Preserving Hyperbolicity in Stochastic Galerkin Method for Uncertainty Quantification

Z. N. Cai[1], R. Li[2]and Y. L. Wang[3,a)]

[1]Center for Computational Engineering Science, RWTH Aachen University, Aachen, Germany

[2]HEDPS & CAPT, LMAM & School of Mathematical Sciences, Peking University, Beijing, China

[3]Institute of Applied Physics and Computational Mathematics, Beijing, China

[a)]Corresponding author, Presenting author: wangyanliwyl@gmail.com

ABSTRACT

We first investigate the structure of the systems derived from the gPC based stochastic Galerkin method for the nonlinear hyperbolic systems with random inputs. This method adopts a generalized Polynomial Chaos (gPC) approximations in the stochastic Galerkin framework, but such approximations to the nonlinear hyperbolic systems do not necessarily yield hyperbolic systems [1]. Thus based on the work in [2], we propose a framework to carry out the model reduction for the general nonlinear hyperbolic system to derive a final global system. Within this framework, the nonlinear hyperbolic system in one space dimension and the symmetric hyperbolic system in multiple space dimensions are reduced into a symmetric hyperbolic system based on the stochastic Galerkin method. We note that the basis functions in the expansion are not restricted to the random-dependent polynomials as that in gPC method and there is no restriction on the dimensions of the random variables neither.

Keywords: uncertainty quantification, hyperbolic system, stochastic Galerkin method, Polynomial chaos, model reduction

References

[1] Després, B., Poëtte, G. and Lucor, D. (2013) Robust uncertainty propagation in systems of conservation laws with the entropy closure method. In *Uncertainty Quantification in Computational Fluid Dynamics*, volume 92 of *Lecture Notes in Computational Science and Engineering*, pages 105–149. Springer International Publishing.

[2] Cai, Z. N., Fan, Y. W. and Li, R. (2015) A framework on moment model reduction for kinetic equation. *SIAM J. Appl. Math.* **75**(5):2001–2023.