

An Improved Pressure Projection Method for Meshfree Modelling of Ideally Incompressible Hyperelasticity

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Abstract

The pressure projection method [1] has been used successfully in meshfree methods to model large deformations of hyperelastic materials, where it stabilises pressure oscillations and overcomes volumetric locking near the incompressibility limit. However, current pressure projection method is limited to the solutions of nearly-incompressible problems, due to use of a finite bulk modulus as a penalty factor to enforce incompressibility [1].

In order to solve ideally incompressible mechanics, we propose an improved pressure projection scheme based on a similar approach used in fluid mechanics called the Polynomial Pressure Projection (PPP) [2]. A stabilisation term similar to the PPP is appended to the Galerkin weak form to penalise the error between the pressure field and the projected pressure field. As a result of this modification, static condensation [3] of the pressure field onto the displacement field is not permissible [2], since the condensed pressure field is the projected pressure field and the stabilization term becomes zero.

We also modified current Lagrangian meshfree framework to solve the modified Galerkin weak form with numerical differencing, which is not permissible previously. This is due to numerical differencing is perturbing the fictitious field approximated from the moving least squares (MLS) shape functions. Therefore, a full transformation [4] between nodal and fictitious values was done and enabled the perturbation of material points for numerical differencing. Furthermore, essential boundary condition can be imposed easily.

We validated the accuracy of our framework against an analytic solution for uniaxial beam extension, and a finite element solution of 2D annulus under inflation and radial torsion. Pressure field was agreeable with compared solutions. Our goal is to apply this framework to simulate large hyperelastic deformations of soft tissues. In particular, we hope to simulate breast reorientation under different gravity loading conditions as well as during mammographic compression.

In conclusion, we have developed and validated a new generic meshfree framework that reliably and robustly predicts large deformations of ideally incompressible soft tissues.

Keywords: Meshfree methods, Hyperelasticity, Incompressibility, Stabilisation method

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