An Integrated Linear Reconstruction for Finite Volume Scheme on Unstructured Grids

Li Chen[1,a)] and Ruo Li[1,b)]

[1]School of Mathematical Sciences, Peking University, Beijing 100871, China

[a)]Presenting author: cheney@pku.edu.cn

[b)]Corresponding author: rli@math.pku.edu.cn

ABSTRACT

Linear reconstruction based on local cell-averaged values is the most commonly adopted techniques to achieve a second-order accuracy when one uses the finite volume scheme on unstructured grids. For solutions with discontinuities appearing in such as conservation laws, a certain limiter has to be applied to the predicted gradient to prevent numerical oscillations. We propose a new formulation for linear reconstruction on unstructured grids, which integrates the prediction of the gradient and the limiter together. By solving on each cell a tiny linear programming problem without any parameters, the gradient is directly obtained which satisfies the monotonicity condition. It can be shown that the resulting numerical scheme with our new method fulfils a discrete maximum principle with fair relaxed geometric constraints on grids. Numerical results demonstrate that our method achieves satisfactory numerical accuracy with theoretical guarantee of local discrete maximum principle.

Keywords: Linear reconstruction, Finite volume method, Simplex method, Linear programming.

Table 1. Convergence test for linear advection equation.

	# cells	L^1 error	Order	L^∞ error	Order	CPU time (s)
Scalar limiter	992	1.73e-1	0.78	4.79e-1	0.67	0.9
	3968	8.62e-2	1.01	2.66e-1	0.85	7.4
	15872	4.44e-2	0.96	1.37e-1	0.95	60.3
	63488	2.41e-2	0.88	7.38e-2	0.90	481.7
Integrated linear reconstruction	992	7.02e-2	1.55	2.86e-1	1.05	1.0
	3968	2.21e-2	1.67	1.34e-1	1.10	8.6
	15872	5.99e-3	1.88	5.80e-2	1.20	68.7
	63488	1.58e-3	1.93	2.45e-2	1.25	553.9

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