# A Non-intrusive Polynomial Chaos For Uncertainty Quantification On Numerical Simulation Of Flows Around Cylinder+

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## Abstract

Uncertainty quantification (UQ) aims at a meaningful characterization of uncertainties in stochastic systems and efficient propagation of these uncertainties for quantitative validation of model predictions from available measurements. In this paper, UQ of flows around cylinder is studied by non-intrusive polynomial chaos (NIPC) method. The flow is described under the uncompressible Navier-Stokes equations as

$$\frac{\partial \mathrm{u}}{\partial t} + (\mathrm{u}\cdot\nabla)\mathrm{u} = -\nabla p + \upsilon\nabla^2\mathrm{u}$$

$$\nabla\cdot\mathrm{u} = 0$$

Where $\mathrm{u}=(u,v)$, and $p$ are the $x-$component of the velocity, the $y-$component of the velocity, and the scalar pressure field, respectively, with at the initial time, zero flow is assumed, the inflow boundary conditions for this case are given by a parabolic inflow profile, modulated by a sine function depending on time as

$$u(x,y,t) = 0.41^2 \sin\left(\frac{\pi}{8}t\right)6(y+0.2)(0.21-y),$$

$$v(x,y,t) = 0,$$

no slip boundary conditions are imposed on the cylinder and cannel walls; at the outflow, a zero pressure is assumed and nature boundary conditions for velocity. We here assume that the inflow x-component of velocity $u$ and viscosity coefficient $\upsilon$ are random parameters. By the initial velocity of inflow, we let its simulating terminated time be $T=8$. In this study we assume that their PDF are uniform distribution. So their bases of polynomial chaos are Legendre polynomials.
Based on the validation with benchmark results, discussions are mainly focused on the statistic properties of horizontal and vertical components of the total force on the cylinder. The influence of random variables on flow parameters decreases with increasing of viscosity coefficient. The results indicate the effect of NIPC method on the simulation of propagation of uncertainty in the flow field.

**Keywords:** Non-intrusive Polynomial Chaos, Uncertainty Quantification, Flows around Cylinder, Numerical Simulation.

### References

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## Appendix:

The spatial domain for this problem is shown in Fig. 1. In the simulation of Fig. 2.-4., the mean value of viscosity coefficient is $10^3$; its standard deviation is 10% of the mean value. In the simulation of Fig. 5.-7., the mean value of velocity $u$ is the boundary condition of inflow; its standard deviation is also 10% of the mean value.

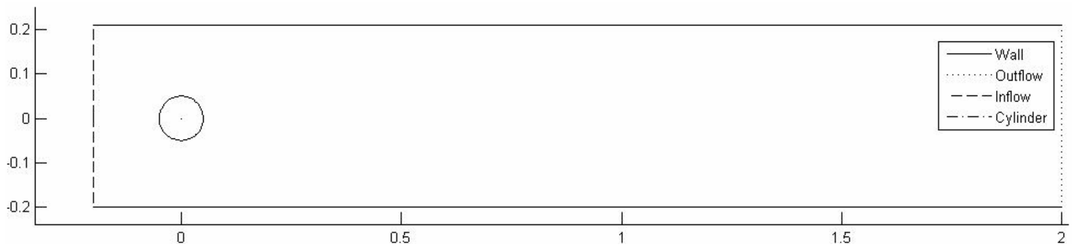


Fig. 1. The spatial domain of the flow around cylinder

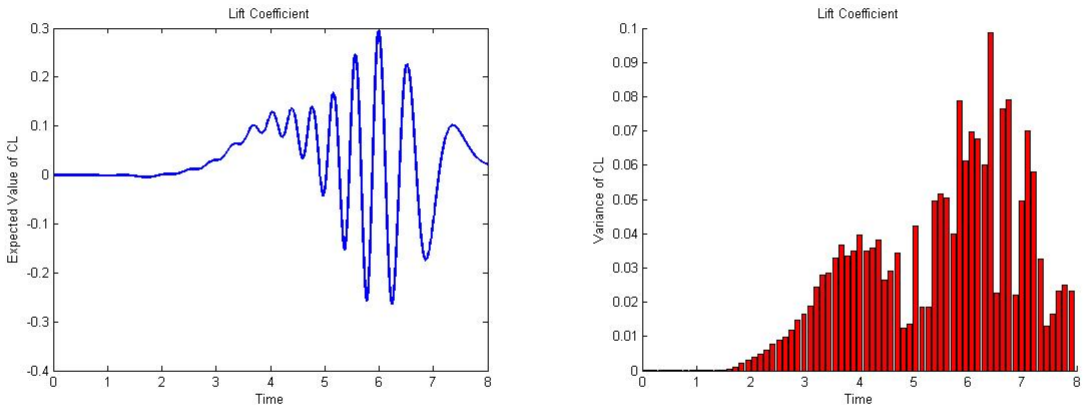


Fig. 2. Vertical components of the total force on the cylinder with mean value $10^3$ of $\upsilon$ (Left: mean values; Right: standard deviations)

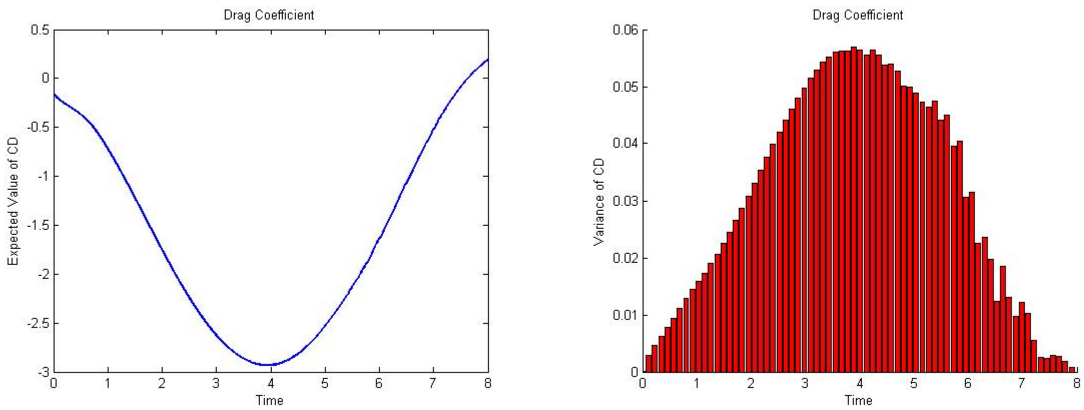


Fig. 3. Horizontal components of the total force on the cylinder with mean value $10^3$ of $\upsilon$ (Left: mean values; Right: standard deviations)

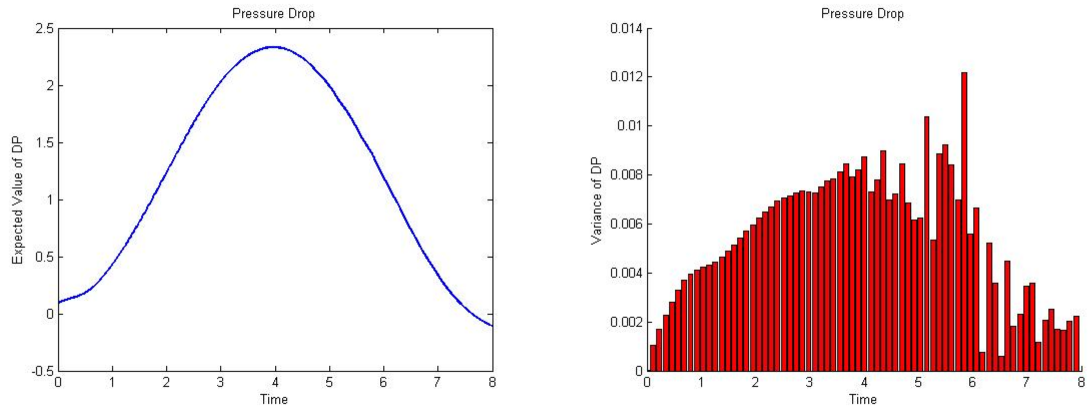


Fig. 4. The drop in pressure between the leading point of the cylinder and trailing point with mean value $10^3$ of $\upsilon$
(Left: mean values; Right: standard deviations)

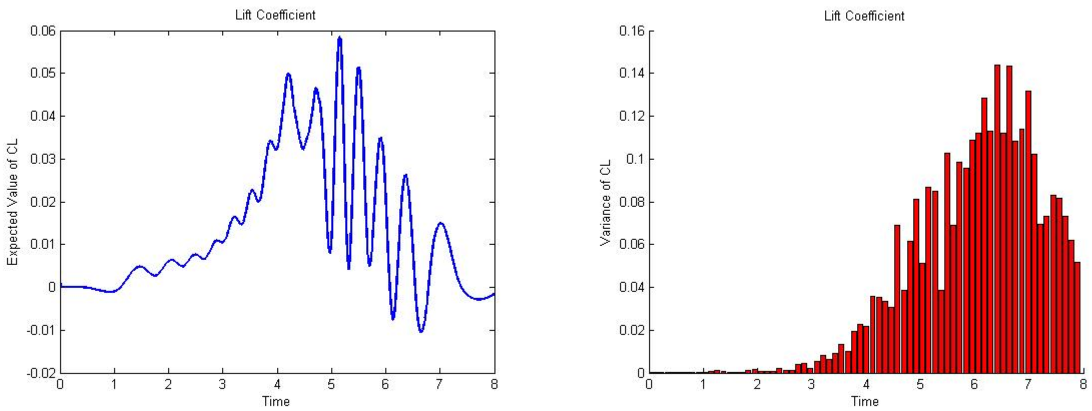


Fig. 5. Vertical components of the total force on the cylinder with random parameter $u$
(Left: mean values; Right: standard deviations)

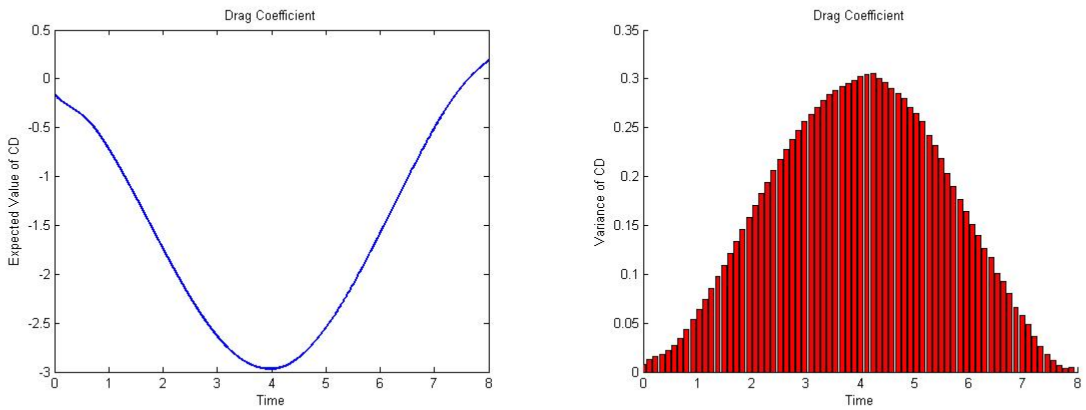


Fig. 6. Horizontal components of the total force on the cylinder with random parameter $u$
(Left: mean values; Right: standard deviations)

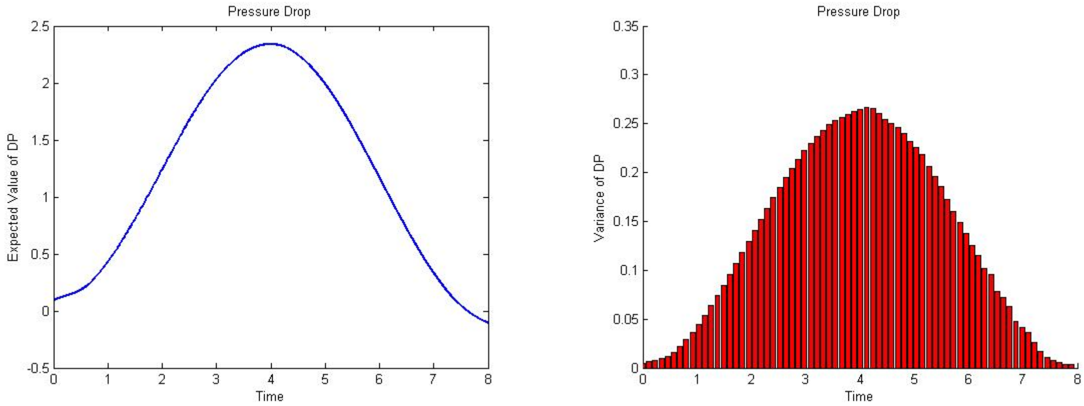


Fig. 7. The drop in pressure between the leading point of the cylinder and trailing point with random parameter $u$
(Left: mean values; Right: standard deviations)