Ill-conditioning with C∞ radial basis functions and asymmetric collocation

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Abstract

C∞ radial basis functions (RBFs) used in asymmetric collocation of partial differential equations(PDEs) and integral equations (IEs),see [1], are by nature, global, and can become very ill-conditioned.. Madych [2] showed such RBFs possess exponential convergence rates. Schaback [3] observed that very rapid convergence came at the expense of ill-conditioned systems of equations. Domain decomposition methods (DDM), widely used in sparse finite element methods both as a pre-conditioner and as a way to achieve parallel computations; some experts promulgated the opinion that DDM were deemed too difficult to implement, even though R.L. Hardy [4] who discovered mu;tiquadric C∞ RBFs was successful in applying as well as in [1]. Madych [2] in 1992 praised C∞ RBFs for their extremely impress rates, but lamented in 1992 computer hardware did not permit exploitation of these basis functions. The inadequacy of computer hardware was documented in the SIAM 100 digit challenge and software pioneers such as David H. Bailey and Pavel Holoborodko . Huang et al, [5] used quadruple precision to solve PDEs with globally supported C∞ RBFs, demonstrating clearly that even though quadruple precision is considerably slower/operation than he double precision calculations using compactly supported finite elements, because many orders of magnitude of elements are required to achieve a specific target accuracy.

Three techniques were used to overcome the ill-conditioning and perhaps a singular problem of the strong form collocation problem for integral and partial differential equations. These are: (1) Domain decomposition (2) The weak formulation of Galperin-Zheng [6]. (3) Software multi-precision arithmetic package in www.advantix.com. Numerical results will be provided.

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