$\frac{\partial}{\partial x}\int N dy = -\frac{\partial}{\partial x}\ln(y + W_3) = -\frac{1}{y + W_3}\frac{dW_3}{dx} \tag{1.042}$$

From the differential that defined the third auxiliary variable as given by equation (1.029), we can write:

$$\frac{dW_3}{dx} = \frac{-2x}{W_3} \tag{1.043}$$

Thus equation (1.042) may be rewritten as follow:

$$\frac{\partial}{\partial x}\int N dy \;=\; -\frac{1}{y+\;W_3}\;\frac{dW_3}{dx} \;=\; \frac{2x}{W_3(y+W_3)} \;=\; \boldsymbol{M(x,y)} \tag{1.044}$$

so that :

$$\int\left(M\;-\;\frac{\partial}{\partial x}\int N dy\right)dx \;=\; \int(M\;-\;\boldsymbol{M})\,dx \;=\; 0 \tag{1.045}$$

Euler's integral formula may now be rewritten in the following final form:

$$\int N dy \;+\; \int\left(M\;-\;\frac{\partial}{\partial x}\int N dy\right)dx \;=\; \int N dy \;=\; \int\frac{-dy}{y+W_3} \;=\; -\ln(y+\;W_3) \tag{1.046}$$

The complete exact solution of the differential form that define "W_4" is obtained by integrating equation (1.030) using the above integral solution.

The results are:

$$-\ln(y\;+\;W_3) \;=\; W_4\;+\;K \tag{1.047}$$

Substituting the expression for "$W_3(x)$" as defined by equation (1.031) into the above equation, we obtain:

$$-\;\ln(\,y\;\pm\;\sqrt{C_3\;-\;2x^2}\,) \;=\; W_4\;+\;K \tag{1.048}$$

Solving for "W_4" :

$$W_4(x,y) \;=\; C_4\;-\;\ln(\,y\;\pm\;\sqrt{C_3\;-\;2x^2}\,) \tag{1.049}$$

The complete inverse *Multivariate Polynomial Transform* of the given implicitly defined equation is obtained by substituting the expression for "$W_1(x)$", "$W_2(y)$", "$W_3(x)$" and "$W_4(x,y)$" into the *Primary Expansion* defined by equation (1.026).

The results are:

$$f(x,y) = \;0\;=\;C_4\;-\;\ln\left(y\;\pm\;\sqrt{C_3\;-\;2x^2}\right)\;+\;2y \tag{1.050}$$

where the constants of integration defined by "C_3" and "C_4" are each determined from the initial condition of "$f(x,y) = 0$".

In *higher dimension* than two, the basic principle behind the main test for exactness is still applicable but requires some very minor modifications in order to account for the multivariate nature of the corresponding general differential form representation.

In view of equation (1.002) through (1.006), an example of a *single* first order multivariate ODE that can be present inside a *Secondary Differential Expansion* may be expressed in the following general form:

$$(M_1 dz_1 + M_2 dz_2 + \dots + M_m dz_m) + (M_{m+1} dx_1 + M_{m+2} dx_2 + \dots +$$

$$+ \dots + M_{m+n} dx_n) = M_{m+n+1} dW_j \tag{1.051}$$

where as by eliminating each auxiliary variable in terms of the dependent and independent variables on the left hand side of this equation, we can also define:

$$M_i = M_i(z_1, z_2, \dots, z_m, x_1, x_2, \dots, x_n) \qquad (1 \le i \le m+n) \tag{1.052}$$

The *right hand* side of this equation can be expressed only in terms of the auxiliary variable "W_j" so that:

$$M_i = M_i(W_j) \qquad (i = m+n+1) \tag{1.053}$$

The auxiliary variable "W_j" is actually a "*multivariate composite function"* and is to be determined assuming of course that an exact expression for each of the auxiliary variables "W_1, W_2 , ... , W_{j-1}" have all been previously obtained in ascending order of complexity.

Equation (1.051) may be rewritten as:

$$dH_1 = dH_2 \tag{1.054}$$

where:

$$dH_1 = (M_1 dz_1 + M_2 dz_2 + \dots + M_m dz_m) + (M_{m+1} dx_1 + M_{m+2} dx_2 + \dots +$$

$$+ \dots + M_{m+n} dx_n) \tag{1.055}$$

and where:

$$dH_2 = M_{m+n+1} W_j \tag{1.056}$$

If each side of equation (1.054) is an exact differential then from the chain rule:

$$dH_1 = \sum_{k=1}^{m} \frac{\partial H_1}{\partial z_k} dz_k + \sum_{k=1}^{n} \frac{\partial H_1}{\partial x_k} dx_k \tag{1.057}$$

and since "$H_2 = H_2(W_j)$" :

$$dH_2 = \frac{\partial H_2}{\partial W_j} dW_j \tag{1.058}$$

It follows from equation (1.055) and (1.057) that :

$$M_1 = \frac{\partial H_1}{\partial z_1} \tag{1.059}$$

$$M_2 = \frac{\partial H_1}{\partial z_2} \tag{1.060}$$

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$$M_m = \frac{\partial H_1}{\partial z_m} \tag{1.061}$$

$$M_{m+1} = \frac{\partial H_1}{\partial x_1} \tag{1.062}$$

$$M_{m+2} = \frac{\partial H_1}{\partial x_2} \tag{1.063}$$

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$$M_{m+n} = \frac{\partial H_1}{\partial x_n} \tag{1.064}$$

It also follows from equation (1.056) and (1.058) that:

$$M_{m+n+1} = \frac{\partial H_2}{\partial W_j} \tag{1.065}$$

From multivariate calculus, the condition that *both* sides of equation (1.054) each define an exact differential is of course when:

$$\frac{\partial M_1}{\partial z_2} = \frac{\partial M_2}{\partial z_1}, \quad \frac{\partial M_1}{\partial z_3} = \frac{\partial M_3}{\partial z_1}, \quad \frac{\partial M_1}{\partial z_4} = \frac{\partial M_4}{\partial z_1}, \quad \dots \quad , \frac{\partial M_1}{\partial z_m} = \frac{\partial M_m}{\partial z_1} \tag{1.066}$$

$$\frac{\partial M_1}{\partial x_1} = \frac{\partial M_{m+1}}{\partial z_1}, \quad \frac{\partial M_1}{\partial x_2} = \frac{\partial M_{m+2}}{\partial z_1}, \quad \dots \quad , \frac{\partial M_1}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_1} \tag{1.067}$$

$$\frac{\partial M_2}{\partial z_3} = \frac{\partial M_3}{\partial z_2}, \quad \frac{\partial M_2}{\partial z_4} = \frac{\partial M_4}{\partial z_2}, \quad \frac{\partial M_2}{\partial z_5} = \frac{\partial M_5}{\partial z_2}, \quad \dots \quad , \frac{\partial M_2}{\partial z_m} = \frac{\partial M_m}{\partial z_2} \tag{1.068}$$

$$\frac{\partial M_2}{\partial x_1} = \frac{\partial M_{m+1}}{\partial z_2}, \quad \frac{\partial M_2}{\partial x_2} = \frac{\partial M_{m+2}}{\partial z_2}, \quad \dots \quad , \frac{\partial M_2}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_2} \tag{1.069}$$

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$$\frac{\partial M_m}{\partial x_n} = \frac{\partial M_{m+n}}{\partial z_m} \tag{1.070}$$

If each of the above conditions are met then the solution for "H_1" and "H_2" is obtained as follow:

For "H_1", we integrate equation (1.059) :

$$H_1 = \int M_1(z_1, z_2, \dots, z_m, \; x_1, x_2, \dots, x_n) \;\; \partial z_1 \tag{1.071}$$

where in this case, z_i and x_j for $1 < i \le m, \; 1 \le j \le n$ and $i \ne 1$ are all treated as constants when evaluating this indefinite integral.

We can also use as another alternative:

$$H_1 \;=\; \int M_k(z_1, z_2, \dots, z_m,\; x_1, x_2, \dots, x_n)\; \partial z_k \tag{1.072}$$

where in this case, z_i and x_j for $2 \leq i \leq m,\ 1 \leq j \leq n$ and $i \neq k$ are all treated as constants when evaluating this indefinite integral.

Other alternatives for the same expression of "H_1" can also be obtained from:

$$H_1 \;=\; \int M_{m+k}(z_1, z_2, \dots, z_m,\; x_1, x_2, \dots, x_n)\; \partial x_k \tag{1.073}$$

where in this case, z_i and x_j for $1 \leq i \leq m,\ 1 \leq j \leq n$ and $j \neq k$ are all treated as constants when evaluating this indefinite integral.

As for the expression of "H_2" defined by equation (1.056), it can be determined using the following integral :

$$H_2 \;=\; H_2(W_j) \;=\; \int M_{m+n+1}(W_j)\; \mathrm{d}W_j \tag{1.074}$$

because "$W_j \;=\; W_j(z_1, z_2, \dots, z_m,\; x_1, x_2, \dots, x_n)$" is a multivariate composite function.

The complete exact solution of the first order multivariate ODE defined by equation (1.054) that would be present inside a *Secondary Differential Expansion* is:

$$H_1(z_1, z_2, \dots, z_m,\; x_1, x_2, \dots, x_n) \;-\; H_2(W_j) \;=\; 0 \tag{1.075}$$

from which "W_j" can be obtained explicitly whenever possible.

Once the complete expression for each auxiliary variable is obtained, they can afterwards be substituted along with each of their initial condition(s) into the *Primary Expansion* for arriving at the required system of implicitly defined equations in the form of "$f_k(z_i, x_j) = 0$" for $1 \leq i \leq m$ and $1 \leq j \leq n$.

The initial condition(s) that belong to each auxiliary variable all take part in satisfying the initial condition(s) of a system implicitly defined equations that can be used for completely representing the exact or approximate analytical solution of a *system of PDEs*.

For inverting a *Multivariate Polynomial Transform* defined in much higher dimension follows the same type of logic as for the simple two dimensional case. The following example illustrates this in greater detail.

Example (1.3). Assuming the following *Secondary Differential Expansion* as a part of a large *Multivariate Polynomial Transform* that would correspond to some large system of implicitly defined equations involving several dependent variables and one single independent variable. Furthermore, for the sake of simplicity, let us assume that every first order ODE present in the *Secondary Differential Expansion* would satisfy the condition for exactness everywhere and thus readily integrable using the method described earlier.

<u>*Secondary Differential Expansion:*</u>

$$dx + 0 \cdot dy_1 + 0 \cdot dy_2 + 0 \cdot dy_3 = dW_1 \tag{1.076}$$

$$0 \cdot dx + dy_1 + 0 \cdot dy_2 + 0 \cdot dy_3 = dW_2 \tag{1.077}$$

$$0 \cdot dx + 0 \cdot dy_1 + dy_2 + 0 \cdot dy_3 = dW_3 \tag{1.078}$$

$$0 \cdot dx + 0 \cdot dy_1 + 0 \cdot dy_2 + dy_3 = dW_4 \tag{1.079}$$

$$W_1^2 dx + W_2^2 dy_1 + W_3^2 dy_2 + W_4^2 dy_3 = W_5^2 dW_5 \tag{1.080}$$

$$W_1 dx + W_2 dy_1 + W_3 dy_2 + 0 \cdot dy_3 = W_6 dW_6 \tag{1.081}$$

$$W_1 W_6^{-1} dx + (W_2 W_6^{-1} + 2W_3) dy_1 + (W_3 W_6^{-1} + 2W_2) dy_2 + 0 \cdot dy_3 =$$

$$= \frac{dW_7}{1 + W_7^2} \tag{1.082}$$

subjected to "$y_1(x_0)$", "$y_2(x_0)$" and "$y_3(x_0)$"

We will now determine its complete inverse where it is assumed that for the sake of simplicity each first order ODE present in the above *Secondary Differential Expansion* have already been factored out in order to filter out any unnecessary multivariate polynomials. These do not contribute in any manner on the overall integration process as they would tend to naturally cancel each other out by computation.

The first step is to naturally begin by integrating in ascending order of complexity each first order ODE present in the *Secondary Differential Expansion* for an expression of each auxiliary variable.

The results are:

For "W_1", $W_1(x) = x$ (1.083)

For "W_2", $W_2(y_1) = y_1$ (1.084)

For "W_3", $W_3(y_2) = y_2$ (1.085)

For "W_4", $W_4(y_3) = y_3$ (1.086)

For "$W_5(x, y_1, y_2, y_3)$", this is more involved.

Equation (1.080) can be rewritten as:

$$M_1 dx \; + \; M_2 dy_1 \; + \; M_3 dy_2 \; + \; M_4 dy_3 \;\; = \;\; M_5 dW_5 \tag{1.087}$$

where:

$$M_1 \; = \; W_1^2 \; = \; x^2 \tag{1.088}$$

$$M_2 \; = \; W_2^2 \; = \; y_1^2 \tag{1.089}$$

$$M_3 \; = \; W_3^2 \; = \; y_2^2 \tag{1.090}$$

$$M_4 \; = \; W_4^2 \; = \; y_3^2 \tag{1.091}$$

$$M_5 \; = \; W_5^2 \tag{1.092}$$

Since "$M_1 = M_1(x)$", "$M_2 = M_2(y_1)$", "$M_3 = M_3(y_2)$", and "$M_4 = M_4(y_3)$", our test for exactness using equation (1.066) through (1.070) reveals that:

$$\frac{\partial M_2}{\partial x} \; = \; \frac{\partial M_1}{\partial y_1} \; = \; 0 \tag{1.093}$$

$$\frac{\partial M_3}{\partial x} \; = \; \frac{\partial M_1}{\partial y_2} \; = \; 0 \tag{1.094}$$

$$\frac{\partial M_4}{\partial x} \; = \; \frac{\partial M_1}{\partial y_3} \; = \; 0 \tag{1.095}$$

$$\frac{\partial M_3}{\partial y_1} \; = \; \frac{\partial M_2}{\partial y_2} \; = \; 0 \tag{1.096}$$

$$\frac{\partial M_4}{\partial y_1} \; = \; \frac{\partial M_2}{\partial y_3} \; = \; 0 \tag{1.097}$$

$$\frac{\partial M_4}{\partial y_2} \; = \; \frac{\partial M_3}{\partial y_3} \; = \; 0 \tag{1.098}$$

so that equation (1.087) is an exact differential equation with solution:

$$\int (x^2 dx \; + \; y_1^2 dy_1 \; + \; y_2^2 dy_2 \; + \; y_3^2 dy_3) \;\; = \;\; \int W_5^2 dW_5 \tag{1.099}$$

or:

$$W_5 = \sqrt[3]{x^3 + y_1^3 + y_2^3 + y_3^3 + c_5} \tag{1.100}$$

For "$W_6(x, y_1, y_2)$", equation (1.081) can be rewritten as:

$$M_1 dx + M_2 dy_1 + M_3 dy_2 = M_4 dW_6 \tag{1.101}$$

where:

$$M_1 = W_1 = x \tag{1.102}$$

$$M_2 = W_2 = y_1 \tag{1.103}$$

$$M_3 = W_3 = y_2 \tag{1.104}$$

$$M_4 = W_6 \tag{1.105}$$

Since "$M_1 = M_1(x)$", "$M_2 = M_2(y_1)$" and "$M_3 = M_3(y_2)$", our test for exactness using equation (1.066) through (1.070) reveals that:

$$\frac{\partial M_2}{\partial x} = \frac{\partial M_1}{\partial y_1} = 0 \tag{1.106}$$

$$\frac{\partial M_3}{\partial x} = \frac{\partial M_1}{\partial y_2} = 0 \tag{1.107}$$

$$\frac{\partial M_3}{\partial y_1} = \frac{\partial M_2}{\partial y_2} = 0 \tag{1.108}$$

so that equation (1.101) is an exact differential equation with solution:

$$\int (x dx + y_1 dy_1 + y_2 dy_2) = \int W_6 dW_6 \tag{1.109}$$

or:

$$W_6 = \sqrt{x^2 + y_1^2 + y_2^2 + c_6} \tag{1.110}$$

For "$W_7(x, y_1, y_2)$", equation (1.082) can be rewritten as:

$$dH_1 = dH_2 \tag{1.111}$$

where:

$$dH_1 = M_1 dx + M_2 dy_1 + M_3 dy_2 \tag{1.112}$$

$$M_1 = W_1 W_6^{-1} \tag{1.113}$$

$$M_2 = W_2 W_6^{-1} + 2W_3 \tag{1.114}$$

$$M_3 = W_3 W_6^{-1} + 2W_2 \tag{1.115}$$

and:

$$dH_2 = M_4(W_7)\, dW_7 \tag{1.116}$$

$$M_4 = \frac{1}{1 + W_7^2} \tag{1.117}$$

It follows that:

$$\frac{\partial M_1}{\partial y_1} = \frac{\partial W_1}{\partial y_1} W_6^{-1} + W_1(-W_6^{-2})\frac{\partial W_6}{\partial y_1} = 0 - W_1 W_6^{-2}(W_2 W_6^{-1}) \tag{1.118}$$

$$= -W_1 W_2 W_6^{-3} \tag{1.119}$$

$$\frac{\partial M_1}{\partial y_2} = \frac{\partial W_1}{\partial y_2} W_6^{-1} + W_1(-W_6^{-2})\frac{\partial W_6}{\partial y_2} = 0 - W_1 W_6^{-2}(W_3 W_6^{-1}) \tag{1.120}$$

$$= -W_1 W_3 W_6^{-3} \tag{1.121}$$

$$\frac{\partial M_2}{\partial x} = \frac{\partial W_2}{\partial x} W_6^{-1} + W_2(-W_6^{-2})\frac{\partial W_6}{\partial x} + 2\frac{\partial W_3}{\partial x} = 0 - W_2 W_6^{-2}(W_1 W_6^{-1}) + 0 \tag{1.122}$$

$$= -W_1 W_2 W_6^{-3} \tag{1.123}$$

$$\frac{\partial M_2}{\partial y_2} = \frac{\partial W_2}{\partial y_2} W_6^{-1} + W_2(-W_6^{-2})\frac{\partial W_6}{\partial y_2} + 2\frac{\partial W_3}{\partial y_2} = 0 - W_2 W_6^{-2}(W_3 W_6^{-1}) + 2 \tag{1.124}$$

$$= -W_2 W_3 W_6^{-3} + 2 \tag{1.125}$$

$$\frac{\partial M_3}{\partial x} = \frac{\partial W_3}{\partial x} W_6^{-1} + W_3(-W_6^{-2})\frac{\partial W_6}{\partial x} + 2\frac{\partial W_2}{\partial x} = 0 - W_3 W_6^{-2}(W_1 W_6^{-1}) + 0 \tag{1.126}$$

$$= -W_1 W_3 W_6^{-3} \tag{1.127}$$

$$\frac{\partial M_3}{\partial y_1} = \frac{\partial W_3}{\partial y_1} W_6^{-1} + W_3(-W_6^{-2}) \frac{\partial W_6}{\partial y_1} + 2\frac{\partial W_2}{\partial y_1} = 0 - W_3 W_6^{-2}(W_2 W_6^{-1}) + 2 \quad (1.128)$$

$$= -W_2 W_3 W_6^{-3} + 2 \quad (1.129)$$

Our test for exactness using equation (1.066) and (1.070) reveals that:

$$\frac{\partial M_2}{\partial x} = \frac{\partial M_1}{\partial y_1} = -W_1 W_2 W_6^{-3} \quad (1.130)$$

$$\frac{\partial M_3}{\partial x} = \frac{\partial M_1}{\partial y_2} = -W_1 W_3 W_6^{-3} \quad (1.131)$$

$$\frac{\partial M_3}{\partial y_1} = \frac{\partial M_2}{\partial y_2} = -W_2 W_3 W_6^{-3} + 2 \quad (1.132)$$

Furthermore:

$$M_4 = M_4(W_7) \quad (1.133)$$

so that equation (1.112) is an exact differential equation with solution:

$$\int dH_1 = \int M_1 \, \partial x = \int \frac{W_1}{W_6} \partial x \quad (1.134)$$

$$= \int \frac{x}{\sqrt{x^2 + y_1^2 + y_2^2 + C_6}} \, \partial x \quad (1.135)$$

Solving for "H_1 ":

$$H_1 = \sqrt{x^2 + y_1^2 + y_2^2 + C_6} + f_1(y_1, y_2) \quad (1.136)$$

$$= W_6 + f_1(y_1, y_2) \quad (1.137)$$

We can also define as a second alternative for "H_1" the following integral equation:

$$\int dH_1 = \int M_2 \, \partial y_1 = \int \left(\frac{W_2}{W_6} + 2W_3\right) \partial y_1 \quad (1.138)$$

$$= \int \left(\frac{y_1}{\sqrt{x^2 + y_1^2 + y_2^2 + C_6}} + 2y_2\right) \partial y_1 \quad (1.139)$$

so that:

$$H_1 = \sqrt{x^2 + y_1^2 + y_2^2 + C_6} + 2y_1y_2 + f_2(x, y_2) \tag{1.140}$$

$$= W_6 + 2W_2W_3 + f_2(x, y_2) \tag{1.141}$$

A third alternative for "H_1" can be derived from:

$$\int dH_1 = \int M_3\,\partial y_2 = \int \left(\frac{W_3}{W_6} + 2W_2\right)\partial y_2 \tag{1.142}$$

$$= \int \left(\frac{y_2}{\sqrt{x^2 + y_1^2 + y_2^2 + C_6}} + 2y_1\right)\partial y_2 \tag{1.143}$$

so that:

$$H_1 = \sqrt{x^2 + y_1^2 + y_2^2 + C_6} + 2y_1y_2 + f_3(x, y_2) \tag{1.144}$$

$$= W_6 + 2W_2W_3 + f_3(x, y_1) \tag{1.145}$$

From equation (1.140) and (1.144) we arrive at the conclusion that:

$$f_2(x, y_2) = f_3(x, y_1) \tag{1.146}$$

The only condition for this equation to be satisfied is of course when:

$$f_2 = f_3 = F(x) \tag{1.147}$$

because "$y_1 \neq y_2$".

Substituting equation (1.147) into equation (1.145), we obtain:

$$H_1 = W_6 + 2W_2W_3 + F(x) \tag{1.148}$$

Since "$f_1(y_1, y_2)$" in equation (1.137) is not a function of "x" then it is safe to assume in equation (1.147) that:

$$F(x) = 0 \tag{1.149}$$

Substituting the expression for "W_2", "W_3", "W_6" and "$F(x)$" into equation (1.148), the expression for "H_1" can now be completely defined as:

$$H_1 \;=\; W_6 \;+\; 2y_1y_2 \;+\; 0 \tag{1.150}$$

$$= \;\sqrt{x^2 + y_1^2 + \; y_2^2 \;+\; C_6} \;\;+\; 2y_1y_2 \tag{1.151}$$

The expression for "H_2" can be determined by integrating equation (1.116) using equation (1.117):

$$H_2 \;=\; \int M_4 dW_7 \;=\; \int \frac{dW_7}{1 \;+\; W_7^2} \;=\; \tan^{-1}(W_7) \;+\; K \tag{1.152}$$

Since from equation (1.111) "$H_1 = H_2$" we thus arrive at the following complete expression for "W_7":

$$\sqrt{x^2 + y_1^2 + \; y_2^2 \;+\; C_6} \;\;+\; 2y_1y_2 \;=\; \tan^{-1}(W_7) \;+\; K \tag{1.153}$$

or:

$$W_7 \;=\; \tan\left(\sqrt{x^2 + y_1^2 \; + y_2^2 \; + \; c_6} \;\;+\; 2y_1y_2 \;+\; c_7\right) \tag{1.154}$$

The complete inverse of the *Multivariate Polynomial Transform* whose *Secondary Differential Expansion* is defined by equation (1.076) through (1.082) is obtained by substituting each of the expression for the auxiliary variables "$W_1(x)$", "$W_2(y_1)$", "$W_3(y_2)$", "$W_4(y_3)$", "$W_5(x, y_1, y_2, y_3)$", "$W_6(x, y_1, y_2)$" and "$W_7(x, y_1, y_2)$" into a *Primary Expansion* that could be described in the following general form:

$$F_k\left(W_j\right) \;=\; 0 \qquad (1 \le k \le 3)\;\; (1 \le j \le 7) \tag{1.155}$$

2. Complete analytical theory of integration under one universal system of computational logic

The universal representation of all mathematical equations presented in the differential expansion form described by equation (1.002) through (1.006) should really be referred to as a general *initially assumed Multivariate Polynomial Transform* (IAMPT) when it comes to solving for DEs and systems of DEs. The only difference between traditional methods of series expansion and the one presented here, is that ours can succeed in arriving at complete *exact* analytical solution to any type of DEs and systems of DEs. All other known traditional methods of series solutions are incapacitated right from the beginning for arriving at *exact* analytical solutions since they were originally meant only to be utilized as part of some functional approximation theory. This being the direct consequence for all tradition methods of series solutions for not having originated from the application of some form of a unified theory of integration.

For those functional expressions that are present inside a DE or a system of DEs, they somehow would have to be totally accounted for in our initially assumed *Multivariate Polynomial Transform*. This is made possible only if we append at the end of our initially assumed expansion the *Multivariate Polynomial Transform* of each functional expression by introducing additional new supplemental auxiliary variables. Each of these additional auxiliary variables in turn are most likely to reappear in the final analytical solution of the DE or system of DEs. This would thus providing us with a real sense of measure in the manner by which such individual functional expressions can succeed in influencing the complete behavior of a physical system.

For including these types of DEs and systems of DEs, our general initially assumed *Multivariate Polynomial Transform* would have to be modified accordingly as follow:

(1). <u>*Primary Expansion:*</u>

$$F_i(W_j) \;=\; 0 \;=\; \sum_r a_{i,r}\left(\prod_j^{p+q} W_j^{E_{i,s}}\right) \quad (1 \le i \;\le k) \tag{2.01}$$

where "W_j" are auxiliary variables, "q" is the total number of auxiliary variables required for defining the *Multivariate Polynomial Transform* of each functional expression that is present in a DE or a system of DEs. The total number of auxiliary variables now grows from "p" to "$p+q$" when functional expressions are present in these types of DEs. Each of the "p" number of auxiliary variables are always assumed raised to some floating point value and finally, "r" is the total number of terms present in each of the "k" number of implicitly defined multivariate polynomial equations.

(2). <u>*Secondary Differential Expansion:*</u>

$$dz_i \;=\; dW_i \qquad (1 \le i \le m) \tag{2.02}$$

$$dx_i \;=\; dW_{m+i} \qquad (1 \le i \le n) \tag{2.03}$$

$$\sum_{t=1}^{m} N_{i(m+n+1)-m-n-1+t}\,dz_t \;+\; \sum_{t=1}^{n} N_{i(m+n+1)-n-1+t}\,dx_t \;=$$

$$=\; N_{i(m+n+1)}\,dW_j \qquad [1 \le i \le p-m-n] \;\; [m+n+1 \le j \le p] \tag{2.04}$$

$$N_c(W_j) \;=\; \sum_{t=(c-1)r+1}^{cr} b_{c,t}\left(\prod_j^{p+q} W_j^{E'_{c,s}}\right) \tag{2.05}$$

$$[1 \le c \;\le i(m+n+1)] \;\; [1 \le i \le p-m-n]$$

$$\sum_{t=1}^{m} T_{i(m+n+1)-m-n-1+t} dz_t \;+\; \sum_{t=1}^{n} T_{i(m+n+1)-n-1+t} dx_t \;=$$

$$= \; T_{i(m+n+1)} dW_j \qquad [1 \le i \le q] \;\; [p \le j \le p+q] \tag{2.06}$$

where "$T_g(W_j)$" are the special multivariate polynomials that would be reserved exclusively for only representing those functional expressions that would be present inside a DE or system of DEs.

Just as we can represent any mathematical equation in universal differential form, we can also express and type of DE and system of DEs also in complete universal differential form.

The *Primary Expansion* representation for the following general system of DEs:

$$g_k\left(\; z_i, \; x_j, \; \frac{\partial}{\partial \mathrm{x}_u}\left(\frac{\partial z_r}{\partial x_v}\right) \; \right) \;=\; 0 \tag{2.07}$$

can be defined as follow:

$$G_k\left(W_t, \; \frac{P_{ruv}}{Q_{ruv}}\right) \;=\; 0 \qquad\qquad [1 \le t \le m+n+q] \tag{2.08}$$

where:

$$\frac{P_{ruv}}{Q_{ruv}} \;=\; \frac{\partial}{\partial \mathrm{x}_u}\left(\frac{\partial z_r}{\partial x_v}\right) \tag{2.09}$$

As for the *Secondary Differential Expansion* representation, it becomes exactly identical to the one present inside the initially assumed *Multivariate Polynomial Transform* that would have been selected for solving the general system of DEs.

Example (2.1). The following system of second order ODEs is used to describe the motion of a dumbbell of length "L" in space consisting of masses "m_1" and "m_2" both rigidly attached at its extremities and free to rotate under the influence of gravity:

$$(m_1 \;+\; m_2)\frac{d^2 x_1}{dt^2} \;-\; m_2 L \frac{d^2\theta}{dt^2}\sin(\theta) \;-\; m_2\, L\left(\frac{d\theta}{dt}\right)^2 \cos(\theta) \;=\; 0 \tag{2.10}$$

$$(m_1 \;+\; m_2)\frac{d^2 y_1}{dt^2} \;-\; m_2 L \frac{d^2\theta}{dt^2}\cos(\theta) \;-\; m_2\, L\left(\frac{d\theta}{dt}\right)^2 \sin(\theta) \;=\; -(m_1 \;+\; m_2)g \tag{2.11}$$

$$L\frac{d^2\theta}{dt^2} \;-\; \frac{d^2 x_1}{dt^2}\sin(\theta) \;+\; \frac{d^2 y_1}{dt^2}\cos(\theta) \;=\; -g\cos(\theta) \tag{2.12}$$

For this system of equations, "x_i" and "y_i" represent the horizontal and vertical linear displacements of mass "m_i" respectively and "θ" is the angle of rotation of the dumbbell with respect to the X-axis. We will assume for the sake of simplicity that the mass of the rod is negligible compared to mass "m_1" and "m_2".

The complete *Multivariate Polynomial Transform* of the system of second order ODEs will now be determine.

For the sake of simplicity, we will need to express the *Sine* and *Cosine* function as a rational combination of the *Tangent* function by selecting:

$$h_1 = \tan(\theta/2) \tag{2.13}$$

so that:

$$\sin\theta = \frac{2\tan(\theta/2)}{1 + \tan^2(\theta/2)} = \frac{2h_1}{1 + h_1^2} \tag{2.14}$$

and

$$\cos\theta = \frac{1 - \tan^2(\theta/2)}{1 + \tan^2(\theta/2)} = \frac{1 - h_1^2}{1 + h_1^2} \tag{2.15}$$

We can arbitrarily select each auxiliary variable as:

$$W_1 = x_1 \tag{2.16}$$

$$W_2 = y_1 \tag{2.17}$$

$$W_3 = \theta \tag{2.18}$$

$$W_4 = \tan\left(\frac{\theta}{2}\right) \tag{2.19}$$

The *Multivariate Polynomial Transform* of the single *external input* "h_1" as defined by equation (2.13) is:

(1). <u>*Primary Expansion:*</u>

$$H_1 = W_4 \tag{2.20}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$0 \cdot dx_1 + 0 \cdot dy_1 + (1 + W_4^2)d\theta = 2\,dW_4 \tag{2.21}$$

Using our standard notation in equation (2.08) and (2.09), we can now define the complete *Multivariate Polynomial Transform* of this entire system of second order ODEs starting with the *Primary Expansion* as:

(1). *Primary Expansion:*

$$G_1 = (m_1 + m_2)\left(\frac{P_{21}}{Q_{21}}\right) - m_2 L\left(\frac{P_{23}}{Q_{23}}\right)\left(\frac{2W_4}{1 + W_4^2}\right) - $$

$$- m_2 L\left(\frac{P_{13}}{Q_{13}}\right)^2\left(\frac{1 - W_4^2}{1 + W_4^2}\right) = 0 \qquad (2.22)$$

$$G_2 = (m_1 + m_2)\left(\frac{P_{22}}{Q_{22}}\right) - m_2 L\left(\frac{P_{23}}{Q_{23}}\right)\left(\frac{1 - W_4^2}{1 + W_4^2}\right) - m_2 L\left(\frac{P_{13}}{Q_{13}}\right)^2\left(\frac{2W_4}{1 + W_4^2}\right)$$

$$+ (m_1 + m_2)g = 0 \qquad (2.23)$$

$$G_3 = L\left(\frac{P_{23}}{Q_{23}}\right) - \left(\frac{P_{21}}{Q_{21}}\right)\left(\frac{2W_4}{1 + W_4^2}\right) + \left(\frac{P_{22}}{Q_{22}}\right)\left(\frac{1 - W_4^2}{1 + W_4^2}\right) + g\left(\frac{1 - W_4^2}{1 + W_4^2}\right) = 0 \qquad (2.24)$$

Where:

$$\frac{P_{n1}}{Q_{n1}} = \frac{d^n x_1}{dt^n} \qquad (2.25)$$

$$\frac{P_{n2}}{Q_{n2}} = \frac{d^n y_1}{dt^n} \qquad (2.26)$$

$$\frac{P_{n3}}{Q_{n3}} = \frac{d^n \theta}{dt^n} \qquad (2.27)$$

The complete *Secondary Differential Expansion* of this system of second order ODEs is the combination of the *Secondary Differential Expansion* of "h_1" as defined by equation (2.21) and the same one present inside the initially assumed *Multivariate Polynomial Transform* that would have been selected for solving this particular system of second order ODEs.

Example (2.2). For the following system of *second order* PDEs,

$$z_2 \frac{\partial z_1}{\partial x_1} + \sin(2x_2) \frac{\partial}{\partial x_1}\left(\frac{\partial z_2}{\partial x_2}\right) + x_1 x_2 = 0 \tag{2.28}$$

$$\left(\frac{\partial^2 z_1}{\partial x_2^2}\right)\left(\frac{\partial^2 z_2}{\partial x_1^2}\right) + \left(\frac{\partial z_1}{\partial x_1}\right)^2 + \left(\frac{\partial z_2}{\partial x_2}\right)^2 + x_1^2 + x_2^2 = 6 z_1 e^{x_1} \tag{2.29}$$

we can define each external input as:

$$h_1 = z_1 \tag{2.30}$$

$$h_2 = z_2 \tag{2.31}$$

$$h_3 = x_1 \tag{2.32}$$

$$h_4 = x_2 \tag{2.33}$$

$$h_5 = \sin(2x_2) \tag{2.34}$$

$$h_6 = 6e^{x_1} \tag{2.35}$$

We can also select each auxiliary variable as:

$$W_1 = z_1 \tag{2.36}$$

$$W_2 = z_2 \tag{2.37}$$

$$W_3 = x_1 \tag{2.38}$$

$$W_4 = x_2 \tag{2.39}$$

$$W_5 = \tan(x_2) \tag{2.40}$$

$$W_6 = e^{x_1} \tag{2.41}$$

The *Multivariate Polynomial Transform* of the first external input "h_1" is:

(1). <u>*Primary Expansion:*</u>

$$H_1 = W_1 \tag{2.42}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$dz_1 + 0 \cdot dz_2 + 0 \cdot dx_1 + 0 \cdot dx_2 = dW_1 \tag{2.43}$$

The *Multivariate Polynomial Transform* of the second external input "h_2" is:

(1). <u>*Primary Expansion:*</u>

$$H_2 \;=\; W_2 \tag{2.44}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$0 \cdot dz_1 \;+\; dz_2 \;+\; 0 \cdot dx_1 \;+\; 0 \cdot dx_2 \;=\; dW_2 \tag{2.45}$$

The *Multivariate Polynomial Transform* of the third external input "h_3" is:

(1). <u>*Primary Expansion:*</u>

$$H_3 \;=\; W_3 \tag{2.46}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$0 \cdot dz_1 \;+\; 0 \cdot dz_2 \;+\; dx_1 \;+\; 0 \cdot dx_2 \;=\; dW_3 \tag{2.47}$$

The *Multivariate Polynomial Transform* of the fourth external input "h_4" is:

(1). <u>*Primary Expansion:*</u>

$$H_4 \;=\; W_4 \tag{2.48}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$0 \cdot dz_1 \;+\; 0 \cdot dz_2 \;+\; 0 \cdot dx_1 \;+\; dx_2 \;=\; dW_4 \tag{2.49}$$

The *Multivariate Polynomial Transform* of the fifth external input "h_5" is:

(1). <u>*Primary Expansion:*</u>

$$H_5 \;=\; \frac{2W_5}{1 \;+\; W_5^2} \tag{2.50}$$

(2). <u>*Secondary Differential Expansion:*</u>

$$0 \cdot dz_1 \;+\; 0 \cdot dz_2 \;+\; 0 \cdot dx_1 \;+\; (1 + W_5^2)dx_2 \;=\; dW_5 \tag{2.51}$$

The *Multivariate Polynomial Transform* of the sixth external input "h_6" is:

(1). *Primary Expansion:*

$$H_6 \;=\; 6W_6 \tag{2.52}$$

(2). *Secondary Differential Expansion:*

$$0 \cdot dz_1 \;+\; 0 \cdot dz_2 \;+\; W_6 dx_1 \;+\; 0 \cdot dx_2 \;=\; dW_6 \tag{2.53}$$

Using the notation defined in equation (2.08) and (2.09), the complete *Multivariate Polynomial Transform* of the entire system of second order PDEs may now be completely defined as:

(1). *Primary Expansion:*

$$G_1 \;=\; H_2\left(\frac{P_{110}}{Q_{110}}\right) \;+\; H_5\left(\frac{P_{212}}{Q_{212}}\right) \;+\; H_3 H_4 \;=\; 0 \tag{2.54}$$

$$G_2 \;=\; \left(\frac{P_{122}}{Q_{122}}\right)\left(\frac{P_{211}}{Q_{211}}\right) \;+\; \left(\frac{P_{110}}{Q_{110}}\right)^2 \;+\; \left(\frac{P_{201}}{Q_{201}}\right)^2 \;+\; H_3^2 \;+\; H_4^2 \;-\; H_1 H_6 \;=\; 0 \tag{2.55}$$

The complete *Secondary Differential Expansion* of this system of second order PDEs is the combination of the *Secondary Differential Expansion* of "h_1" through "h_6" and the same one present inside an initially assumed *Multivariate Polynomial Transform* that would have been selected for solving this particular system of second order PDEs.

By substituting an initially assumed *Multivariate Polynomial Transform* into any type of DE or system of DEs would always result into defining a complete system of *Nonlinear Simultaneous Equations* to solve for. Each *exact numerical* solution set obtained will always define a complete *exact analytical* solution of the DE or system of DEs by *inverting* the corresponding initially assumed *Multivariate Polynomial Transform.* This is provided of course that each of the first order ODEs present inside the *Secondary Differential Expansion* have all been determined as being exact differentials and therefore always completely integrable.

Some of the *unknown coefficients* present inside an initially assumed *Multivariate Polynomial Transform* would be reserved exclusively for defining all the basis function that are to be present inside the *analytical* solution of a DE or a system of DEs. Others would be mainly responsible for assuring that the boundary conditions of the DE or system of DEs would be completely satisfied.

As a consequence of the fundamental laws of algebra, a completely differentiable mathematical equation as well as its many equivalent differential form representation in terms of a *Multivariate Polynomial Transform* can always appear in various disguise form. That is, any mathematical equation as well as its equivalent differential form representation can always have many alternative equivalent representations. However, to an observer each may appear quite distinct from one another and yet are completely identical with each other purely from a computational point of view.

Such a unique mathematical property about equations in general would guarantee that there will always be an infinite number of *numerical solution* sets of the *Nonlinear Simultaneous Equations* corresponding to a DE or a system of DEs. As a result of this, we acquire the ability of being able to select among an infinite number of *numerical solution* sets obtained only those that would *translate* into defining much simpler *Secondary Differential Expansion* to integrate. This would have the effect of significantly facilitating the entire integration process involved in the *Secondary Differential Expansion* when attempting to invert an initially assumed *Multivariate Polynomial Transform* for acquiring an exact analytical solution to a DE or a system of DEs.

No analytical method of integration has ever been devised in the history of Calculus that could offer us with this much flexibility for selecting out of an *infinite* number of integrals only those that are considered more friendly to evaluate then others while in the process of attempting to solve for a DE or a system of DEs. Other well known traditional methods of analytical integration have shown weaknesses in that area mainly as a result of some major integrability issues due to a very restricted number of integrals that could be resolved in the end while leaving behind a vast majority of them as completely unsolved.

When the *Nonlinear Simultaneous Equations* cannot be solved in terms of an *exact numerical solution set*, this in turn would indicate that the *exact analytical solution* of the DE or system of DEs in question cannot be resolved as some exact combination of algebraic and elementary basis functions whether explicitly or implicitly defined. It is then always possible to establish some form of a measure on the degree of accuracy that a particular *numerical solution set* can satisfy a system of *Nonlinear Simultaneous Equations* by using various well known methods of optimization techniques. This in turn would provide us with some real measure of accuracy on how well the resultant *analytical solution* obtained can satisfy the DE or system of DEs. Of course only when an <u>*exact numerical solution set*</u> of the *Nonlinear Simultaneous Equations* has been found then this would automatically indicate that the DE or system of DEs in question can be completely resolved in terms of an <u>*exact analytical solution*</u>. All of this is provided of course that each first order ODE present in the *Secondary Differential Expansion* are determined to be exact and thus always completely integrable.

The simplicity in appearance for the analytical solution of a particular DE or a system of DEs is very crucial towards a complete understanding of a physical system so that only those appearing in its simplest form would be of greatest interest to the physical science. If we were to apply this very general principle directly into the world of physics under the new proposed unified theory of integration, then Albert Einstein's assertion that "*God does not play with dices*" could certainly be put to the real test with potential major historical implications !

3. A universal method of proof for the quadratic equation and the superposition theorem

As a direct consequence of having established a unified theory of integration, a universal method of proof can be devised for proving a variety of classical theorems that were once proven under old traditional methods of pure mathematical logic. Only those theorems that can be formulated through some form of a DE or a system of DEs would be included.

The simple quadratic formula would fall into such category of theorems since it can always be reformulated computationally using a method that is based entirely on the use of successive partial differentiation. In this case, the unique computational method of proof for the quadratic equation begins by first computing the various partial derivatives of an initially assumed *Multivariate Polynomial Transform* that has been selected solely on the basis of representing only the class of *multivariate mathematical equations* that are defined in <u>*explicit*</u> form only.

This would correspond to the case for "$k = m = 1$" in equation (1.002) through (1.006) such that instead of assuming a *Primary Expansion* in the form of "$f(z_i, x_j) = 0$", we would instead assume an *explicit version* in the form of "$z = z(x_j)$" as being a ratio of two general multivariate polynomials. Note that since an explicitly defined equation is just a special case of an implicitly defined equation, we could have selected the original implicit form representation in the *Primary Expansion* and still arrive at an explicitly defined analytical solution in the end.

(1). *Primary Expansion:*

$$z(W_j) \;=\; \frac{P(W_j)}{Q(W_j)} \qquad\qquad (1 \le j \le p) \tag{3.01}$$

where "P" and "Q" are each multivariate polynomials each consisting of a total number of "p" auxiliary variables each of which are raised to some floating point value.

(2). *Secondary Differential Expansion:*

$$dz \;=\; dW_1 \tag{3.02}$$

$$dx_i \;=\; dW_{i+1} \qquad\qquad (1 \le i \le n) \tag{3.03}$$

$$\sum_{t=1}^{1} N_{i(n+2)-n-2+t}\,dz \;+\; \sum_{t=1}^{n} N_{i(n+2)-n-1+t}\,dx_t \;=$$

$$=\; N_{i(n+2)}\,dW_j \qquad [1 \le i \le p-1-n]\;\; [n+2 \le j \le p] \tag{3.04}$$

$$N_c(W_j) \;=\; \sum_{t=(c-1)r+1}^{cr} b_{c,t}\left(\prod_{j}^{p} W_j^{E'_{c,s}}\right) \tag{3.05}$$

$$[1 \le c \;\le i(n+2)]\;\; [1 \le i \le p-1-n]$$

The computed values for the various partial derivatives of "$Z = Z(W_j)$" would then be equated with the various partial derivatives that are calculated based entirely on a very unique change of variables involving the coefficients and the root of the quadratic equation.

This unique change of variable would include the root of the quadratic equation “r = r(A,B,C)" that would be regarded as the *dependen*t variable while the coefficients A, B and C would be defined as the *independent* variables. This would correspond to "$m = 1$" and "$n = 3$" in the above differential expansion form representation. We would setup our complete system of *Nonlinear Simultaneous Equations* to solve for by simply equating the various partial derivatives of "r(A,B,C)" with respect to each of the coefficient A, B and C with the various partial derivatives of our initially assumed *Multivariate Polynomial Transform* that was setup to only represent all multivariate mathematical equations defined in explicit form only. We can also apply the same logic for determining the root formulas corresponding to *higher degree* polynomials.

By restricting our initially assumed *Multivariate Polynomial Transfor*m to represent all mathematical equations in explicit form, this will guarantee the presence of *exact* numerical solution sets corresponding to the *Nonlinear Simultaneous Equations* to solve for. Each of these exact numerical solution sets obtained would lead towards the formation of many complete snapshots of the actual general formula such that by some very special algebraic manipulations, will enable confirmation of its very unique existence. The type of algebraic manipulation involved that is to be conducted will be referred to in the following section as being a special type of mathematical interpolation.

Another and far more interesting example mainly for the physical sciences is arriving at the famous *superposition theorem* by once again beginning with an initially assuming *Multivariate Polynomial Transform.* This time we would be selecting our differential expansion strictly in terms of representing all *univariate* mathematical equations defined in *explicit* form only.

This would correspond to the case for "k = m = n = 1" in equation (1.002) through (1.006) such that instead of assuming a *Primary Expansion* in the form of "$f(z_i, x_j) = 0$", we would instead assume the explicit version of "$y = y(x)$" as being a ratio of two general multivariate polynomials:

(1). *Primary Expansion:*

$$y(W_j) = \frac{P(W_j)}{Q(W_j)} \qquad (1 \le j \le p) \tag{3.06}$$

where "P" and "Q" are each multivariate polynomials each consisting of a total number of "p" auxiliary variables each of which are raised to some floating point value.

(2). *Secondary Differential Expansion:*

$$dx = dW_1 \tag{3.07}$$

$$dy = dW_2 \tag{3.08}$$

$$N_{3i-2}dx + N_{3i-1}dy = N_{3i}dW_j \qquad [1 \le i \le p-2] \; [3 \le j \le p] \tag{3.09}$$

$$N_c(W_j) = \sum_{t=(c-1)r+1}^{cr} b_{c,t} \left(\prod_{j}^{p} W_j^{E'_{c,s}} \right) \tag{3.10}$$

$$[1 \le c \le 3i)] \; [1 \le i \le p-2]$$

We would define the *Nonlinear Simultaneous Equations* to solve for by substituting the above generally assumed *Multivariate Polynomial Transform* into the following general class of *second order* ODEs.

$$h_1(x)\frac{d^2y}{dx^2} + h_2(x)\frac{dy}{dx} + h_3(x)y = h_4(x) \tag{3.11}$$

Next, we would be performing a very complete and detailed analysis on all computational results obtained by solving for the corresponding *Nonlinear Simultaneous Equations.*

The generalized form of this second order ODE would have been selected purely on the basis of its reoccurrence in describing various types of linear mechanical and electrical models.

As in the case of the quadratic equation, by restricting our initially assumed *Multivariate Polynomial Transform* to represent all mathematical equations in *explicit form* only, the presence of exact numerical solution sets corresponding to the *Nonlinear Simultaneous Equations* to solve for will confirm the unique explicit nature of the superposition theorem. It is with some very special type of algebraic manipulations to be discussed in the next section that we will succeed in identifying a number of *subclasses* of ODEs by which the general explicitly defined analytical solution obtained would be applicable to. By method of conjecture this would eventually lead us directly towards a purely *computational* proof of the famous *superposition theorem* thereby completely bypassing all forms of non-computationally based mathematical methods of analysis !

4. A new form of mathematical interpolation as a means of establishing a main pathway by which a unified theory of physics may be obtained

Only from the relentless application of the new unified theory of integration on a very large scale over a substantial class of DEs and systems of DEs can we expect to begin slowly unravelling many potentially new and yet undiscovered theorems similar to the *superposition theorem.* It is only from the long term cumulative effect of gathering a large collection of such universal theorems that can only lead towards the development of some unified theory of physics. This would be the result of having meticulously consolidate each of the most fundamental theorems ever discovered into one gigantic universal theory of physics.

All existing experimentally based methods of physics could never succeed in achieving such a monumental objective for the physical sciences. That is because during the process of gathering the physical data there would be a severe loss of continuity that only mathematical equations are capable of maintaining throughout.

There are of course more advanced examples that can be selected other than the ones involving the computational proof of the quadratic equation and the superposition theorem especially from someone with a remarkable understanding of mathematics and the physical sciences. But no matter what example in whatever subject matter anyone decides to choose from, the bottom line is that *by following a very unique brand of mathematical ideology such as the one being proposed in this article, the new unified theory of integration will always computationally arrive at the same mathematical equations that all traditional methods have succeeded in arriving at in the past.* By doing so, this would undoubtedly provide just the ideal fertile testing ground for any real future software development related to the unified theory of integration.

It is expected that we would be following an extremely long computational trajectory for achieving in some cases the same exact results as with traditional methods of analysis. However, it should be very obvious to everyone of the enormous potential benefits involved especially on a long term basis.

"Our unique computational approach will always certainly succeed in solving those "<u>other</u>" problems by which classical methods of analysis have completely failed as a result of not having provided an adequate solution to certain key DEs or systems of DEs" **.**

So in order to take full advantage of what the unified theory of integration can offer to everyone, it must be implemented in a very methodological manner. That is, each DE and system of DEs that is being solved for must absolutely undergo a very thorough examination in terms of determining the best analytical solution that can be extracted from the relentless numerical application of the initially assumed *Multivariate Polynomial Transform* described in equation (1.002) through (1.006). All boundary conditions related to the DE or system of DEs must also become included as part of this gigantic computational process.

We would therefore need to construct some form of a very unique presentation by which a very special type of mathematical database would have to be created for storing all empirical results obtained. This would then be entirely converted in the form of pure mathematical equations. Beyond this computational stage, much further scrutiny would then be necessary for potentially recognizing certain key fundamental theorems that over time would eventually contribute towards the complete development of some unified theory of physics.

The exact nature of such a presentation that would be applicable for solving *all types* of DEs and systems of DEs under the new unified theory of integration can be described through the following general mathematical template.

$g\left(x, y, \frac{dy}{dx}\right) = 0$		
Initial Condition	*Coefficient Values*	*Exact analytical solution obtained using the Multivariate Polynomial Transform method*
x_0, y_0	$a_0, b_0, c_0, \ldots$	$U_1(x, y) = 0$
x_0, y_0	$a_1, b_0, c_0, \ldots$	$U_2(x, y) = 0$
x_1, y_1	$a_0, b_2, c_2, \ldots$	$U_3(x, y) = 0$
x_1, y_1	$a_3, b_0, c_0, \ldots$	$U_4(x, y) = 0$
x_2, y_2	$a_4, b_3, c_2, \ldots$	$U_5(x, y) = 0$
. . .	. . .	. . .
. . .	. . .	. . .
. . .	. . .	. . .

Table 4.1

This tailored designed template was produced to accommodate only first order ODEs. However due to the universality nature of the fundamental logic behind introducing such a new type of template in mathematics, it can easily be modified to accommodate other far more complex types of DEs and systems of DEs.

In the following example, we have included a very simple *live* demonstration by which the proposed unified theory of integration would succeed in resolving a randomly selected "*general*" first order ODE uniquely in terms of a complete "*general*" analytical solution.

Only by following this example very closely would it become very apparent that our unique mathematical template has succeeded in developing a more *generalized* approach for arriving at *general analytical solutions to any type of DEs and systems of DEs*. This would certainly go a long way towards uncovering the many well hidden potential mathematical theorems that lay very deep beneath many unresolved DEs and systems of DEs.

"It is only by being in complete possession of a very large collection of powerful mathematical theorems that can succeed in carving a whole new pathway by which a unified theory of physics can eventually be uncovered. "

Example (4.1). Starting with the following *general* first order ODE,

$$x\frac{dy}{dx} + ay + bx^n y^2 = 0 \tag{4.01}$$

we can begin by constructing the following table:

$x\frac{dy}{dx} + ay + bx^n y^2 = 0$		
Initial Condition	*Coefficient Values*	*Exact analytical solution obtained using the Multivariate Polynomial Transform method*
$x_0 = 1$ $y_0 = 1$	$a = 1.0$ $b = 1.0$ $n = -1.0$	$(-3x + x^{-1})y + 2 = 0$
$x_0 = 1$ $y_0 = 2$	$a = 1.2$ $b = -1.0$ $n = 2.0$	$(1.4x^{1.2} - x^2)y - 0.80 = 0$
$x_0 = 1$ $y_0 = -1$	$a = 1.2$ $b = 1.5$ $n = -2.0$	$(1.7x^{1.2} + 1.5^{-2})y + 3.2 = 0$
$x_0 = 1$ $y_0 = 2$	$a = 2.0$ $b = -1.0$ $n = 2.0$	$x^2 y(0.5 - \ln(x)) - 1 = 0$
$x_0 = 1$ $y_0 = -2$	$a = 1.5$ $b = 2.0$ $n = 3.0$	$(-2.75x^{1.5} + 2x^3)y - 1.5 = 0$
$x_0 = 1$ $y_0 = 1$	$a = 1.0$ $b = 1.0$ $n = 1.0$	$xy(1 + \ln(x)) - 1.0 = 0$
$x_0 = 1$ $y_0 = -1$	$a = -1.0$ $b = 1.5$ $n = -1.0$	$x^{-1}y(-1 + 1.5\ln(x)) - 1.0 = 0$

Table 4.2

The evidence gathered purely by observation from this table suggests by *conjecture* that:

$$f_1(x,y) = \ 0 \ = \ (Ax^B \ + \ Cx^D)y \ + \ E \tag{4.02}$$

and:

$$f_2(x,y) = \ 0 \ = \ x^A y(B \ + \ C\ln(x)) \ + \ D \tag{4.03}$$

both appear to be perfect candidates for the general exact solution of the ODE where the coefficients "A", "B", "C", "D" and "E" are to be expressed in terms of the coefficients "a", "b", "n" and the initial condition of the ODE.

For the first expression defined by "$f_1(x,y) = 0$", we substitute this equation into the ODE and equate like terms to zero.

The first derivative of equation (4.02) is defined as:

$$\frac{dy}{dx} \ = \ -\frac{\partial f_1}{\partial x}\Big/\frac{\partial f_1}{\partial y} \ = \ \frac{-y(ABx^{B-1} \ + \ CDx^{D-1})}{Ax^B \ + \ Cx^D} \tag{4.04}$$

Substituting this equation into the ODE defined by equation (4.01), we obtain:

$$\frac{-xy(ABx^{B-1} \ + \ CDx^{D-1})}{Ax^B \ + \ Cx^D} \ + \ ay \ + \ bx^n y^2 \ = \ 0 \tag{4.05}$$

From equation (4.02):

$$y \ = \ \frac{-E}{Ax^B \ + \ Cx^D} \tag{4.06}$$

Substituting this equation into equation (4.05) and simplifying the results we arrive at the following general expression:

$$A(a \ - \ B)x^B \ + \ C(a \ - \ D)x^D \ - \ bEx^n \ = \ 0 \tag{4.07}$$

Based ***purely on empirical observations only***, we can conjecture from entry 1, 2, 3 and 5 of table (4.2) and equation (4.02) that:

$$B \ = \ a \tag{4.08}$$

and:

$$D \ = \ n \tag{4.09}$$

Under this purely hypothetical assumption based entirely on the empirical data obtained, equation (4.07) can now be rewritten as:

$$A(a \ - \ B)x^a \ + \ C(a \ - \ D)x^n \ - \ bEx^n \ = \ 0 \tag{4.10}$$

Since the initial condition of the ODE is always known in advance, we can also include the following additional equation by substituting "$B = a$", "$D = n$", "$x = x_0$" and "$y = y_0$" into equation (4.02).

The results are:

$$(Ax_0^a + Cx_0^n)y_0 + E = 0 \tag{4.11}$$

The complete system of nonlinear simultaneous equations to solve for where the unknown coefficients are now reduced to "A", "C" and "E", can now be obtained by equating like terms to zero in equation (4.10) and by including equation (4.11) for satisfying the initial condition of the ODE.

The results are:

$$A(a - B) = 0 \tag{4.12}$$

$$C(a - n) - bE = 0 \tag{4.13}$$

$$(Ax_0^a + Cx_0^n)y_0 + E = 0 \tag{4.14}$$

One complete solution set to this system of three nonlinear equations in four unknowns is:

$$A \neq 0 \tag{4.15}$$

$$B = a \tag{4.16}$$

$$C = \frac{-Abx_0^a y_0}{a + bx_0^n y_0 - n} \tag{4.17}$$

and from equation (4.13):

$$E = \frac{(a - n)C}{b} \qquad (a \neq n) \tag{4.18}$$

where in equation (4.02), we can set the "A" coefficient as arbitrary defined provided that it is not equal to zero. Note that the expression for "C" in equation (4.17) was derived by multiplying both sides of equation (4.14) with "b", adding the result with equation (4.13) and finally solving for "C".

If for example, we select "$A = -2.75$", "$a = 1.5$", "$b = 2$", "$n = 3$","$x_0 = 1$" and "$y_0 = -2$" then using equation (4.15) through (4.18) we find that:

$$B = a = 1.5 \tag{4.19}$$

$$C = \frac{-Abx_0^a y_0}{a + bx_0^n y_0 - n} = \frac{-(-2.75)(2)1^{1.5}(-2)}{1.5 + 2(1)^{3.0}(-2) - 3} = \frac{-11}{-5.5} = \mathbf{2} \leftarrow \tag{4.20}$$

$$D = n = 3 \tag{4.21}$$

and:

$$E = \frac{(a - n)C}{b} = \frac{(1.5 - 3)(2)}{2} = \mathbf{-1.5} \leftarrow \quad (4.22)$$

Substituting these coefficient values into equation (4.02), we arrive at the same expression as the one defined in the fifth entry of table (4.2).

Equation (4.02) represents an exact solution that appears to only satisfy a limited range of values for the coefficients present in the ODE. However, evidence suggests from table (4.2) that the exact solution obtained in entries 4, 6 and 7 are not in the same format as in equation (4.02).

As a result of this observation, more digging is required before a more complete general exact solution satisfying all the initial conditions and the coefficients present in the ODE is obtained.

For the second candidate "$f_2(x, y) = 0$" as defined by equation (4.03), a relationship for the coefficients "A", "B", "C" and "D" expressed in terms of the coefficients "a", "b", "n" and the initial condition of the ODE can be determined by simply substituting equation (4.03) into the ODE and equating like terms to zero.

The first derivative of equation (4.03) is defined as:

$$\frac{dy}{dx} = -\frac{\partial f_2}{\partial x} \Big/ \frac{\partial f_2}{\partial y} = \frac{-y(ABx^{A-1} + ACx^{A-1}\ln(x) + Cx^{A-1})}{x^A(B + C\ln(x))} \quad (4.23)$$

Substituting this equation into the ODE we get:

$$\left[\frac{-x^A y(AB + AC\ln(x) + C}{x^A(B + C\ln(x))}\right] + ay + bx^n y^2 = 0 \quad (4.24)$$

From equation (4.03):

$$y = \frac{-D}{x^A(B + C\ln(x))} \quad (4.25)$$

Substituting this equation into equation (4.24) and simplifying the results we arrive at the following general expression to solve for:

$$y[(-AB + aB - C)x^A + C(a - A)x^A\ln(x) - bDx^n] = 0 \quad (4.26)$$

based purely on empirical observations, we can conjecture from the fourth, sixth and seventh entry of table (4.2) that:

$$A = n \quad (4.27)$$

Thus, on the basis of this purely hypothetical assumption, equation (4.26) becomes:

$$y[(-nB + aB - C - bD)x^n + C(a - n)x^n\ln(x)] = 0 \quad (4.28)$$

Since the initial condition of the ODE is always known in advance, we can also include the following additional equation by substituting "$x = x_0$", "$y = y_0$" and "$A = n$" into equation (4.03).

The results are:

$$x_0^n y_0(B + C\ln(x_0)) + D = 0 \tag{4.29}$$

The complete system of nonlinear simultaneous equations to solve for where the unknown coefficients are B, C and D can be obtained by equating like terms to zero in equation (4.28) and by including equation (4.29) for satisfying the initial condition of the ODE.

The results are:

$$B(a - n) - C - bD = 0 \tag{4.30}$$

$$C(a - n) = 0 \tag{4.31}$$

$$x_0^n y_0(B + C\ln(x_0)) + D = 0 \tag{4.32}$$

Equation (4.31) is a critical equation that specifies under which condition for the parameters in the ODE are "$f_1(x,y) = 0$" and "$f_2(x,y) = 0$" a valid exact solution. This condition is clearly visible since from equation (4.31), we know that "$C \neq 0$" which ultimately leads us to conclude that "$n = a$".

Thus as a result of equation (4.31),

$$f_2(x,y) = 0 = x^A y(B + C\ln(x)) + D \tag{4.33}$$

satisfies the ordinary differential equation _IF AND ONLY IF_ "$n = a$".

By extending table (4.2) to include additional exact solutions corresponding to a different set of values for the initial conditions and the coefficients present in the ODE, we can easily deduce that:

$$f_1(x,y) = 0 = (Ax^B + Cx^D)y + E \tag{4.34}$$

satisfies the ODE when "$n \neq a$".

The complete solution set of this system of three equations in three unknowns is:

$$D \neq 0 \tag{4.35}$$

$$C = -bD \tag{4.36}$$

$$B = \frac{-D}{x_0^n y_0} - C\ln(x_0) = \frac{-D - Cx_0^n y_0 \ln(x_0)}{x_0^n y_0} \tag{4.37}$$

where from equation (4.33), we can set the "D" coefficient as arbitrary defined provided that it is not equal to zero.

If we select for example, "D = -1", "a = 2", "b = -1", "n = a = 2", "$x_0 = 1$" and "$y_0 = 2$" then using equation (4.27), (4.36) and (4.37), we find that:

$$A = n = \mathbf{2} \leftarrow \tag{4.38}$$

$$C = -bD = -1(-1)(-1) = \mathbf{-1} \leftarrow \tag{4.39}$$

and:

$$B = \frac{-D - Cx_0^n y_0 \ln(x_0)}{x_0^n y_0} = \frac{-(-1) - (-1)(1)^2(2)\ln(1)}{(1)^2(2)} = \mathbf{0.50} \leftarrow \tag{4.40}$$

Substituting these coefficient values into equation (4.33), we arrive at the same expression as the one defined in the fourth entry of table (4.2).

The results of having performed such an indebt computational analysis from the application of a unified theory of integration on this particular general first order ODE has provided a very substantial amount of detailed information. In fact, this would go much further beyond the capability of any traditional *non-universal* method of computational analysis.

A typical report that a numerical analyst might be presenting to management would appear as follow:

"... thus, our empirical findings has indicated to us that for this first order ODE there are two recognizable general exact solutions. The first one is for the case when "$n = a$" and the other is when "$n \neq a$". The general exact solutions obtained can be expressed as a combination of algebraic and elementary basis functions defined only in explicit form. Furthermore, we have established that there is according to the empirical results presented in table (4.2) an explicit relationship involving the initial condition (x_0, y_0) of the ODE, the coefficients (a, b, n) of the ODE and the coefficients in our two initially assumed general exact solutions."

It is expected that many such reporting systems applied on a very large variety of DEs and systems of DEs would inevitably lead to the discovery of many new fundamental theorems similar to the superposition theorem.

5. A universal system of implicit numerical interpolation

Finite and infinite expansion series were traditional used for many centuries as a means of approximating certain types of functions. Many forms of approximation were developed in the past but the Taylor's and Fourier's expansion series still remain the most widely used today.

We have described an entirely new *universal* differential expansion form capable of representing far more complex mathematical functions than what is possible under the Taylor's and Fourier's expansion series method. It now becomes a matter of much further and deeper investigation to determine how well can such a type of new differential expansion form succeed in approximating a general mathematical equation.

There are two major requirements for an initially assumed *Multivariate Polynomial Transform* to be used as a practical method of approximation. The first, is of course that there must be some type of DE or system of DEs associated in the process of completely defining the mathematical equation that is being approximated. The second, is that the *Secondary Differential Expansion* must become completely integrable upon having successfully arrived at some fairly *good approximate* numerical solution set of the relevant system of *Nonlinear Simultaneous Equations*.

When both of these conditions are met then this could potentially open the door for achieving a far more complex system of approximations than what other traditional methods can offer in mathematics. In our case, we would go much beyond the use of the more conventional types of approximation series by allowing *only* the *computational aspect* of our initially assumed *Multivariate Polynomial Transform* decide what basis functions are contained in the approximation solution and also whether it is explicit or implicit by nature. This would also include *computationally* arriving at the correct combination of composite functions without imposing any limits whatsoever on each of their degree of composition.

We demonstrate in the following example a case by which a simple exponential function was being successfully approximated by a very complex implicitly defined mathematical equation consisting of at least one high degree composite function. It must be emphasized that the exact nature of the composite function and the very implicit nature of the entire approximation solution obtained were entirely established purely my method of computational analysis only.

Example (5.1). If we substitute the following *initially assumed Multivariate Polynomial Transform*:

(1). *Primary Expansion:*

$$Y = \frac{a_1 W_1 + a_2}{a_3 W_1 + a_4} \tag{5.01}$$

(2). *Secondary Differential Expansion:*

$$dX = \frac{b_1 W_1 + b_2}{b_3 W_1 + b_4}\, dW_1 \tag{5.02}$$

into the first order ODE that define the following exponential function:

$$y = 1.5e^{-0.5x} \tag{5.03}$$

then by solving for the relevant system of *nonlinear simultaneous equations*, we arrive at the following initially assumed *Multivariate Polynomial Transform* to invert:

(1). *Primary Expansion:*

$$Y = \frac{-0.16301958\,W_1 + 0.26986711}{1.82320996\,W_1 + 0.07715033} \qquad (5.04)$$

(2). *Secondary Differential Expansion:*

$$dX = \frac{0.83816740\,W_1 + 1.31793167}{0.64312753\,W_1 + 0.0271196}\; dW_1 \qquad (5.05)$$

The complete inverse *Secondary Differential Expansion* can be obtained by first integrating both sides of equation (5.05) for "$W_1(x)$" using the following general integral formula for partial fractions:

$$\int \frac{au + b}{pu + q}\, du = \frac{au}{p} + \left[\frac{bp - aq}{p^2}\right] \ln(pu + q) \qquad (5.06)$$

The next step afterwards is to substitute the expression obtained for "$W_1(x)$" into the *Primary Expansion* defined by equation (5.04).

The solution of equation (5.05) using the above integral formula is written as:

$$x = \left[\frac{0.838W_1}{0.6431}\right] + \left[\frac{\{1.318(0.643) - 0.838(0.0271)\}}{0.643^2}\right] \ln(0.643W_1 + 0.0271) + $$

$$+ \quad K \qquad (5.07)$$

$$= 1.303W_1 + 2.00\ln(0.643W_1 + 0.0271) + K \qquad (5.08)$$

where:

$$K = x_0 - 1.303W_{01} - 2.00\ln(0.643W_{01} + 0.0271) \qquad (5.09)$$

It can be shown that if:

$$y = \frac{A_1W_1 + A_2}{A_3W_1 + A_4} \qquad (5.10)$$

then:

$$W_1 = \frac{-A_4y + A_2}{A_3y - A_1} \qquad (5.11)$$

so that from our *Primary Expansion* as defined by equation (5.04), we can directly express "W_1" as a function of "y" to obtain

$$W_1 = W_1(y) = \frac{-0.0771503y + 0.26986711}{1.82320996y + 0.16301958} \tag{5.12}$$

where:

$$A_1 = -0.16301958, \qquad A_2 = 0.26986711$$

$$A_3 = 1.82320996, \qquad A_4 = 0.07715033$$

It follows that:

$$W_{01} = W_1(y_0) = \frac{-0.0771503y_0 + 0.26986711}{1.82320996y_0 + 0.16301958} \tag{5.13}$$

$$= \frac{-0.077(1.5) + 0.270}{1.823(1.5) + 0.163} = \frac{0.1545}{2.8975} \tag{5.14}$$

$$= 0.0533 \tag{5.15}$$

The constant of integration defined by equation (5.09) may now be evaluated as:

$$K = 0 - 1.303(0.0533) - 2.00\ln[(0.643)(0.0533) + 0.0271] \tag{5.16}$$

$$= 0 - 0.0694 - 2.00\ln(0.06137) = 5.512 \tag{5.17}$$

By substituting equation (5.12) into equation (5.08) and simplifying the results, we arrive at an implicitly defined equation in the form of:

$$f(x,y) \;=\; 0 \;=\; 1.303\left[\frac{-0.077y \;+\; 0.270}{1.823y \;+\; 0.163}\right] \;+$$

$$+\;\; 2.0\ln\left[\frac{0.178}{1.823y \;+\; 0.163}\right] \;-\; x \;+\; 5.512 \qquad (5.18)$$

x	y_{exact}	w_1	$f(x, y_{exact})$
-5.0	18.273741	-0.034049	-5.861676E-003
-4.5	14.231604	-0.031716	-5.583780E-003
-4.0	11.083584	-0.028729	-5.233661E-003
-3.5	8.631904	-0.024910	-4.795080E-003
-3.0	6.722534	-0.020031	-4.249876E-003
-2.5	5.235514	-0.013808	-3.579069E-003
-2.0	4.077423	-0.005885	-2.765262E-003
-1.5	3.175500	0.004179	-1.797217E-003
-1.0	2.473082	0.016924	-6.780194E-004
-0.5	1.926038	0.033003	5.611024E-004
0.0	1.500000	0.053192	1.838294E-003
0.5	1.168201	0.078390	2.985896E-003
1.0	0.909796	0.109606	3.697588E-003
1.5	0.708550	0.147920	3.452581E-003
2.0	0.551819	0.194418	1.412990E-003
2.5	0.429757	0.250076	-3.704934E-003
3.0	0.334695	0.315614	-1.377577E-002
3.5	0.260661	0.391310	-3.140153E-002
4.0	0.203003	0.476811	-5.999799E-002
4.5	0.158099	0.570992	-1.037754E-001

Table 5.1

Many of the numerical solution sets obtained not shown here satisfied the relevant system of *Nonlinear Simultaneous Equations* to a fairly high degree of accuracy. In fact so much so that we decided to conduct a more indebt numerical analysis by comparing the results with the implicitly defined equation obtained from having inverting the corresponding initially assumed *Multivariate Polynomial Transform*. This has created just the perfect environment by which an implicitly defined analytical solution was able to approximate to a fairly reasonable level of accuracy the simple ordinary exponential function defined by equation (5.03).

6. Mathematica's own approach to analytical integration

Mathematica is a very popular software package that maintains a collection of symbolic and numerical methods for dealing with the entire aspect of differentiation and integration. Their general approach to integration is nowhere near the one described in this article which is based entirely on the application of multivariate polynomials as well as the differential of multivariate polynomials for finding analytical solutions to all types of DEs that would also include systems of DEs as well. Their online documentation does not present a single instance by which multivariate polynomials and the differential of multivariate polynomials have ever being applied for solving any particular type of DE or system of DEs.

Wolfram's general symbolic approach to solving DEs has the greatest drawback that it cannot be applied universally right across all types of DEs and systems of DEs. Under the new proposed unified theory of integration presented in this article, all DEs and systems of DEs are first subjected to a very rigorous computational process designed specifically for acquiring the type of data that would be transformed in terms of analytical solutions involving the algebraic and elementary basis functions only. Depending on the nature of the data acquired, each analytical solution obtained would be expressed in either explicit or in implicit form involving the use of composite functions with no limit whatsoever on each of their degree of composition.

The exact computational process involved would be the result of substituting the initially assumed *Multivariate Polynomial Transform* described by equation (1.002) through (1.006) into a DE or a system of DEs and afterwards solving for the relevant system of *Nonlinear Simultaneous Equations* that are generated from this process. Each numerical solution set of the *Nonlinear Simultaneous Equations* become the data by which all analytical solutions are constructed from.

As part of the general procedure, this would always involve the *exact* integration of a series of *first order* ODEs that are present in the *Secondary Differential Expansion* of an initially assumed *Multivariate Polynomial Transform* as described by equation (1.003) through (1.006). They will always appear as first order ODEs <u>*regardless*</u> of the type of DE or system of DEs that is being solved for. Since only first order ODEs are always involved then each are subjected to passing the fundamental test of exactness for determining whether or not any one of them is an exact differential. If so, then the integration process becomes considerably simplified for all those differentials that succeed in passing the critical test of exactness.

The final stage of the process would require that exact analytical solutions obtained from this unique integration process be substituted into the *Primary Expansion* as defined by equation (1.002). It is at this point that the various boundary conditions of the original DE or system of DEs are being matched with the ones that are naturally present throughout the complete integration process of the *Secondary Differential Expansion* of an initially assumed *Multivariate Polynomial Transform.*

With *Mathematica* you cannot just simply enter <u>*any type*</u> of DE or system of DEs, especially of the PDE type and expect that an analytical solution whether exact or approximate be returned to you in either explicit or in implicit form. Also, you cannot expect an analytical solution to be constructed entirely from composite functions with no limits on each of their degree of composition just from the use of the algebraic and elementary basis functions. *"That is only possible under a true unified analytical theory of integration which is currently not present anywhere within all of Mathematica".* So in no way does Wolfram appear to follow this type of ideology in mathematics mainly because the computational complexities involved would also have been far too overwhelming for execution on just a regular PC.

By writing a general computer program for implementing such a proposed unified theory of integration in a <u>*complete automated setting*</u> would represent a far better alternative than using *Mathematica's general non-universal approach to analytical integration*.

7. The development of a new type of physics for maintaining uniform continuity throughout

The new proposed mathematical ideology restricts all analysis on mathematical equations at the differential level in order to insure that the concept of continuity be always maintained throughout. This would suggest that the application of a true unified theory of integration for solving *any type* of DEs or system of DEs could hypothetically lead us towards the creation of some infinitely perfect universe over its entire composition. This is provided of course that we are able to maintain complete continuity in mathematical equations throughout the entire process of finding analytical solutions to DEs and systems of DEs. Such an infinitely perfect and continuous universe would be quite feasible to construct but only on the general assumption that "*the mathematical properties of a straight line equation will always remain the same regardless of your exact physical location inside this perfect universe and regardless to what time frame you are specifically referring to*".

A true unified analytical theory of integration will guarantee that every type of DE or system of DEs has some analytical solution behind it whether considered as being exact or approximate. Furthermore, if the theory is to retain all the basic features of universality then it must be applicable to all cases involved without any exceptions whatsoever. The only way for this to be entirely possible is that such a unified analytical theory of integration must *absolutely* be "***computationally-base***" for arriving at complete *analytical solutions* to any type of DE and systems of DEs. So at this point there can be no doubt that the new proposed mathematical ideology being presented in this article does indeed appear to define some sort of a unified theory of integration.

This very powerful assertion made about analytical integration in general has mutated itself into a new kind of physics that I would like to introduce everyone as being an "***idealistic physics***".

The fundamental principle behind this new type of physics is that we can use an infinitely perfect universe for modelling our own *imperfect* physical universe as long as we are able to maintain complete continuity in mathematical equations by solving all DEs and systems of DEs under a single unified theory of integration. Other imperfect physical universes similar to our own may be modeled like clay from the same infinitely perfect controlled universe. Each would then differ from one another only in terms of some mathematical variation representing a measure on how energy is being distributed within the basic atomic structure of matter.

Without some way of maintaining complete continuity in mathematical equations it would virtually become impossible to establish some very fundamental links that can exists between mathematical equations. It's only through the complete consolidation of each of these fundamental links between mathematical equations that in the end would play a vital role for arriving at some unified theory of physics. All of this of course becomes absolutely invisible under any form of experimentally based theory.

In an idealistic physics, discrete variables would have no meaning whatsoever since everything would exist inside an infinitely perfect dynamical structure involving infinitesimal measurements of space and time. All forms of navigation inside this perfect universe would be moving along a pathway of DEs with the new mathematical ideology acting as the main propulsion engine. The only access entry point inside such an infinitely perfect universe is by computation and not based entirely on the use of our imperfect sense of human physical observation that everyone was expecting to succeed during the complete historical development of classical and modern physics.

" To always remain a part of this reality, we need to listen very attentively to what mathematics is telling us and not what we always want to hear. "

The complete understanding of our own imperfect physical universe could never become reality unless we take advantage of the basic tools offered by the new proposed mathematical and physical ideology being introduced in this article. Under this new system of logic, all references made from within this infinitely perfect universe would be driven strictly by computation which would virtually eliminate any risk of encountering the type of contradictions that today are so prevalent everywhere in classical and quantum physics.

If we were to succeed in arriving at some unified theory of physics then no doubt we would have at our fingertips a complete and very detailed understanding of our own physical universe that maybe one day might bring us one step closer to its original creator.

and so ...

"what we are able to understand could give us the capacity to change it for the better."

8. The complete unification of all of physics under one computer software development

A unified theory of physics has true meaning only in relation to some unified theory of analytical integration. It is based on the general assumption that everything in this physical universe can be described by the use of DEs and systems of DEs. They in turn would be completely solvable as some exact or approximate algebraic combination of elementary and algebraic basis functions by following a very unique system of computational logic such as the one being introduced in this article.

Such a grand theory of physics would be constantly referring to the existence of some type of a gigantic universal algebraic system, the very same one in which *Albert Einstein* himself always believed had to exist for completely describing reality. It would stand up at the very top of the hierarchy of all other know existing theories of physics that would include the theory of general relativity, quantum physics and including string theory as well.

All traditional theories in physics lack a great deal of universality, the type that can only lead to the unification of all physics under a single unified theory of integration. By following the same common mathematical ideology that would be entirely based on the fundamental continuity property of all mathematical equations, there would be no risk of encountering any type of contradictions whatsoever. That is because everything would be presented on a computational platform driven entirely from the relentless application of the fundamental laws of differentiation from which the proposed unified theory of integration is entirely based on.

Methods of computation are so important in our everyday lives. The current existing global monetary structure which drives our entire world economy completely depends on it just as much as our technology could not exists without it. None of this would be possible without the use of some form of a *"system of computational logic"* applied to mathematical equations that would have originated from the application of some type of a mathematical ideology.

A *highly automated* computer software program can always be written for the complete implementation of the process involved in solving for any type of DEs and systems of DEs that would be entirely based on the application of the new proposed mathematical ideology. Such a new type of software development would undoubtedly be regarded as being ***"the complete unified theory of physics"*** but only in its most raw state. Human intervention would then only be necessary for complete translation of all computer results that would appear in the form of *exact* numerical computations into practical decipherable mathematical equations. They in turn would be used exclusively for describing the very fundamental structure of our entire physical universe.

Everyone would have complete access to this computer software over the internet for execution on the most advanced super computers of our time. This software would then be regarded as the main pillar by which all of theoretical physics may now be reconstructed *without leaving the impression that we are attempting to reinvent the wheel.* This I believe is possible since we would be finding ourselves moving along a pathway that would be describing an entirely new ideology in both mathematics and physics, the type that has never been investigated by anyone in the past. Much along the same line of reasoning as *CERN* was built around every part of *experimental* nuclear physics. In our case, we would be implementing a very unique technology by which every part of *theoretical* physics would now be investigated under a single common unified theory of integration.

This may perhaps one day have a very profound effect in the manner by which the prestigious *Nobel Prize* would be being presented for major contributions into physics. There would be two such major prizes offered instead of one. The first, would be for exceptional contributions to all aspects of *experimental* physics while the other, for outstanding new contributions into all aspects of the new proposed *idealistic* physics under a *complete unified theory of integration*. Eventually at some point in time, both types of physics will be expected to intersect at the same common point of intersection by which a theory of everything may one day become reality for all of mankind.

9. Engineering science under one universal system of computational logic

The new proposed mathematical ideology can also be transformed into a very unique method of engineering analysis by which all DEs and systems of DEs may now be more closely scrutinized for arriving at a much greater variety of analytical solutions. This would not be feasible by following any other existing traditional methods of analysis since the vast majority of analytical solutions obtained are generally limited to very simple functional expressions that are mostly expressed in explicit form.

Today, methods of solving for DEs and systems of DEs particularly of the PDE type are mostly based on the use of various forms of finite element methods of computational analysis. Under the new proposed mathematical ideology, all forms of engineering analysis would be initiated from the direct application of the initially assumed *Multivariate Polynomial Transform* that was defined by equation (1.002) through (1.006) above.

So rather than presenting a solution to a particular physical problem as a part of some traditional numerical database, our very unique approach would consist of building an entirely new different type of database that would have been constructed on the principle of substituting an initially assumed *Multivariate Polynomial Transform* into *any type* of DEs and system of DEs. The same computer program described earlier as representing the complete unified theory of physics in raw computational form would also be applicable for solving those well know DEs of engineering science that have proven very similar in appearance to those encountered in theoretical physics. In both cases involved, the proposed initially assumed *Multivariate Polynomial Transform* would become the main center stage by which all forms of theoretical analysis would be conducted in the future.

Most particularly important to the engineering science are the need for approximation methods of analysis that are based on the use of highly imperfect control volumes. For these types of engineering problems, we would then be adopting a more *approximate* analytical method of analysis that would be sharing the same common principles as those introduced in section (5).

It is expected that the same computer program originally built for handling all problems in *theoretical physics* would no doubt provide us with the greatest opportunity yet for revisiting all those problems in engineering science that have remained in cold storage. They all have remained there for quite some time now mainly due to a lack of a unified theory of analytical integration

10. Conclusions

You have now all witness a very unique circumstance by which a new mathematical ideology has mutated itself into some form of a new ideology for the physical sciences. The new proposed mathematical ideology is entirely *computational-based* so that the entire process of arriving at some *analytical* solution for resolving *any type* of DEs and systems of DEs can be entirely automated through the development of a *unified computer program*. The proposed initially assumed universal differential expansion as described by equation (1.002) through (1.006) is a testament that all forms of pure *analytical* integration may now be handled under one gigantic unified *computational-based algebraic theory*. The development of such a unified theory of integration would not have been possible without the complete preservation of the fundamental continuity property of all mathematical equations. It is only through the use of differential expansion forms defined in the very special format as described by equation (1.002) through (1.006) that we are able to maintain complete continuity of all mathematical equations throughout the entire process of solving for any type of DEs and systems of DEs. Since virtually all of theoretical physics is founded on mathematical equations, it would be safe to assume that a universal computer program that would be build around such a proposed unified theory of integration would have to be regarded as being some sort of a *"**unified theory of physics**"* in its most raw numerical state. Human intervention would then only be necessary for translating all computer results that would appear in the form of *exact* numerical computations into practical decipherable mathematical equations. The very unique computational structure of our standard initially assumed differential expansion form would offer an unlimited variety of mathematical equations for conducting all forms of *exact* theoretical analysis not only in the field of theoretical physics but also in the engineering and biological sciences as well.

It would be conducted on a scale never imagined possible under any other known traditional methods of analysis. Such a new exact method of analysis could one day offer the best hope yet for arriving at some unified theory of physics without the risk of incurring any form of contradictions that are so prevalent in modern physics today. Also, by introducing such a unified theory of integration into the physical sciences, it is expected in the long term that both physicists and engineers would become much less dependent on pure experimental method of analysis for achieving much greater design reliability of commercial products.

Mathematics has no boundaries; its really our inability to understand it that creates such boundaries (12/14/97).

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