Car-following model with considering vehicle's backward looking effect and its stability analysis

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Abstract

In this paper, an extended car-following model is derived by considering vehicle's backward looking effect which is based on the optimal velocity model and the optimal velocity (OV) function is extended by introducing variable safety distance. Also, a new control signal including more comprehensive information is introduced on the viewpoint of feedback control. Furthermore, the stability condition for the model is derived and the numerical simulation is carried out to investigate the advantage of the proposed model with control signal which can alleviate the traffic jams efficiently. The results are also consistent with the theoretical analysis correspondingly.

Keywords: Car-following model, Feedback control method, Stability condition, Variable safety distance.

Introduction

In recent decades, traffic flow theories have attracted much attention of scientists' and researchers' in the study of mathematical physics and control theory. Because the traffic congestion has closely influenced human's daily life up to present, such as traffic accident, fuel consumption and air pollution. As for traffic behavior, many approaches have been introduced to investigate the properties of traffic flow, and obtained some significant results [1-5].

Modern traffic is one of the most significant symbols of social modernization which provides much convenience for our daily life. However, traffic congestion problem is also being increasingly deteriorated because of the huge traffic flux. Back to 1953, Pipes [6] developed a car following model to restrain the traffic congestion and provided some relevant results through theoretical analysis, which assumed that the behind vehicle adjusted its behavior following the preceding vehicle's action in the same lane. After that, Newell [7] proposed a car-following model with a differential equation and gave some graphic description for the optimal velocity (OV) function in 1961. Then it's worth pointing out that an vital extended car-following model called optimal velocity model (OVM) was introduecd by Bando et al. [8]. In the OVM, the acceleration of the vehicle at the same time was determined by the difference between actual velocity and an optimal velocity. Based on this (OVM), a great deal of car-following models have been extended by adding more comprehensive information into the real traffic system [9-12].

In 1999, Konishi et al. [13] developed a chaotic car-following model by setting time delay feedback control signals, and studied single-lane traffic operation without reverse phenomenon under an open boundary condition. In 2007, Han et al. [14] put forward to a modified CM car-following model and found that their model could promote the stability of traffic flow. Recently, Zheng et al. [15] presented an improved car-following model with considering lateral effect and its feedback control research, and the obtained results were correspond to the theoretical analysis. Additionally, other researches related to the control scheme have been carried out in a piecemeal form gradually [16][17].

Even in the physical community, the car-following model is still a hot topic. But up to now, we can hardly see studies concerning car-following in a viewpoint of control methods. So in this paper, it's necessary to provide a modified car-following model considering vehicle's backward looking effect based on the control theory which means a new control scheme that takes more comprehensive information into account is proposed. Detail definitions are in the section 3.

The outline of this paper is organized as follows. In Sec. 2, the modified car-following model considering vehicle's backward looking effect is presented, and its stability condition is analyzed via control method. In Sec. 3, the model including control signal is established and feedback control theory is used to analyze the stability conditions. In Sec. 4, several numerical simulations are carried out to verify the theoretical results. Conclusions are given in Sec. 5.

Car-following model and its stability analysis

Modified model

This research is based on OVM [8] in 1995. The dynamic equation is described as

$$\frac{dv_n(t)}{dt} = \frac{1}{\tau}\left[pV_p(\Delta x_n(t), v_n(t)) + qV_b(\Delta x_{n-1}(t), v_{n-1}(t)) - v_n(t)\right] \tag{1}$$

Where $\Delta v_n(t) = v_{n+1}(t) - v_n(t)$ is the velocity difference between $\Delta x_n(t) = x_{n+1}(t) - x_n(t)$, the n-th considering vehicle and the preceding vehicle; $x_n(t)$ is the real position of the n-th considering car at time t; $a = \frac{1}{\tau}$ is the sensitivity of driver and is the inverse of delay time τ . $V_p(\Delta x_n(t), v_n(t))$ is the improved optimal velocity (OV) function for forward looking and $V_b(\Delta x_{n-1}(t), v_{n-1}(t))$ is the modified optimal velocity (OV) function for backward looking; $p, q(p \geq q)$ stands for the relative weights of two OV functions. The two OV functions are given as:

$$V_p(\Delta x_n(t), v_n(t)) = \frac{v_{max}}{2}[\tanh(\Delta x_n(t) - h_n^v) + \tanh(h_n^v)] \tag{2}$$

$$V_b(\Delta x_{n-1}(t), v_{n-1}(t)) = \frac{v_{max}}{2}[\tanh(\Delta x_{n-1}(t) - h_{n-1}^v) + \tanh(h_{n-1}^v)] \tag{3}$$

$$h_n^v = d_1 T_s v_n(t) + h_c ; h_{n-1}^v = d_2 T_s v_{n-1}(t) + h_c \tag{4}$$

where v_{max} is the maximum velocity and h_c is the traditional safety distance; T_s is the time step unit, and d is the reaction coefficient for $v_n(t)$.

Stability analysis

The dynamical equation is rewritten as follows:

$$\begin{cases} \dfrac{dv_n(t)}{dt} = a\left[pV_p\left(y_n(t), v_n(t)\right) + qV_b\left(y_{n-1}(t), v_{n-1}(t)\right) - v_n(t)\right], \\ \dfrac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t), \end{cases} \tag{5}$$

where $y_n(t) = \Delta x_n(t)$.

We suppose the desired velocity of vehicles and comprehensive distance are v^* and y^*, so the steady state of the following vehicles is

$$[v_n(t), y_n(t)]^T = [v^*, y^*]^T. \tag{6}$$

Then, consider an error system around steady state (6), that is,

$$\begin{cases} \dfrac{d\delta v_n(t)}{dt} = a\left[p\delta y_n(t)\Lambda_1 + p\delta v_n(t)\Lambda_2 + q\delta y_{n-1}(t)\Lambda_3 + q\delta v_{n-1}(t)\Lambda_4 - \delta v_n(t)\right] \\ \dfrac{d\delta y_n(t)}{dt} = \delta v_{n+1}(t) - \delta v_n(t) \end{cases} \tag{7}$$

where $\delta v_n(t) = v_n(t) - v^*$, $\Lambda_1 = \dfrac{\partial V(y_n(t), v_n(t))}{\partial y_n(t)}\Big|_{y_n(t)=V_p^{-1}(v_0)}$, $\Lambda_2 = \dfrac{\partial V(y_n(t), v_n(t))}{\partial v_n(t)}\Big|_{v_n(t)=v_0}$, $\Lambda_3 = \dfrac{\partial V(y_{n-1}(t), v_{n-1}(t))}{\partial y_{n-1}(t)}\Big|_{y_{n-1}(t)=V_b^{-1}(v_0)}$, $\Lambda_4 = \dfrac{\partial V(y_{n-1}(t), v_{n-1}(t))}{\partial v_{n-1}(t)}\Big|_{v_{n-1}(t)=v_0}$, $\delta y_n(t) = y_n(t) - y^*$.

After Laplace transformation for traffic system (7), we can get

$$\begin{bmatrix} V_n(s) \\ Y_n(s) \end{bmatrix} = \frac{1}{p(s)} \begin{bmatrix} s & ap\Lambda_1 \\ -1 & s + a - ap\Lambda_2 \end{bmatrix} \begin{bmatrix} aq\Lambda_4 & 0 & aq\Lambda_3 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{n-1}(s) \\ V_{n+1}(s) \\ Y_{n-1}(s) \end{bmatrix} \tag{8}$$

$$p(s) = s^2 + a(1 - p\Lambda_2)s + ap\Lambda_1 \tag{9}$$

where $V_n(s) = L(\delta v_n(t))$, $Y_n(s) = L(\delta y_n(t))$, $L(.)$ denotes the Laplace transform and s is a complex variable.

In reality, based on the control theory, we obtain the transfer function $G(s)$, that is

$$G(s) = \frac{(aq\Lambda_3 + aq\Lambda_4)s + ap\Lambda_1}{s^2 + a(1 - p\Lambda_2)s + ap\Lambda_1} \tag{10}$$

Thus, traffic jams will never occur in the traffic flow system if $p(s)$ is stable and $\|G(s)\|_{\infty} \le 1$. In fact, based on the Hurwitz stability criterion, we can get that $p(s)$ is stable. So, the stability condition is given by

$$a \ge \frac{2p\Lambda_1 + q(\Lambda_3 + \Lambda_4)}{(1 - p\Lambda_2)^2} \tag{11}$$

Feedback control scheme

In this part, an extended feedback control signal including more comprehensive information is added into system (1), so we have

$$\frac{dv_n(t)}{dt} = a\left[pV_p(\Delta x_n(t), v_n(t)) + qV_b(\Delta x_{n-1}(t), v_{n-1}(t)) - v_n(t)\right] + u_n(t) \tag{12}$$

$$u_n(t) = \lambda \Delta v_n(t) + \gamma^2 H(y_n(t) - h_n^v)(y_n(t) - h_n^v), \tag{13}$$

where λ is the reaction coefficient for the relative velocity $\Delta v_n(t)$ and γ is another reaction coefficient for the $H(y_n(t) - h_c)(h_c - y_n(t))$. Function $H(.)$ is described as

$$H(y_n(t) - h_n^v) = \begin{cases} 0, & y_n(t) - h_n^v > 0, \\ 1, & y_n(t) - h_n^v \le 0, \end{cases} \tag{14}$$

As $y_n(t) - h_n^v \le 0$, our feedback control signal $u_n(t)$ is

$$u_n(t) = \lambda \Delta v_n(t) + \lambda^2 (y_n(t) - h_n^v), \tag{15}$$

Under this condition, the dynamical Eq.(12) can be described as

$$\begin{cases} \dfrac{dv_n(t)}{dt} = a\left[pV_p(y_n(t), v_n(t)) + qV_b(y_{n-1}(t), v_{n-1}(t)) - v_n(t)\right] \\ +\lambda(v_{n+1}(t) - v_n(t)) + \gamma^2(y_n(t) - h_n^v), \\ \dfrac{dy_n(t)}{dt} = v_{n+1}(t) - v_n(t), \end{cases} \tag{16}$$

Similar to the analysis of second part, the transfer function $\widetilde{G}(s)$ can be obtained after Laplace transform.

$$\widetilde{G}(s) = \frac{(aq\Lambda_3 + aq\Lambda_4 + \lambda)s + (ap\Lambda_1 + \gamma^2)}{s^2 + (a + dT_s\gamma^2 + \lambda - ap\Lambda_2)s + ap\Lambda_1 + \gamma^2} \tag{17}$$

$$\widetilde{p}(s) = s^2 + (a + dT_s\gamma^2 + \lambda - ap\Lambda_2)s + ap\Lambda_1 + \gamma^2 \tag{18}$$

In fact, the traffic jams will be weaken if $\widetilde{p}(s)$ is stable and $\|\widetilde{G}(s)\|_{\infty} \le 1$. Furthermore, $\widetilde{G}(j\omega)$ must be smaller than 1 for all positive ω^2 to ensure stability. Hence, the stability criterion of the extended mode is given by

$$Aa^2 + Ba + C \geq 0 \tag{19}$$

where $A=(1-p\Lambda_2)$, $B=2(1-p\Lambda_2)(dT_s\gamma^2+\lambda^2)-(2p\Lambda_1+q\Lambda_3+q\Lambda_4)$, $C=(dT_s\gamma^2+\lambda^2)^2-(2\gamma^2+\lambda)$.

Numerical simulations

In this simulations, the parameters for the improved car-following model are set as $y^* = 5.0m$, $a = 2s^{-1}$, $v^* = 20m/s$, $p = 0.8$, $q = 0.2$, $d = 0.3$ and $T = 0.1s$. It is assumed that all vehicles have the same parameters. The initial condition is the steady state for the model, and the initial positions and speeds are set as $y_n(0) = y^*$, $v_n(0) = v^*$, and $N = 120$ is the total number of vehicles. We consider a case where the leading vehicle stops suddenly for $v_n(0) = 0, t = nT = 100-103$.

Figure. 1 shows the velocity-time patterns of the 1st, the 25th and the 50th vehicles with different parameter values of γ. It can be seen from Fig. 1 that with the control signal, as the reaction coefficient γ decreases from 0.85 to 0.35, the stability of the traffic system is strengthened. And we can find that vehicles can reach steady running state in relatively short time as the reaction coefficient γ decreases. The amplitude of the velocity for the 25th vehicle decreases and the 50th vehicle runs smoothly.

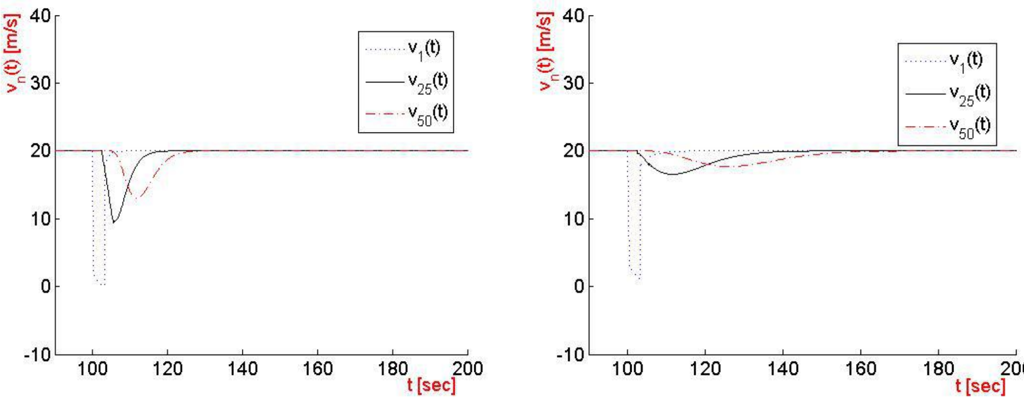

Figure 1. Numerical simulations for the modified car-following model with $\lambda = 0.65$, $v_{max} = 25m/s$, $\gamma = 0.85$ **(left);** $\lambda = 0.65$, $v_{max} = 25m/s$, $\gamma = 0.35$ **(right)**

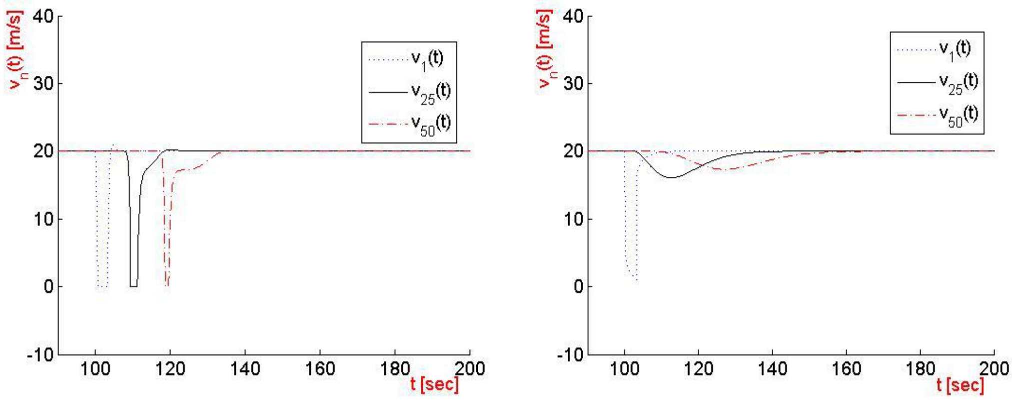

Figure 2. Numerical simulations for the modified car-following model with $\gamma = 0.35$, $v_{max} = 25m/s$, $\lambda = 0.15$ **(left);** $\gamma = 0.35$, $v_{max} = 25m/s$, $\lambda = 0.65$ **(right)**

Figure. 2 shows the velocity-time patterns of the 1st, the 25th and the 50th vehicles with different parameter. It can be seen from Fig.2 that with the control signal, as the reaction coefficient increases from 0.15 to 0.65, the stability of the traffic system is strengthened. And we can find that vehicles can reach steady running state in relatively short time as the reaction coefficient increases. The amplitude of the velocity for the 25th vehicle decreases and the 50th vehicle runs placidly. The simulation results of Fig. 1 and Fig. 2 illustrate that feedback control plays an vital role in vehicle dynamic driving behavior.

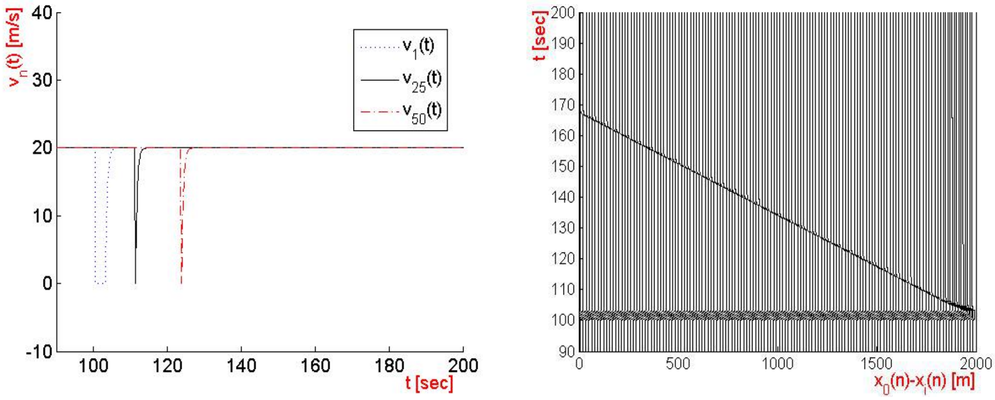

Figure 3. (a) Space-time plot of the traffic system (b) Temporal velocity behavior of the first,25th and 50th vehicles ($\gamma = 0, \lambda = 0, v_{max} = 20m/s$)

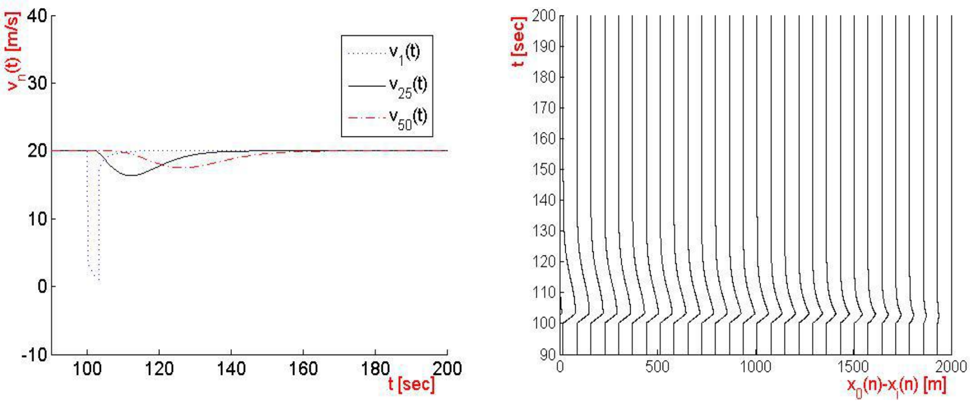

Figure 4. (a) Space-time plot of the traffic system (b) Temporal velocity behavior of the first, 25th and 50th vehicles ($\lambda = 0.75, \gamma = 0.35, v_{max} = 25m/s$)

Then, we simulate the system with the modified control scheme. As the stability condition in Eq. (11) and Eq. (18) is met, a comparison between the results in Figs. 3-4 illustrate that with control signal, although the maximum speed is larger compared with Fig. 3, as we choose the right parameters ($\lambda = 0.75, \gamma = 0.35, v_{max} = 25m/s$), it can be seen that vehicles can reach more steady running state in relatively short time. The amplitude of the velocity for the 25th vehicle decreases and the 50th vehicle runs more smoothly. Thus, it can be concluded that the proposed car-following model is useful for suppressing the increasingly serious traffic jams.

Conclusions

In this paper, an extended car-following model is established considering vehicle's backward looking effect. The optimal velocity (OV) function is extended by introducing variable safety distance. The effect of some important information (such as the relative velocity and the difference between safety variable distance and headway) on the traffic current and the jamming transition has been investigated with the use of numerical and analytic methods. The stability condition is obtained for the new model via control method. The numerical simulation is used to show the advantage of the proposed model with control scheme. The results are consistent with the theoretical analysis.

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