Shape identification of steady-state viscous flow fields to prescribe flow velocity distribution

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Abstract

This paper presents a numerical solution to shape identification problem of steady-state viscous flow fields. In this study, a shape identification problem is formulated for flow velocity distribution prescribed problem, while the total dissipated energy is constrained to less than a desired value, in the viscous flow field. The square error integral between the actual flow velocity distributions and the prescribed flow velocity distributions in the prescribed sub-domains is used as the objective functional. Shape gradient of the shape identification problem is derived theoretically using the Lagrange multiplier method, adjoint variable method, and the formulae of the material derivative. Reshaping is carried out by the traction method proposed as an approach to solving shape optimization problems. The validity of proposed method is confirmed by results of 2D numerical analysis.

Keywords: Inverse problem, Shape identification, Optimum design, Flow control, Traction method

Introduction

Shape optimization problems of viscous flow fields for improving performance are important in mechanical engineering fields. The theory of shape optimization for incompressible viscous flow fields was initiated by Pironneau [Pironneau(1973; 1974; 1984)], who formulated a shape optimization problem for an isolated body located in a uniform viscous flow field to minimize the drag power on this body. The distributed shape sensitivity, which is called the shape gradient, was derived with respect to the domain variation by means of an adjoint variable method based on optimal control theory. The adjoint variable method introduces adjoint variables into variational forms of the governing equations as variational variables; it also determines the adjoint variables using adjoint equations derived from criteria defining an optimality condition with respect to the domain variation.

The present authors have proposed an approach for the shape optimization of such channels or bodies based on a gradient method using the distributed shape sensitivity. In previous studies, the present authors presented a numerical method for the minimization of the dissipation energy of steady-state viscous flow fields [Katamine and Azegami(1995); Katamine et al.(2005)] and extended this method to 3D problems [Katamine et al.(2009)]. Also, the present authors applied this method to the shape optimization solution for the drag minimization and lift maximization of an isolated body located in a uniform viscous flow field [Katamine and Matsui(2012)].

The present study describes the extension of this method for solving a shape identification problem of flow velocity distribution prescribed problem in sub-domains of steady-state viscous flow fields. Reshaping is accomplished using the traction method [Azegami el al.(1995; 1997); Azegami(2000)], which was proposed as a means of solving boundary shape optimization problems of domains. In the traction method, domain variations that minimize the objective functional are obtained as solutions of pseudo-linear elastic problems for continua defined in the design domain. These continua are loaded with pseudo-distributed traction in proportion to the shape gradient in the design domain.

In this study, the shape identification problem is formulated for flow velocity distribution prescribed problem, while the total dissipated energy is constrained to less than a desired value, in the viscous flow field. The square error integral between the actual flow velocity distributions and the prescribed flow velocity distributions in the prescribed sub-domains is used as the objective functional. Shape gradient of the shape identification problem is derived theoretically using the Lagrange multiplier method, adjoint variable method, and the formulae of the material derivative. The validity of proposed method is confirmed by results of 2D numerical analysis.

Flow velocity distribution prescribed problem

Let Ω be a viscous flow fields in a steady state. The fluid flows in from sub-boundaries Γ_0 and flows out from sub-boundaries Γ_1, where we write velocity vector $u = \{u_i\}_{i=1}^n$ and pressure p. A domain variation problem where the flow velocity distribution u is specified with u_D in sub-domains $\Omega_D \subset \Omega$ can be regarded as a shape optimization problem. For simplicity, we assume that the sub-domains Ω_D, sub-boundaries Γ_0 and Γ_1 are invariables. The flow velocity distribution prescribed problem considering constraint for dissipation energy is formulated as

$$\begin{aligned}
\text{Given} \quad & \Omega & (1)\\
\text{find} \quad & \Omega_s & (2)\\
\text{that minimizes} \quad & E(u-u_D,\ u-u_D) & (3)\\
\text{subject to} \quad & a^V(u,w)+b(u,u,w)+c(w,p)=l(w) \qquad \forall w\in W & (4)\\
& c(u,q)=0 \quad \forall q\in Q & (5)\\
& a^V(u,u)\le a^V_M & (6)
\end{aligned}$$

where Eqs.(4) and (5) are variational forms, or weak forms, using adjoint velocity $w = \{w_i\}_{i=1}^n$ and adjoint pressure q a for the state equations. Eq.(6) is the constraint with respect to the dissipation energy, and a^V_M is the limit of dissipation energy. The flow velocity square error integral $E(u-u_D,\ u-u_D)$ and the terms such as the $a^V(u,w)$ are defined as

$$\begin{aligned}
&E(u-u_D,\ u-u_D) = \int_{\Omega_D} (u_i - u_{Di})\cdot(u_i-u_{Di})\, dx,\\
&a^V(u,w) = \frac{2}{Re}\int_\Omega \varepsilon_{ij}(u)\varepsilon_{ij}(w)\, dx = \frac{1}{Re}\int_\Omega w_{i,j}(u_{i,j}+u_{j,i})\,\mathrm{d}x,\\
&b(v,u,w) = \int_\Omega w_i v_j u_{i,j}\,\mathrm{d}x, \quad c(w,p) = -\int_\Omega w_{i,i}p\,\mathrm{d}x, \quad l(w) = \int_{\Gamma_1} w_i\hat{\sigma_i}\,\mathrm{d}\Gamma
\end{aligned}$$

where $\varepsilon_{ij}(u) = \frac{1}{2}(u_{i,j}+u_{j,i})$, Reynolds number Re and the traction $\hat{\sigma_i}$ are given as known values or functions.

Applying the concept of the Lagrange multiplier method and the adjoint variable method, this problem can be rendered as a stationary problem for the Lagrange functional $L(u,p,w,q,\Lambda)$:

$$\begin{aligned}
L = \quad & E(u-u_D,\ u-u_D)\\
& -a^V(u,w) - b(u,u,w) - c(w,p) + l(w) - c(u,q) + \Lambda(a^V(u,u)-a^V_M) \qquad (7)
\end{aligned}$$

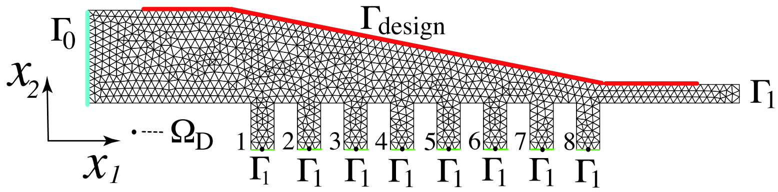

Figure 1: 2D numerical analysis model

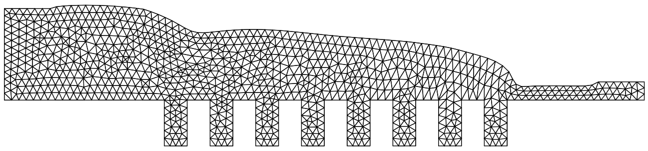

Figure 2: Identified shape

where Λ is the Lagrange multiplier with respect to the dissipation energy constraint. The derivative $\dot{L}$ with respect to domain variation for shape optimization is calculated. Letting this $\dot{L} = 0$, the Kuhn-Tucker conditions with respect to u, p, w, q, Λ are obtained by

$$a^V(u, w') + b(u, u, w') + c(w', p) = l(w') \quad \forall w' \in W \tag{8}$$

$$c(u, q') = 0 \quad \forall q' \in Q \tag{9}$$

$$a^V(u', w) + b(u', u, w) + b(u, u', w) + c(u', q) = 2E(u - u_D,\ u') + 2\Lambda a^V(u, u') \quad \forall u' \in W \tag{10}$$

$$c(w, p') = 0 \quad \forall p' \in Q \tag{11}$$

$$\Lambda \geq 0, \quad a^V(u, u) \leq a^V_M, \quad \Lambda(a^V(u, u) - a^V_M) = 0 \tag{12}$$

that indicate the variational forms of the original state equations for u and p, the variational forms of the adjoint equations for w and q which we call adjoint equations, respectively. Where $(\,\cdot\,)'$ is the shape derivative for domain variation of the distributed function fixed in spatial coordinates. Under the condition satisfying Eqs.(8)- (12), the derivative $\dot{L}$ agrees with the linear form $< G\nu, V >$ with respect to the velocity function V of domain variation:

$$\dot{L}|_{u,p,w,q,\Lambda} =< G\nu, V >= \int_{\Gamma} G\nu_i V_i \, d\Gamma, \tag{13}$$

$$G = -\frac{1}{Re} w_{i,j}(u_{i,j} + u_{j,i}) + \Lambda \frac{1}{Re} u_{i,j}(u_{i,j} + u_{j,i}) \tag{14}$$

where ν is an outward unit normal vector on the boundary.
The coefficient vector function $G\nu$ in Eq.(13) has the meaning of a sensitivity function relative to domain variation and is so-called the shape gradient function. The scalar function G is called the shape gradient density function. Since the shape gradient function is obtained, the traction method[Azegami el al.(1995; 1997); Azegami(2000)] can be applied to this shape identification problem.

Numerical results

We present the results of a numerical analysis for a 2D shape identification problem using the traction method and the shape gradient derived as described in the above sections.

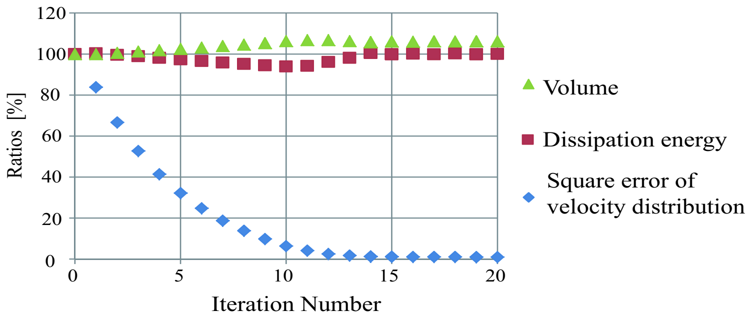

Figure 3: Iterative histories

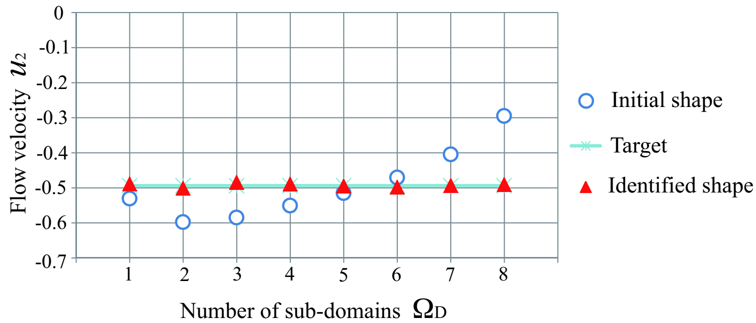

Figure 4: Flow velocity distribution on 8 lower-side sub-domaines Ω_D

We analyzed the 2D problem as one fundamental problem, as shown in Figure 1 The fluid flows in from left-side sub-boundary Γ_0 and flows out from a right-side and 8 lower-side sub-boundaries Γ_1. The sub-domain Ω_D to prescribe the flow velocity distribution was set as 8 lower-side sub-domains. The purpose of this analysis is to determine the shape for which the flow velocity distribution in the 8 lower-side sub-domains becomes as uniform as possible.

In this numerical analysis of the flow field, we used the Hood-Taylor type finite element. That is, the complete polynomial series of the second-order terms was used to provide the interpolation functions for u and w, while the linear polynomial series was used to provide the interpolation functions for p and q. Further, finite elements with six nodes for u and w and three nodes for p and q were also used. The total numbers of nodes and elements were 3,902 and 1,803, respectively. For the analyses of the domain variation V, we used the finite element method with second-order finite elements. The Reynolds number is 100. The dissipation energy is less than the initial shape measure.

The numerical results for the shape identification are shown in Figures 2, 3 and 4. Figures 2 shows the obtained identified shape. Figure 3 shows the iterative history ratios of the square error of velocity distribution $E(u - u_D,\ u - u_D)$, the dissipation energy, and the volume normalized by their respective initial values. Figure 4 shows the flow velocity distribution in the 8 lower-side sub-boundaries Γ_1 for the target, the initial shape, and the identified shape. These results confirm that the flow velocity distribution of the identified shape analyzed by the proposed method approached the target uniform distribution and that the value for the objective functional became zero. The validity of the present method was confirmed based on the numerical results obtained for the basic problems described above.

Conclusions

In the present study, we formulated a shape identification problem in which the square error integral between the actual flow velocity distributions and the prescribed distributions in the prescribed sub-domains on viscous flow fields was used as the objective functional. The shape gradient of the shape identification problem was derived theoretically. The validity of the proposed method was confirmed based on the results of a 2D numerical analysis. The present study was supported in part by JSPS KAKENHI Grant Numbers 26420161.

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