Computing contact forces of elastic structure based on entropy in statistical physics

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Abstract

We extend entropy and its use in statistical physics to evaluate contact forces in the continuum mechanics of elastic structures. Each potential contact node of an elastic structure discretized by finite elements along with the normalized contact force on the node is considered as a system and all potential contact nodes together with their normalized contact forces are considered as a canonical ensemble, with the normalized contact force of each node representing the microstate of the node. The product of non-penetration conditions for potential contact nodes and the normalized nodal contact forces then act as an expectation that its value will be zero, and maximizing the entropy under the constraints of the expectation and the minimum potential energy principle results in an explicit probability distribution for the normalized contact forces that shows the relation between contact forces and displacements in a formulation similar to the formulation for particles occupying microstates in statistical physics. Moreover, an iterative procedure that solves a series of isolated systems to find the contact forces is presented. Finally, an example is examined to verify the correctness and efficiency of the procedure.

Keywords: Contact forces, Entropy, Ensemble, Finite elements, Statistical physics.

Introduction

Entropy is one of the core concepts in statistical physics and information theory, and the parallelism between the two subjects goes beyond replacing one name with another, as each gives us a new way of thinking about the other. The concept of entropy has been used in various fields of science and engineering and has received much attention in problems aimed at determining the disorder, possibility, or uncertainty in a physical ensemble. At the same time, entropy is also thought of as merely a very useful formalism, that is, a convenient means for understanding and representing physical principles in mathematical terms.

In this work, the elastic bodies are discretized by finite elements, and the finite element nodes are separated into two groups: one includes the potential contact nodes on the potential contact boundary, and the other includes the nodes on the rest of the body. We consider the contact part as a canonical ensemble, as in statistical physics. Table 1 presents a comparison of entropy in statistical physics and in contact mechanics.

Iterative procedure

In the finite element analysis of the displacement of a structure with a contact condition, the principle of minimum potential energy is given by

$$\min_{\mathbf{u}\in\mathscr{U}} \pi(\mathbf{u}),$$

Table 1. The descriptions of entropy in statistical physics and contact mechanics.

Description in statistical physics	Description in contact mechanics
a simple system or a particle	a potential contact node
microstate of a system	normalized contact force of a node
a complex system	a number of contact nodes
a microstate of a complex system	a contact state of a number of potential contact nodes
an ensemble of complex systems	all potential contact nodes and their contact states
probability of a microstate	normalized contact force of a node
temperature	displacement
non-isolation concerns temperature	non-isolation concerns displacement
expectation of energy	expectation of work by contact forces

with $\pi(\mathbf{u}) = \frac{1}{2}\mathbf{u}^T\mathbf{K}\mathbf{u} - \mathbf{c}^T\mathbf{u}$ and $\mathscr{U} = \{\mathbf{u} \in \Re^n | d(\mathbf{u}) \in \Re^m, d(\mathbf{u}) \leq \mathbf{0}\}$, where $\mathbf{K} \in \Re^{n\times n}$ is a positive definite stiffness matrix, n gives the degrees of freedom, and $\mathbf{c} \in \Re^n$ is the vector combining the body force and the external force. The corresponding Lagrangian function for the principle of minimum potential energy is

$$L(\mathbf{u}, \lambda, \alpha) = \pi(\mathbf{u}) + \alpha\lambda^T d(\mathbf{u}).$$

With some derivations, the iterative procedure is listed as follows:

(1) *Define the tolerance value* δ*, and set* $k=0$*, calculate* $\lambda^k = \frac{1}{m}\mathbf{1}$;

(2) *Calculate* $\alpha^k = \frac{\lambda^{kT}(\mathbf{A}\mathbf{K}^{-1}\mathbf{c}-\mathbf{g})}{\lambda^{kT}\mathbf{A}\mathbf{K}^{-1}\mathbf{A}^{kT}\lambda}$, $\mathbf{u}^k = \mathbf{K}^{-1}\left(\mathbf{c} - \alpha^k\mathbf{A}^T\lambda^k\right)$*, and*

$$\lambda^{k+1} = \frac{\exp\left(\alpha^k \exp(k)\left(\mathbf{A}\mathbf{u}^k - \mathbf{g}\right)\right)}{\mathbf{1}^T\exp\left(\alpha^k \exp(k)\left(\mathbf{A}\mathbf{u}^k - \mathbf{g}\right)\right)};$$

(3) *If* $\|\lambda^{k+1} - \lambda^k\|$ *is less than or equal to* δ*, then terminate the iteration, or else set* $k = k+1$*, then go to step 2.*

Verifiction

An elastic cylinder that is in the state of plane strain comes to contact with a foundation as shown in Figure 1. Figure 2 shows the results of contact forces on the potential contact nodes or node pairs for the cases of a rigid foundation and an elastic foundation, respectively. The numbers on the lines represent the iteration number, and the square marks show the reference solution produced by the commercial code. Six and five iterations, respectively, are needed to calculate the solutions for the two cases. Tables 2 and 3 give the results of the entropy, the total nodal contact force α, and the potential energy PE in each iteration for the two cases.

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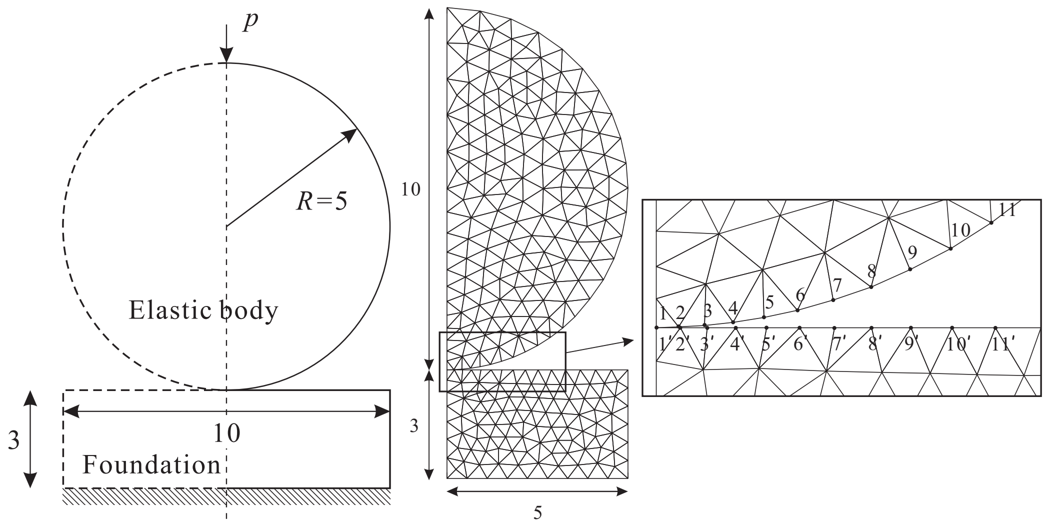

Figure 1. An elastic cylinder coming into contact with a foundation.

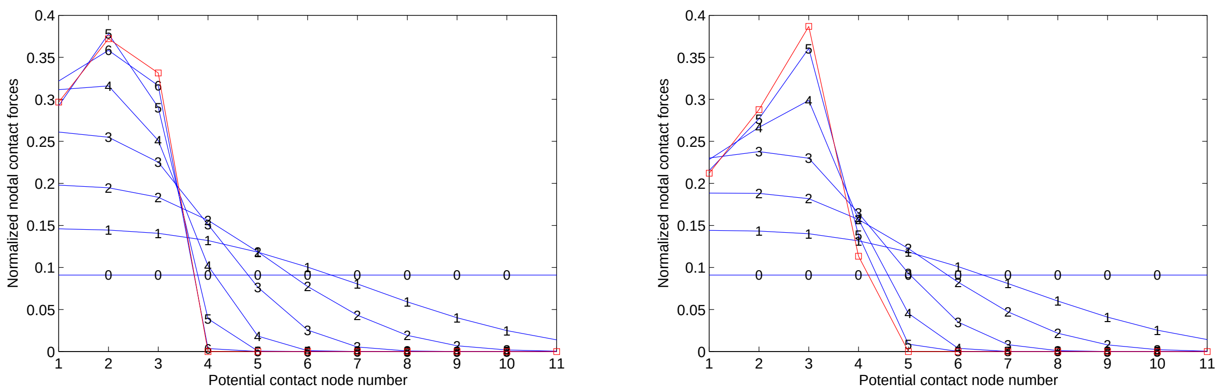

Figure 2. The results of nodal contact forces in the case of rigid (left) and elastic (right) foundations.

Table 2. The results for the case of a rigid foundation.

Iter. no.	0	1	2	3	4	5	6
Entropy	2.398	2.237	1.952	1.643	1.388	1.216	1.117
α	80.000	80.000	80.000	80.000	80.000	80.000	80.000
PE	-26.530	-15.254	-10.352	-8.257	-7.802	-7.612	-7.582

Table 3. The results in the case of an elastic foundation.

Iter. no.	0	1	2	3	4	5
Entropy	2.398	2.241	1.975	1.700	1.504	1.369
α	80.000	80.000	80.000	80.000	80.000	80.000
PE	-27.485	-16.743	-12.262	-10.558	-10.136	-10.028