Adjoint program generated by automatic differentiation of a meteorological simulation program, and its application to gradient computation

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Abstract

A source computer program that simulates the atmospheric flow is differentiated by Automatic Differentiation tool TAPENADE. Numerical experiments are presented for the adjoint program.

Keywords: Adjoint program, Automatic differentiation, Atmospheric flow simulation, Gradient

Introduction

Automatic Differentiation (AD) transforms a source computer program P that computes a mathematical vector function into a new source program that computes derivatives of this function. In this paper, a source computer program that simulates the atmospheric flow is differentiated by the AD tool TAPENADE [Hascoet and Pascual (2004)]. The generated adjoint program is employed to compute the gradient of a cost function.

Automatic Differentiation

Given a vector argument $\mathbf{X} \in \mathbf{R}^n$, a source computer program P computes some vector function $\mathbf{Y} = \mathbf{F}(\mathbf{X}) \in \mathbf{R}^m$. The AD tool generates a new source program that, given the argument $\mathbf{X}$, computes some derivatives of $\mathbf{F}$. P represents a sequence of instructions, which is identified with a composition of vector functions. Thus

$$\text{P is } \{I_1; I_2 : \ldots I_P\}$$

$$\mathbf{F} = \mathbf{f}_P \circ \mathbf{f}_{P-1} \circ \cdots \circ \mathbf{f}_1$$

Here each $\mathbf{f}_k$ is the elementary function implemented by instruction I_k.

The chain rule gives the Jacobian $\mathbf{F}'$ of $\mathbf{F}$. Using the Jacobian, for a small perturbation $\delta\mathbf{X}$ in $\mathbf{X}$, the corresponding perturbation $\delta\mathbf{Y} = \mathbf{F}(\mathbf{X} + \delta\mathbf{X}) - \mathbf{F}(\mathbf{X})$ in $\mathbf{Y}$ is computed with quadratic errors:

$$\delta\mathbf{Y} = \mathbf{F}'(\mathbf{X}) \times \delta\mathbf{X} = \mathbf{f}'_P(\mathbf{X}_{P-1}) \times \mathbf{f}'_{P-1}(\mathbf{X}_{P-2}) \times \ldots \times \mathbf{f}'_1(\mathbf{X}_0) \times \delta\mathbf{X} \tag{1}$$

The tangent program computes this perturbation; the program is generated by differentiating the source program in tangent mode. On the other hand, a scalar linear combination $\mathbf{Y}^T \times \bar{\mathbf{Y}}$ is defined as the new result of the source program; $\bar{\mathbf{Y}}$ is the weighting vector. The gradient of $\mathbf{Y}^T \times \bar{\mathbf{Y}}$ is

$$\mathbf{F}'^T(\mathbf{X}) \times \bar{\mathbf{Y}} = \mathbf{f}'^T_1(\mathbf{X}_0) \times \mathbf{f}'^T_2(\mathbf{X}_1) \times \ldots \mathbf{f}'^T_{P-1}(\mathbf{X}_{P-2}) \times \mathbf{f}'^T_P(\mathbf{X}_{P-1}) \times \bar{\mathbf{Y}} \tag{2}$$

The adjoint program is generated by differentiating the source program in reverse mode, and computes the gradient.

Numerical Experiments

Numerical experiments are presented for a three-dimensional downburst. A downburst is a strong downdraft which induces an outburst of damaging winds on or near the ground. A downburst is simulated using the finite difference scheme [Horibata (2012)]. Then, the simulation program is differentiated in reverse mode by AD tool TAPENADE. The cost function is defined by

$$J(X_0) = \sum_{i=0}^{N} c_q \mathbf{q}_{ri} \left(\mathbf{q}_{ri} - \mathbf{q}_{ri}^{ob}\right)^T \left(\mathbf{q}_{ri} - \mathbf{q}_{ri}^{ob}\right) + \sum_{i=0}^{N} c_u \left(\mathbf{u}_i - \mathbf{u}_i^{ob}\right)^T \left(\mathbf{u}_i - \mathbf{u}_i^{ob}\right)$$

Here $\mathbf{q}_r$ and $\mathbf{u}$ are the mixing ration of rainwater and the x component of the wind velocity, respectively; $\mathbf{q}_r^{ob}$ and $\mathbf{u}^{ob}$ are their observations, respectively. N is the number of time steps. The gradient of the cost function with respect to the field variables is computed by the adjoint program. Figures 1 and 2 compare the gradients with respect to rainwater field and the x-component field of the wind velocity with ones computed by the finite difference method, respectively.

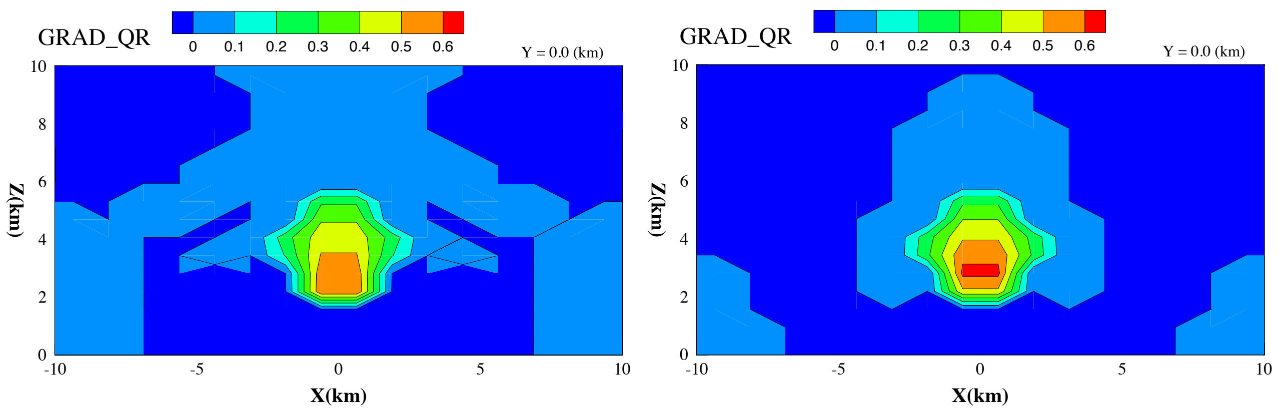

Figure 1. Gradient with respect to the rainwater field computed by the adjoint program (left) and the finite difference method (right) in the x-z cross section at y=0

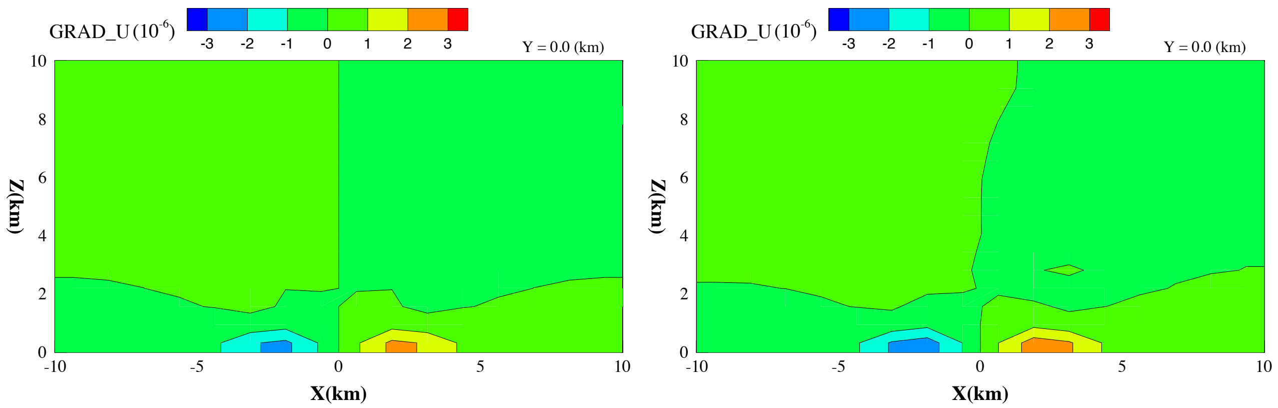

Figure 2. Gradient with respect to the x component field of the wind velocity computed by the adjoint program (left) and the finite difference method (right) in the x-z cross section at y=0

References

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