A Symmetric Moving Particle Semi-implicit Method for the simulation of free surface flows

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Introduction

Meshfree particles methods, such as Moving Particle Semi-implicit (MPS) method [Koshizuka and Oka (1996)] and the Smoothed Particle Hydrodynamics (SPH) method [Lucy (1977)], have been widely applied in modeling free surface flows. They are superior to grid-based methods since it's not necessary to introduce extra equations and special treatments for interface tracking or reconstruction. In general, the basic idea of MPS and SPH methods is to discretize the flow by numerous particles moving in Lagrangian frame. Operators in N-S equations are calculated by different particle interaction models. However, there are some differences between the two methods. In the SPH method, spatial derivatives are obtained by superposition of the derivatives of the kernel function. Comparatively speaking, in the MPS method, a differential operator is approximated using a weighted average of the differential operator between two neighboring particles. Comparing to the kernel function of the SPH method, the simple and crude kernel function of the MPS method will lead to low accuracy. The other difference is that the pressure term is solved implicitly by the Poisson equation to implement real incompressibility in MPS method rather than a fully explicit one in original SPH method.

Application of the particle methods was still limited due to the problem of compressible stress instability, which usually occurs in presence of repulsive forces when particles are approaching to each other. As pointed out by previous researchers [Yang et al. (2014)], the fundamental reason of the compressible stress instability is that the repulsive force first increases and then decreases as two particles get close in the compressible state, and the low accuracy of the numerical schemes for pressure gradient plays a negative role. To date, the available modification is the introduction of artificial stress, which can't solve the problem fundamentally. In this paper, a new MPS-based numerical scheme is proposed to fundamentally improve the stability in the compressible state. Two key modifications are proposed. Firstly, a modified quintic kernel function is adopted to guarantee the repulsive force always increases as two particles get close. Secondly, a symmetric form of the pressure gradient is employed based on the Taylor series expansion, which improves the accuracy of the gradient operator of the standard MPS method.

Numerical algorithm

Firstly, the common used SPH quintic kernel function is modified and introduced in the MPS method as

$$w(r,h)=\frac{7}{488\pi h^2}\begin{cases}2q^5-60q+84 & (0<q<1)\\(3-q)^5-6(2-q)^5 & (1\le q<2)\\(3-q)^5 & (2\le q<3)\\0 & (q\ge 3)\end{cases}\tag{1}$$

As shown in Figure 1, for the modified kernel function, the second derivative is non-negative in the compressible state. That means the repulsive force always increases as two particles get close. Therefore the first reason of the compressible stress instability is removed.

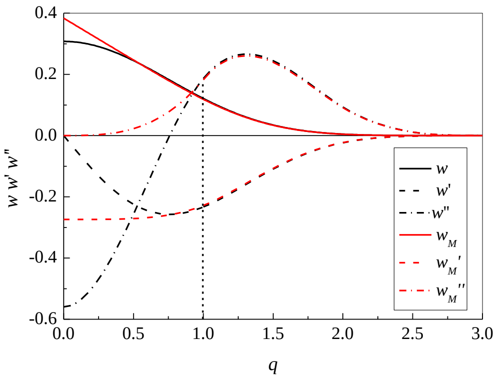

Figure 1 Kernel functions and their first and second derivatives. *w*: the common used SPH quintic kernel; w_M: the modified quintic kernel

Next, a symmetric form of the gradient based on the modified kernel function and the Taylor series expansion are obtained to compute the gradient operator of pressure, which is the key part of velocity correction computation. In the 2D case, for the pressure p of the particle $\boldsymbol{r}_i$. Multiplying both sides of the Taylor series expansion with $(x^\beta_j-x^\beta_i)w_{ij}$, neglecting the second and high order derivatives and integrating the resulting equation over the domain, a series of equations can be obtained:

$$\int\left(x_j^\beta - x_i^\beta\right) w_{ij}\, p(\boldsymbol{x})\, d\boldsymbol{x} = p_i \int\left(x_j^\beta - x_i^\beta\right) w_{ij}\, d\boldsymbol{x} + p_{\alpha,i}\int\left(\boldsymbol{x}^\alpha - \boldsymbol{x}_i^\alpha\right)\left(x_j^\beta - x_i^\beta\right) w_{ij}\, d\boldsymbol{x} \tag{2}$$

α and β are varied between 1 to 2. Pressure p is a known quantity while its first order derivatives $p_{x,i}$ and $p_{y,i}$, are unknowns to be solved. Thus, the pressure gradient can be determined by the following matrix equation:

$$\langle\nabla p\rangle_i = \sum_N \frac{1}{\sum_{k=1} w_{jk}}\left(p_j - p_i\right) \cdot \begin{bmatrix} \sum_{j=1}^{N}\left(x_j - x_i\right)^2 w_{ij} \Big/ \sum_{k=1} w_{jk} & \sum_{j=1}^{N}\left(x_j - x_i\right)\left(y_j - y_i\right) w_{ij} \Big/ \sum_{k=1} w_{jk} \\ \sum_{j=1}^{N}\left(x_j - x_i\right)\left(y_j - y_i\right) w_{ij} \Big/ \sum_{k=1} w_{jk} & \sum_{j=1}^{N}\left(y_j - y_i\right)^2 w_{ij} \Big/ \sum_{k=1} w_{jk} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \left(x_j - x_i\right) w_{ij} \\ \left(y_j - y_i\right) w_{ij} \end{bmatrix} \tag{3}$$

From Eq. (3), the force between two particles is attractive or repulsive is determined by (p_j-p_i). If (p_j-p_i) is positive, the force is repulsive, which is good for the numerical stability. Therefore the minimum p in the neighbor particles of particle i is used in place of p_i. Then the modified pressure gradient used in this paper is

$$\langle\nabla p\rangle_i = \sum_N \frac{1}{\sum_{k=1} w_{jk}}\left(p_j - p_i^{\min}\right) \cdot \boldsymbol{L}^{-1} \nabla_i^{Sy} w_{ij} \tag{4}$$

where

$$\boldsymbol{L} = \begin{bmatrix} \sum_{j=1}^{N}\left(x_j - x_i\right)^2 w_{ij} \Big/ \sum_{k=1} w_{jk} & \sum_{j=1}^{N}\left(x_j - x_i\right)\left(y_j - y_i\right) w_{ij} \Big/ \sum_{k=1} w_{jk} \\ \sum_{j=1}^{N}\left(x_j - x_i\right)\left(y_j - y_i\right) w_{ij} \Big/ \sum_{k=1} w_{jk} & \sum_{j=1}^{N}\left(y_j - y_i\right)^2 w_{ij} \Big/ \sum_{k=1} w_{jk} \end{bmatrix} \tag{5}$$

and

$$\nabla_i^{Sy} w_{ij} = \left[\left(x_j - x_i\right) w_{ij} \quad \left(y_j - y_i\right) w_{ij}\right]^T \tag{6}$$

Since the matrix in Eq. (5) is symmetric, the proposed method is named Symmetric Moving Particle Semi-implicit (SMPS) method.

Results and conclusion

A numerical example of droplet impact is simulated by the SMPS method. As shown in Figure 2, the SMPS method shows the good stability and the compressible stress instability is removed.

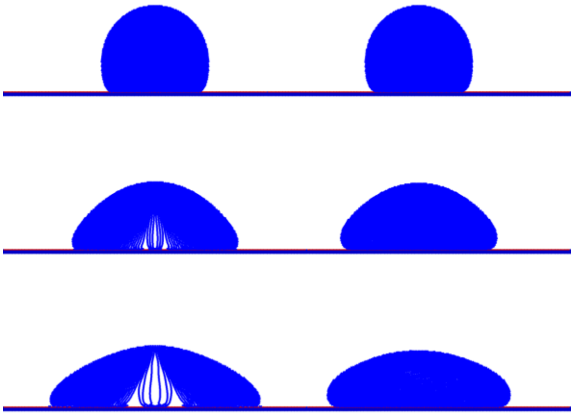

Figure 2 The simulation result of droplet impact. Left: original MPS; Right: SMPS.

Furthermore, the evolution of the dimensionless droplet width is illustrated in Figure 3. It can be seen the result by SMPS is in good agreement with the FDM. Slight difference with original SPH is observed, which may due to the particle inconsistency problem by artificial compressibility of the original SPH algorithm.

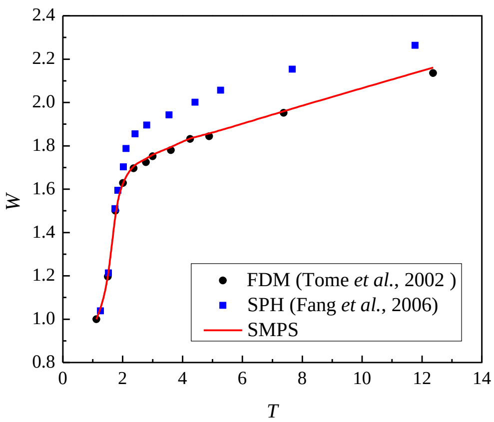

Figure 3 Evolution of the dimensionless droplet width

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