Computation of vadoze zone moisture profiles for successive irrigation scheduling

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Abstract

A numerical simulation model has been developed, to compute vadoze zone soil moisture content profiles under transient field conditions by coupling soil moisture flow equation with a non linear root water uptake model. The model has been tested for the sensitivity of its non linear uptake parameter, for obtaining its optimal value. Computation takes into account a variable transpiration rate and a field measured initial moisture content. Rainfall, irrigation and evaporation have been treated as sources of non-uniform potential surface flux. Solutions to the computation have been obtained numerically by a fully implicit finite difference scheme, involving a non linear system of equations, which has been linearized using Picard's iterations. Field crop data of maize (Zea mays), which is among the most important crops in India and several other countries in the world, has been used to evaluate the results of the simulation. Determining the water requirements of crops is important for improved scheduling of irrigation, which in turn requires accurate measurement of crop evapotranspiration (ET_c). As the first objective, daily and seasonal ETc of maize are computed using Lysimeter set up in an experimental field from May 2006 to September 2006 at Roorkee, India. The average daily ETc of maize varied from a range of 1.4 to 3.4 mm day^{-1} in the early growing period to 8.3 mm day^{-1} at peak that occurred 9 weeks after sowing (WAS) at the silking stage of maize, when leaf area index (LAI) was 4.54. Average daily ETc declined sharply to 2.57 mm day^{-1} during late season stage of crop. The measured seasonal ET_c of maize was 495 mm. Development of computation based schedules of irrigation is the second objective of the study. Plant parameters like root depth and crop height have been continuously observed throughout the crop period. Top 0.3 m depth of root zone is considered to represent the soil moisture status governing the schedules of irrigation. Application of the computation technique to field conditions and comparison of the results with filed measured data shows very good agreement.

Introduction

The availability of water for plant roots is an important topic, which has been explored by a number of investigators (Feddes et al., 1978; Molz, 1981; Kang et al., 2001). Recently the attention is being given to irrigation management, by optimizing the frequency of irrigation, particularly in arid and semi-arid regions. Such management strongly depends upon knowledge of soil moisture movement through the root zone of the crops. Prediction of available moisture for plant roots also has significant effect on irrigation scheduling. The studies in this direction followed basically two approaches; microscopic, where a single root is assumed to be represented by a narrow infinitely long cylinder of constant radius which absorbs water (Afshar and Marino, 1978) and macroscopic, which focus on the removal of moisture from the differential volume of soil as a whole, without considering the effect of individual roots (Feddes et al., 1978). However, the basic assumptions along with the drawbacks and the difficulties involved in microscopic scale models under natural field conditions have restricted their applicability for field situations.

Soil moisture dynamics under cropped conditions are affected by soil, plant and climatic factors. The boundary between soil and the root system of plants is a major hydrologic interface across which well over 50% of evapotranspiration moves. Mathematical models of soil moisture dynamics on a macroscopic scale are mostly employed for predicting soil moisture distribution in the crop root zone on a day-to-day basis. Root water uptake in the crop root zone is represented as a sink term in the soil moisture flow equation. There are many different forms of sink term functions developed till date, of which, hypothetical linear distribution pattern of 40, 30, 20, 10 % moisture uptake in each quarter of root zone by Molz and Remson (1970), Feddes et al. (1978)ôs constant rate model, Prasad (1988)ôs linear rate model and Ojha and Rai (1996)ôs non linear root water uptake model are the prominent ones. Precise estimation of soil moisture depletion in the crop root zone, accurately determines the soil moisture availability for the plant use. It has been established by many recent studies that plant moisture uptake involves considerable non-linearity owing to the non-linear root density distribution in the root zone (Ojha and Rai, 1996; Kang et al., 2001).

Present work couples Ojha and Rai (1996) non-linear root water uptake model, with Richards (1931) equation. A numerical simulation model is developed to compute the soil moisture dynamics in the crop root zone. Requisite soil and crop data is obtained by conducting the field crop experiments. Maize, which is a major crop in this region, has been grown during relevant crop season. Variation of crop evapotranspiration during the crop season has been determined. The first objective of the work is to accurately predict the soil moisture profiles in crop root zone. Based on the simulated soil moisture depletion in root zone, study also aims to compute optimal irrigation schedules for the crop grown in the field at different allowable moisture depletion levels.

Materials and Methods

Water Movement in Soil

The mixed form of Richardsôs equation governing water flow in the unsaturated zone, considering root water uptake can be written as

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right)\right] - S(z, t) \tag{1}$$

Where, ɗ is the volumetric moisture content of soil, ɣ is the pressure head, t is the time, z is the vertical coordinate taken positive upwards, K is hydraulic conductivity, and S(z, t) is the water uptake by roots expressed as volume of water per unit volume of soil per unit time. Richardsôs equation is highly non linear due to changes in pressure head and hydraulic conductivity in unsaturated soils. In order to solve Richardsôs equation, it is required to specify constitutive relationships between the dependent variable (moisture content in this case) and the non linear terms (pressure head and hydraulic conductivity). Present study uses K-θ-ψ relationships proposed by Van Genuchten's (1980), given as:

$$\Theta = \left[\frac{1}{1+\left\|\alpha\ \psi\right\|^{n}}\right]^{m} \quad \text{For } \psi \le 0 \tag{2}$$

$$= 1 \qquad\qquad \text{for } \psi > 0$$

In equation (2), α and n are unsaturated soil parameters with m = 1-(1/n) and Θ is effective saturation defined as

$$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{3}$$

Where, θ_s is saturated moisture content and θ_r is residual moisture content.

Based on Mualem's (1976) model the relation between moisture content and hydraulic conductivity is given by (Van Genuchten, 1980)

$$K = K_{sat}\Theta^{1/2}\left[1-\left(1-\Theta^{1/m}\right)^{m}\right]^{2} \tag{4}$$

Where K_{sat} = saturated hydraulic conductivity of soil

Root Water Uptake

Ojha and Rai (1996), non-linear root water uptake model, referred as O-R model hereafter, has been used to represent the sink term in Eqn (1). According to O-R model, for potential transpiration conditions, the potential rate of soil moisture extraction S_{max} is given by the relation

$$S_{max} = \left[\frac{T_j}{z_{rj}}(\beta+1)\left(1-\frac{z}{z_{rj}}\right)^{\beta}\right]; \qquad 0 \le z \le z_{rj} \tag{5}$$

Where, β is model parameter, z is depth below soil surface, and z_{rj} is root depth on the j^{th} day. For $z = z_{rj}$, S_{max} is zero as per (5) and at z = 0, S_{max} attains a maximum value. Thus (5) satisfies the desired extraction conditions, that extraction is maximum at the top and zero at the bottom of the root. It is to be noted that for β = 0, (9) reduces to a constant rate extraction model of Feddes et al. (1978) with $S_{max} = T_j/z_{rj}$ while for β = 1, (9) reduces to linear extraction model of Prasad (1988) with $S_{max} = 2T_j/z_{rj} - 2T_j\ (z/z_{rj}^2)$. Present work considers the moisture uptake under potential moisture condition.

Initial and Boundary Conditions

Measured pressure head values in the soil profile at the start of crop season have been used as the initial condition, i.e.

$$\psi = \psi_0 (z, 0) \qquad\qquad 0 \le z \le L \qquad (6)$$

Where ψ_0 is the measure pressure head value at corresponding soil depth. For intermediate depths values are linearly interpolated.

The upper boundary condition is a prescribed flux boundary condition accounting for the evaporation taking place from the top soil and a Drichlet boundary condition, during irrigation or rainfall. Thus

$$\psi\ (L, t) = \psi_s \qquad\qquad \text{during irrigation/rainfall} \qquad (7a)$$

$$-K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right) = E \qquad z = L\text{, in absence of irrigation/rainfall} \qquad (7b)$$

Where ψ_s is the pressure head corresponding to the saturated soil moisture condition. E is the evaporation from the top soil.

At lower boundary gravity drainage type condition has been assumed, where a unit hydraulic gradient is considered.

$$-K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right) = -K(\psi) \qquad\qquad \text{for } t \ge 0,\ z = 0 \qquad (8)$$

Numerical Model

A numerical model has been developed to solve equation (1) along with the sink term subjected to initial and boundary conditions (6) to (8), and employing the constitutive relationships (2) to (4). The numerical model is based on a mass conservative, fully implicit finite difference scheme proposed by Celia et al. (1990). The non linear system of equations is linearized using Picardôs methods (Paniconi et al., 1991) and resulting system of equations are solved using Thomas algorithm. The model yields spatial distribution of pressure head and moisture content at successive advancing times in the soil. From the model computed moisture contents, the moisture depletion values at different zones of crop root at different times are computed by numerical integration.

Field Crop Experiments

Field crop experiments have been conducted at the field experimental station of Civil Engineering Department, Indian Institute of Technology, Roorkee, India, from April to September, 2006. The average annual rainfall at Roorkee is 1032 mm, of which about 75 % is usually received between July and September. The required meteorological data for the computation of corresponding crop evapotranspiration using crop coefficient approach is obtained from the Department of Hydrology, Indian Institute of Technology Roorkee. For measuring the soil moisture profile throughout the crop season soil moisture measurement sensors have been embedded at 0.15, 0.30, 0.45, 0.60, 0.75, 0.90, 1.05 and 1.20 m, however at the ground surface the moisture content is measured using TDR soil moisture meter.

Crop details

Maize (Variety K-99 HYBRID) was sown uniformly in Lysimeters and the surrounding field so that the field conditions could be simulated in and around the Lysimeters. Crop period of Maize lasted from May 20th to September 1st, 2006 (105 days). The sampling site for different plant parameters such as leaf area index (LAI) and root length is about 4 to 5 m away from the Lysimeter. The entire crop growth period for the crops is divided into four stages; I-Initial, II-Crop Development, III-Mid Season and IV-Late Season. Growth stages have been considered on the basis of study by Doorenbos and Pruit (1977). Initial stage corresponds to the germination and early growth when the soil surface is not or is hardly covered by the crop (ground cover < 10 %). Crop development stage starts from the end of initial stage to attainment of effective full ground cover (ground cover: 70-80 %). Mid season commences from the attainment of effective full ground cover to time of start of maturing as indicated by discoloring of leaves or leaves falling off and late season stage begins from end of mid-season until full maturity or harvest. Duration of stage I, II, III and IV accordingly has been found to be 17, 30, 34 and 24 days respectively. Irrigations have been provided on 24th, 33rd and 42nd day of the crop period.

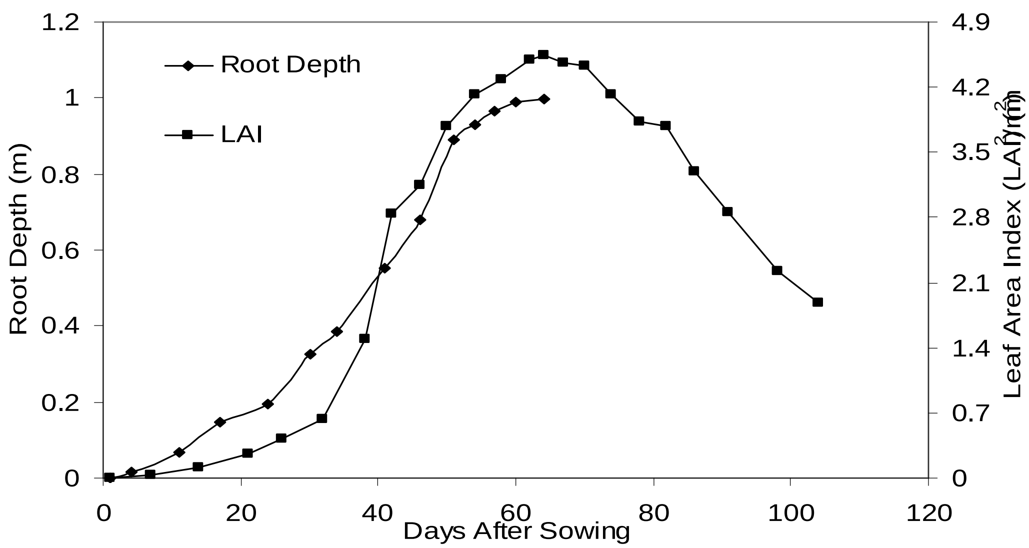


Figure 1. Field observed plant parameters for the Maize

Two major parameters; LAI, and root depth have been recorded at discrete time intervals throughout the growth period. Leaf area index (LAI) required for the partitioning of the crop evapotranspiration into plant transpiration and soil evaporation, was measured by direct method suggested by Jesus et al., (2001). Leaf area measurements are made once in a week during the initial stage, once in five days during development stage, twice a week during middle stage and once a week during last stage. Root depth has been measured by trench profile method described by Wolfgang (1979). At initial stages of crop growth root depth has been measured at 7-10 days interval, where as in later stages this interval has been reduced to 5 day interval. Figure 1 show the variation of root depth and LAI measurements with crop growth period for maize.

Soil parameters

Representative soil samples were obtained from the 0-0.3 m, 0.3-0.6 m, 0.6-0.8 m, 0.8-1.0 m and 1.0-1.2 m depths, in the experimental site for testing the soil properties. The cumulative particle size curves obtained through grain size and hydrometer analysis reveal that the soil profile up to 1.2 m is fairly uniform in texture. The upper 0-0.3 m depth however, shows a slight deviation from the general trend with higher silt and lower clay fractions being indicated, but it is within limits and hence a uniform soil textural classification is considered for 0-1.2 m depth. USDA soil textural class for the experimental field soil is sandy loam. The bulk density, particle density and porosity for the field soil are 1.62 g/cm^3, 2.61 g/cm^3 and 0.38 respectively.

Soil-moisture characteristic curve provides a convenient method for describing the moisture retention properties of different soils (Winter 1974). In-situ determination of SMC has been performed, which involves simultaneous measurement of soil matric potential (ɣ) and moisture content (ɗ) at 0.3, 0.6, 0.9 and 1.2 m depths below the ground level. No clear depth-wise relationship is discernible, indicating the similarity of the retention characteristics of the soil profile and as such a single SMC has been used for the entire zone. Van Genuchten Relationship (1980) described by Eqns (2)-(4) has been used to determine the soil hydraulic characteristics.

The saturated moisture content ɗ$_s$ in eqn. (3) is assumed to be equal to the measured soil porosity (0.38 cm^3 cm^{-3}). A standard residual moisture content value equal to 0.065 cm^3 cm^{-3} (Carsel and Parrish, 1988) for sandy loam soil (soil type for experimental plot) has been considered. A non linear optimization algorithm E04FDF (N.A.G., 1990) has been used to estimate the Van Genuchten parameters Ŭ and n, which are 6.2 m^{-1} and 1.68 respectively. The value of average field saturated hydraulic conductivity (K_{sat}) determined at different depths using Guelph type Permeameter is 3.9 cm/hour. Experimentally obtained value of field capacity (ɗ$_{fc}$ = 0.208) and SMC deduced value of wilting point (ɗ$_{pwp}$ = 0.068) has been used in the present study. The available moisture which is the difference of ɗ$_{fc}$ and ɗ$_{pwp}$ is 0.14. The irrigation has been provided at 50% depletion of the available moisture in the effective root zone.

Computation of Crop Evapotranspiration (ET_c)

Crop evapotranspiration has been determined as the product of daily crop coefficient and reference evapotranspiration. Reference evapotranspiration (ET_0) is a complex phenomenon and depends on several climatological factors, such as temperature, humidity, wind speed, radiation, and, type and growth stage of crop. During the study period ET_0 (mm/day), has been computed by Penman Monteith method. The Penman-Monteith equation for the ET_0 is given as (Allen et al., 1998)

$$ET_0 = \frac{0.408\Delta(R_n - G) + \gamma \frac{900}{T+273} u_2 (e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)} \tag{9}$$

Where, R_n = net radiation at the crop surface [MJ m^{-2} day^{-1}], G = soil heat flux density [MJ m^{-2} day^{-1}], T = mean daily air temperature at 2 m height [°C], u_2 = wind speed at 2 m height [m s^{-1}], e_s = saturation vapour pressure [kPa], e_a = actual vapour pressure [kPa], (e_s - e_a) = saturation vapour pressure deficit [kPa], φ = slope vapour pressure curve [kPa $°C^{-1}$], ɔ = psychrometric constant [kPa $°C^{-1}$].

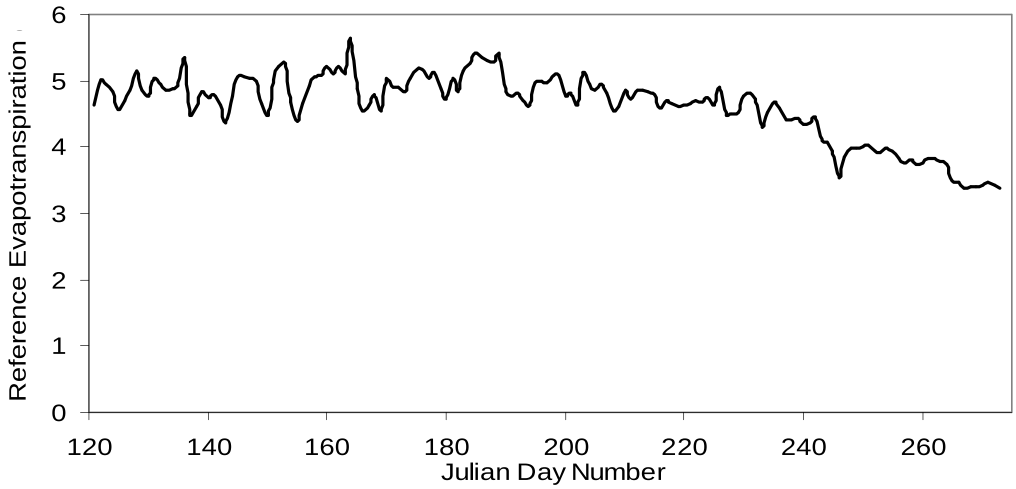


Figure 2. Daily reference evapotranspiration during study period

Different parameters involved have been computed using the mathematical formulations provided by Allen et al. (1998). Fig. 2 shows the daily ET_0 (mm/day) computed using Penman-Monteith method for the study period.

The crop coefficient (K_c) value represents crop-specific water use and is needed for accurate estimation of irrigation requirements of different crops. Comprehensive list of stage-wise crop coefficients is available in literature (Allen et al, 1998). The crop coefficients for initial, development, mid-season and end-season stages are denoted as $K_{c\ ini}$, $K_{c\ dev}$, $K_{c\ mid}$ and $K_{c\ end}$ respectively. In case the local calibration of the crop coefficients is not possible then a procedure has been outlined by Allen et al. (1998), to modify the reported crop coefficients for the local climatic conditions, and crop and irrigation practices. FAO proposed $K_{c\ ini}$, $K_{c\ mid}$ and $K_{c\ end}$ values are 0.3, 1.2 and 0.6 for Maize. These values have been modified for the local climatic, crop and soil characteristics according to the procedure outlined in FAO guidelines. The modified values of $K_{c\ ini}$, $K_{c\ mid}$ and $K_{c\ end}$ are 0.33, 1.126 and 0.55 respectively.

From the stage wise crop coefficients, daily K_c values during the growing period are determined either graphically or numerically (Allen et al., 1998). The daily crop coefficient depends on the plant characteristics as well as the meteorological factors, which are represented in the stage specific crop coefficients. Allen et al. (1998) had observed that K_c values remain constant for early and mid season stages. However, during the crop development and late season stage, K_c varies linearly between the K_c at the end of the previous stage ($K_{c\ prev}$) and the K_c at the beginning of the next stage ($K_{c\ next}$), which is $K_{c\ end}$ in the case of the late season stage. Following Allen et al. (1998), the crop coefficient for an i^{th} day in a particular stage is computed as:

$$K_{ci} = K_{c,prev} + \left[\frac{i - \sum(L_{prev})}{L_{stage}}\right](K_{c,next} - K_{c,prev}) \tag{10}$$

Where, i is the day number within the growing season, $K_{c\ i}$ crop coefficient on day i, L_{stage} is length of the stage under consideration [days], and L_{prev} is the sum of the lengths of all previous stages [days]. Using equation (10) daily crop coefficients for Maize are determined.

Daily crop evapotranspiration is determined as the product of daily K_c value and reference evapotranspiration. Further, the daily crop evapotranspiration is partitioned into plant transpiration and soil evaporation using eqn. (11) method proposed by Belmans et al. (1983), where soil evaporation (E_s) is calculated as a fraction of the ET_c using the LAI of the soil surface.

$$E_s = f * EXP(-c * LAI)\ ET_c \tag{11}$$

Where, f and c are regression coefficients, with f = 1.0, and c = 0.6. This relation gives an acceptable estimation of soil evaporation (Belmans et al., 1983). Plant transpiration is part of the ET_c, and it can be calculated after E_s is determined from Eqn. (12). Since $ET_c = E_s + T_p$, plant transpiration (T_p) is

$$T_p = ET_c - E_s \tag{12}$$

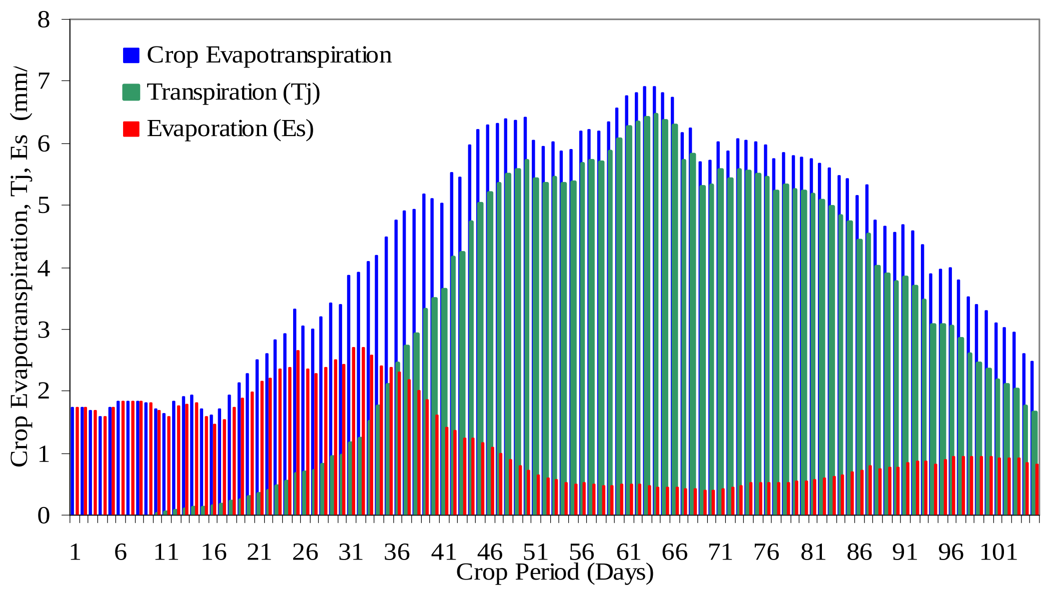


Figure 3. Daily Crop Evapotranspiration, Evaporation and Transpiration for Maize.

The plant transpiration is used as the sink term in the Richards equation and the soil evaporation is used as the boundary condition at the ground surface. Fig. 3 shows the variation of crop evapotranspiration and its components, evaporation and transpiration for Maize throughout the crop period. The average daily crop evapotranspiration of Maize varied from a range of 1.4 to 3.4 mm day^{-1} in the early growing period to 7.2 mm day^{-1} at peak that occurred 9 weeks after sowing (WAS) at the silking stage of maize, when leaf area index (LAI) was 4.54. Average daily ETc declined sharply to 2.57 mm day^{-1} during late season stage of crop.

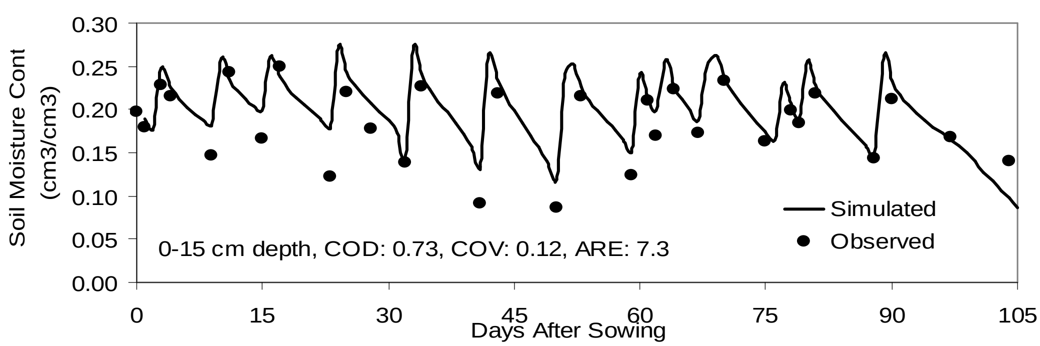
Results and Discussion

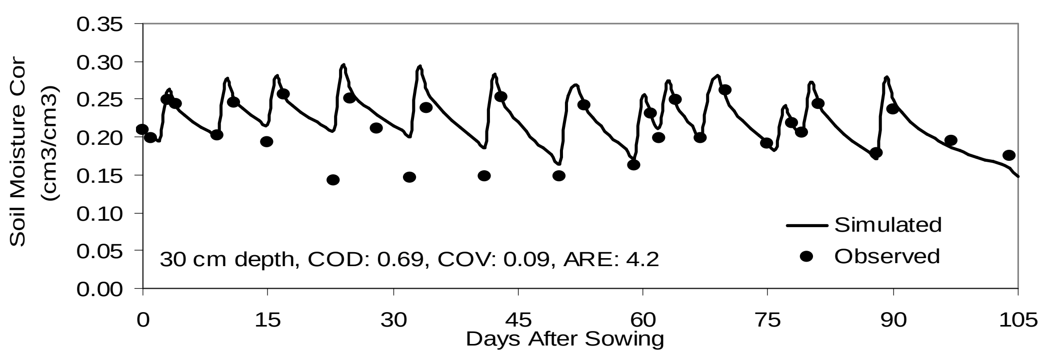
The obtained soil moisture characteristics, crop evapotranspiration and root depth variation over the crop period applied to the numerical model formulated by coupling Richards equation with O-R model to simulate plant moisture uptake. Initially the optimal value of the non-linearity parameter б of O-R model is determined using observed and simulated soil moisture depletion pattern. The optimal value of б for Maize has been found to be 1.5. Observed and simulated soil moisture profiles in the vadoze zone on discrete days and soil moisture status during the crop period of Maize has been compared.

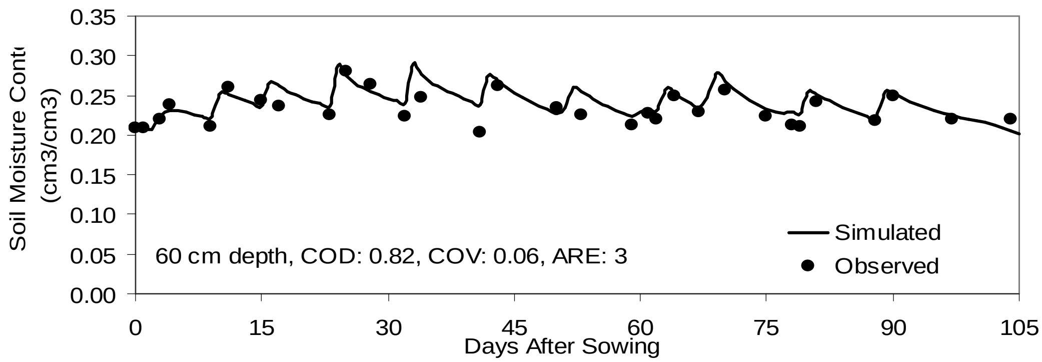
Figs 4, 5 and 6, show the observed and simulated soil moisture status during crop period, and Figs 7 and 8, show the observed and simulated soil moisture profiles on discrete days in crop period of Maize.

It can be observed from the Figs 4-8, that there exists a reliable agreement between simulated and observed values. However, for quantitative evaluation, error statistics e.g. coefficient of determination (COD), coefficient of variation (COV) and average relative error (ARE) (Ambrose and Roesch, 1982) are used for each set of values.

$$\text{COD} = 1 - \frac{\sum_{i=1}^{n}\left(\theta_{mi} - \theta_{si}\right)^2}{\sum_{i=1}^{n}\left(\theta_{mi} - \theta_{avg}\right)^2} \tag{13}$$





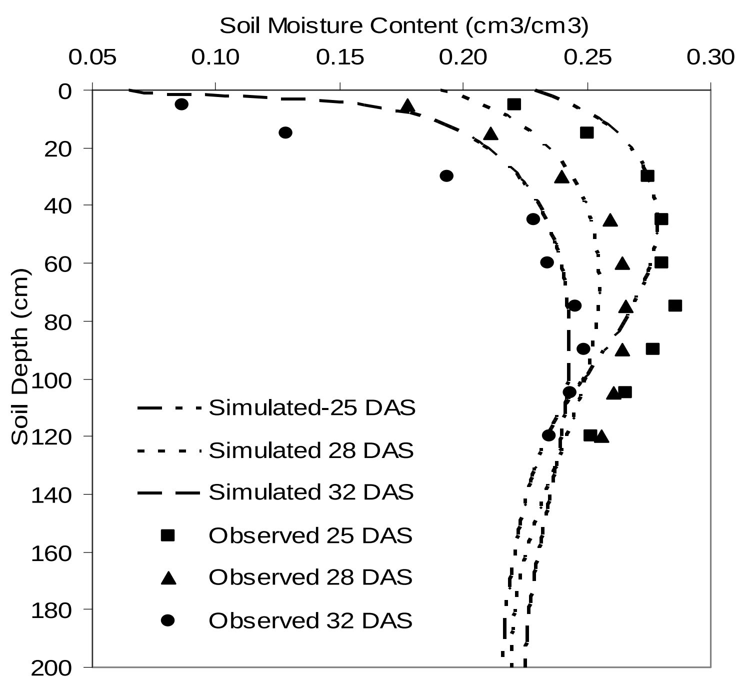


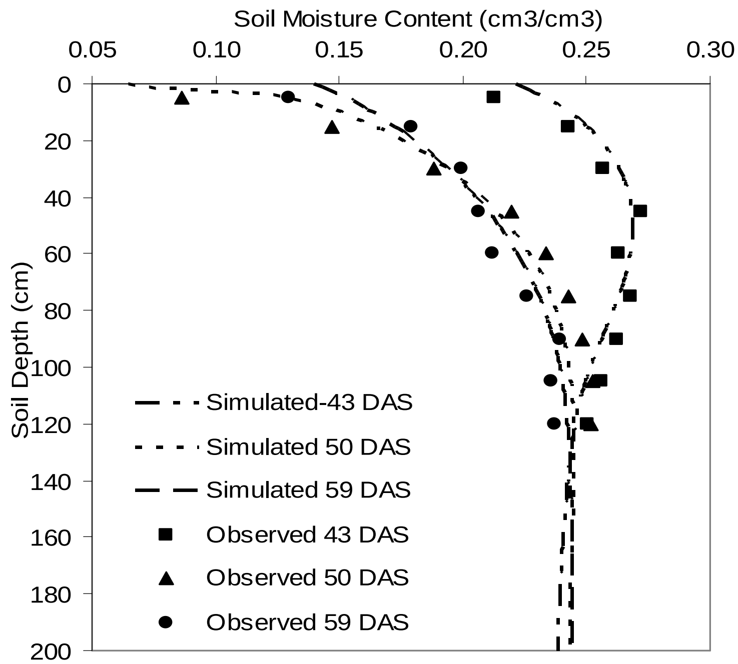
Figures 4, 5 and 6. Moisture status during crop period at 0-15, 30 and 60 cm depths in root zone

$$\mathrm{COV} = \frac{\left[\sum_{i=1}^{n} \frac{(\theta_{si} - \theta_{mi})^2}{n}\right]^{0.5}}{|\theta_m|} \tag{14}$$

$$\mathrm{ARE}\ (\%) = \frac{\sum_{i=1}^{n} (\theta_{si} - \theta_{mi})}{n|\theta_m|} \times 100 \tag{15}$$

Where, θ_{si} is the simulated sil moisture content at ith point, θ_{mi} is the corresponding field observed value, θ_m is the average of the field measured values, and n is the number of observations. A value of COD close to the unity indicates a high degree of association between



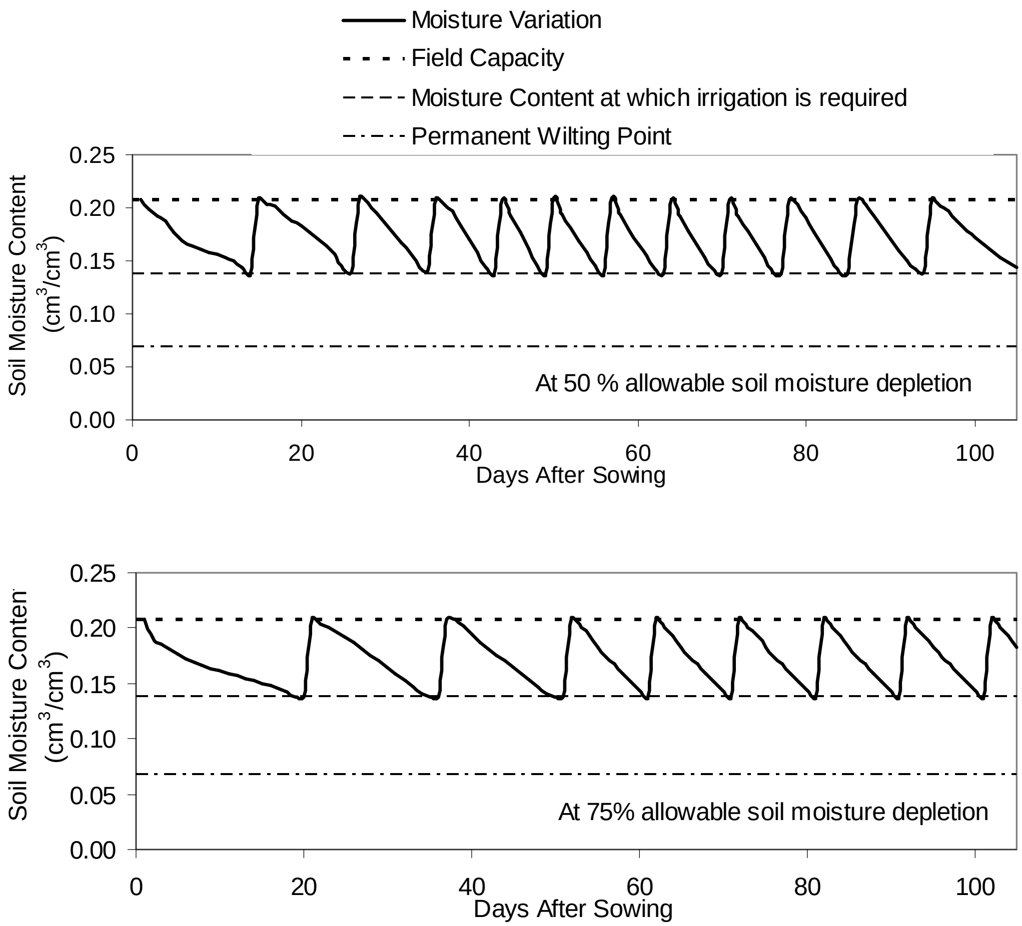


Figures 7 and 8. Vadoze zone soil moisture profiles on discrete days in the crop period

The observed and simulated values, The COV quantifies the amount of ñrandom scatter of the simulated and measured values about 1:1 line and ARE quantify the extent to which model simulations overestimate (positive ARE) or underestimate (negative ARE) the measured values. Corresponding values of error statistics for observed and simulated soil moisture at different depths are shown in the Figs 5-7. In case of observed and simulated soil moisture profiles the COD, COV and ARE values range between 0.74-0.92, 0.08-0.32 and -5.4-9.6 respectively. The values of error statistics fall in satisfactory-high agreement range.

It can be postulated from the above discussion that numerical model involving O-R model coupled with soil moisture flow equation, when applied to precisely determined soil parameters, crop data and crop evapotranspiration accurately simulates the soil moisture dynamics in the crop root zone. This provides the exact soil moisture availability for the plant moisture uptake in the crop root zone. Generally the irrigation is practiced when the average moisture content with in the root zone depth attains certain value between the field capacity and permanent wilting point (Prasad, 1988). This value of moisture content is called the allowable depletion level.

For different depletion levels required scheduling of irrigation is carried out. For optimal scheduling, adequate scheduling criterion is an important parameter in determining the frequency of irrigation events. The two parameters which contribute to assigning an adequate scheduling criterion are; allowable moisture depletion level and root depth considered for accounting the average soil moisture level. The hypothetical condition of no-rainfall is considered during the crop period of Maize. Though, allowable moisture depletion level is dependent on the type of crop and the moisture retention capacity of the soil, 50 % and 75 % moisture depletion levels are considered in the present study. The effective root depth considered for accounting the average soil moisture status is 0.3 m. The optimal irrigation schedule at 50 and 75 % allowable moisture depletion level are given in Fig. 9 and 10.



Figures 9 and 10. Irrigation schedule for Maize at different allowable moisture depletion levels.

Summary and Conclusions

A numerical model has been formulated to compute the soil moisture content profiles under transient field conditions. A non-linear root water uptake model has been used as sink term to represent plant moisture uptake. Numerical model takes into account a variable transpiration rate and non-uniform initial soil moisture content. Rainfall, irrigation and evaporation are treated as sources of non-uniform potential surface flux. Plant control on water uptake when soil moisture is a limiting factor is not considered. The input parameters have been precisely determined using the field crop experiments.

Non-linear root water uptake model involving the optimal non-linearity coefficient has been found to represent the actual plant moisture uptake dependably. Application of the numerical model to field conditions and comparison of the results with field measured data showed good agreement. Precisely determined crop evapotranspiration is the dominant factor in predicting soil moisture dynamics. The practical significance of the study lies in the computation of optimal irrigation schedules for field condition using the numerical model coupled with adequate scheduling criterion. Accurately computed soil moisture profiles result in generating optimal frequency of the irrigation and hence, results in irrigation water saving.

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