Acoustic simulation using a gradient-weighted finite element method

*G. Wang^{1,2}, X.Y. Cui^{1,2}, and †G.Y. Li^{1,2}

¹State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, China. ²Colloborative Innovation Center of Intelligent New Energy Vehicle, Shanghai City, China.

> *Presenting author: wanggangny@163.com †Corresponding author: gyli@hnu.edu.cn

Abstract

It is well-known that the traditional finite element method (FEM) fails to provide accurate results to the Helmholtz equation with the increase of wave number due to the "pollution error" caused by the numerical dispersion. In order to overcome this deficiency, a gradient-weighted finite element method (GW-FEM) that combines Shepard interpolation and linear shape functions is proposed in this work. Three-node triangular (for 2D space) and four-node tetrahedral (for 3D space) elements that can be generated automatically for any complicated geometries are adopted to discretize the problem domain. For each independent element, a compacted support domain is formed based on the element itself and its adjacent elements sharing common edges (or faces). With the aid of Shepard interpolation, a weighted acoustic gradient field is then formulated, which will be further used to construct the discretized system equations through the generalized Galerkin weakform. As the gradient weighted operation can provides the GW-FEM model a close-to-exact stiffness, thus significantly reduces the dispersion error in computational acoustics. Numerical examples, including both benchmark and practical engineering cases, have been studied using the present algorithm. The results demonstrate that the GW-FEM model possesses the following interesting properties compared to the standard FEM and the other high-performance acoustic elements: (1) insensitive to the increase of wave number; (2) super-accuracy and super-convergency; (3) higher computational efficiency; (4) performs well even for very distorted meshes.

Keywords: Numerical methods, Acoustic problems, Shepard interpolation, Linear shape function, Discretization error.