A reliability optimization allocation method considering differentiation of functions Based on Goal Oriented method

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Abstract

A new reliability optimization allocation for multifunction systems considering differentiation of functions based on GO methodology is proposed in this paper. First, constraints considering differentiations of functions are proposed based on GO method, which are function importance factor constraint, and system reliability constraint, respectively. Then, the objective function of optimization allocation problem is built to minimize the system cost. Based on above, the mathematic model of reliability optimization allocation problem for multifunction systems considering differentiations of functions is established. In addition, an improved Ant Colony Optimization (ACO) is proposed to solve this mathematic model. Furthermore, the process of the new method is formulated. Finally, the new method is applied in reliability optimization allocation of Power-Shift Steering Transmission whose goal is to minimize the system cost. Compared with the results by using basic ACO, it is shown that the new method is reasonable, advantageous, and feasible for the reliability optimization allocation problem with differentiation of functions. Clearly, this study solves the disadvantages of the existing reliability optimization allocation methods efficiently so that it can quickly, efficiently, and directly allocate the system reliability index to design units for complex systems. All in all, this paper not only provides a new approach to conduct reliability optimization allocation for multifunction systems considering differentiation of functions, but also improves the theory and widens the application of GO methodology. In addition, this paper can also provide guidance for the similar reliability optimization problem

Keywords: reliability optimization, differentiation of functions, importance factor, multifunction systems, Ant Colony Optimization

Introduction

The aim of reliability optimization allocation is that the system reliability index is allocated to design units considering restrictions, which are cost, size, and weight etc., in order to provide guidance for reliability design of product. Nowadays, a large number of studies on reliability optimization allocation are mainly as follows: (i) fault-tolerance mechanism, (ii) active and cold-standby redundancy, (iii) optimization techniques, (iv) multi-objective optimization, (v) optimization techniques: [Kuo et al., (2007)]. With development of technology, the multifunction systems are often applied in Engineering, and have a key role. While, a large number of research works of reliability allocation for multifunction systems are only considered one single main function of system, and ignored other functions. Clearly, it will lead to an unreasonable and a bias of reliability allocation result. Thus, some researchers are focus on the reliability optimization of multifunction systems. Lim et al. proposed the allocation of the equipment path in a multi-stage manufacturing process: [Lim et al. (2015)]. An improved AGREE method is proposed to solve reliability allocation of multi-mission networked avionic system without considering resource constraint and system structure: [Li et al. (2015)]. For reliability optimization allocation of multifunction systems, Yi et al. proposed the reliability optimization allocation method for units designed and units selected versions [Yi et al. (2015a-b); Yi et al. (2016a)]. While, above reliability optimization allocation methods have three disadvantages, as follows: (i) The product is finalized production through multiple design revisions, but the above method is difficult to conduct reliability re-allocation quickly and efficiently at the situation of design changes, (ii) The reliability models used in above methods are hard to reflect product structure, working principles, (iii) It is difficult to quickly, efficiently, and directly allocate the system reliability index to design units for complex systems containing series structure and redundant structure. In addition, the optimization technologies, such as genetic algorithm, ant colony algorithm, and neural network algorithm etc. are used to solve the problem of reliability optimization allocation effectively. And Kuo et al. overviewed the optimization techniques for reliability optimization allocation: [Kuo and Rin (2007)]. And there are three concerns of the optimization technologies for reliability optimization allocation problem, as follows: (i) To obtain satisfactory convergence effects and efficiency, (ii) To avoid local extremum problem, (iii) For specific problems, the basic optimization algorithm need to improve. It is meaningful to improve the basic algorithm so that it is applicable for specific problems and can obtain the optimal solution efficiently: [Wang and Lee (2015); Alavidoost et al. (2015); Zhao et al. (2015); Yi et al. (2016a)].

Above all, a reliability optimization allocation method for multifunction systems is described through develop reliability optimization allocation problem, and solve this optimization allocation problem by optimization technologies. Goal Oriented (GO) methodology is a success-oriented method for reliability analysis of complex systems [Yi et al. (2014a-b); Yi et al. (2015c-e); Yi et al. (2016b)]. Moreover, the reliability analysis results can be obtained through GO operation according to GO algorithm and GO model. The GO model is developed directly using product schematic diagrams, its structure, and its functional hierarchy. And GO algorithm has a high efficiency and easy to operate. Thus, GO method can be suitable for reliability optimization allocation to overcome above disadvantages of the existing reliability optimization allocation method. Furthermore, Ant Colony Optimization (ACO) has been used widely.

In view of advantages of GO method in aspects of establishing system model and reliability analysis, a reliability optimization allocation for multifunction systems considering differentiation of functions based on GO methodology is firstly proposed in this paper. First, constraints considering differentiations of functions are proposed, which are function importance factor constraint, and system reliability constraint, respectively. The function importance factor constraint is consist of the predicted function importance factors by using allocated reliability of unit based on GO method, and the allocated function importance factors. And the system reliability constraint function is consist of the target reliability of system, and the predicted reliability of system by using allocated reliability of unit based on GO method. Then, the objective function of optimization allocation problem is built to minimize the system cost. Based on above, the mathematic model of reliability optimization allocation problem for multifunction systems considering differentiations of functions is established. In addition, an improved ACO is proposed to solve this mathematic model. Furthermore, the process of the new method is formulated. Finally, the new method is applied in reliability optimization allocation of Power-Shift Steering Transmission (PSST) whose goal is to minimize the system cost. To verify the advantages and engineering applicability of the new method, the results of the new method are compared with the results by using basic ACO.

Reliability optimization allocation method considering differentiation of functions based on GO method

A reliability optimization allocation for multifunction systems considering differentiation of functions based on GO method is proposed in aspects of description of reliability optimization allocation problem, and solving algorithm.

Description of reliability optimization allocation problem

The reliability optimization allocation problem is described through corresponding mathematic model, which contains constraints considering the differentiation of functions, objective function.

Constraints considering the differentiation of functions

(1) Reliability of function and system based on GO operation

For reliability optimization allocation of multifunction systems considering differentiation of functions, the reliabilities of function and system can be obtained by using the success probability of design unit and GO algorithm to conduct GO operation according to GO model. Thus, GO model and GO operation are key elements in GO method. GO model is directly using product schematic diagrams, its structure, and its functional hierarchy, and is consist of GO operator and signal flow. GO operator represents design unit and logical relation in system, and signal flow represents the specific fluid, logical process, and the direction of GO operation. GO algorithm is key element of GO operation, and there are two kinds of GO algorithm, which are suitable for GO model with shared signal flow [Shen et al. (2000)], and GO model without containing shared signal flow [Shen et al. (2001)].

Therefore, the reliability of signal flow based on GO method for reliability optimization allocation is defined, as follows:

$$R_x = F_x(R_{x1}, R_{x2}, \cdots, R_{xy}) \tag{1}$$

where, R_x is the predicted reliability of *xth* signal flow, R_{xy} is allocated reliability of *yth* unit for calculating *xth* signal flow, $F_x(\cdot)$ is the success probability of *xth* signal flow obtained by using allocated reliability of unit to conduct GO operation according to GO model and GO algorithm.

In GO model, the reliability of output signal flows for functions and system represents the reliability of system functions and system.

(2) Constraints considering the differentiation of functions

To deal with the differentiation of functions in the process of reliability allocation, the function importance factor constraint is proposed combined the predicted function importance factors by using allocated reliability of unit based on GO method with the allocated function importance factors, and the system reliability constraint are proposed combined the predicted reliability of system by using allocated reliability of unit based on GO method with target reliability of system. Assumed that a multifunction system is consists of *m* units, and can execute *n* functions.

The higher importance factor of function, the higher reliability should be allocated. The function importance factor constraint indicates predicted function importance factor by using allocated reliability of unit based on GO method should meet the allocated function importance factor, as shown in Eq. (2).

$$\begin{cases} R_{Gw} = F_w(R_{w1}, R_{w2}, \cdots, R_{wj}) \\ g(R_{Gw}) = e^{(R_{Gw}-1)} \geq g(R_{Gw}^*) = e^{(R_{Gw}^*-1)} \end{cases} \tag{2}$$

where, R_{Gw} is the predicted reliability of *wth* function, R_{wj} is allocated reliability of *jth* unit for *wth* function, $F_w(\cdot)$ is the reliability of output signal flow represented *wth* function by using allocated reliability of unit based on GO method, $g(R_{Gw})$ is the predicted function importance factor of *wth* function, $g(R_{Gw}^*)$ is the allocated function importance factor by the estimation of function importance factors [Yi et al. (2016a)], $w = 1, 2, \cdots, n$, $1 \leq j \leq m$.

The system reliability constraint indicates predicted reliability of system by using allocated reliability of unit based on GO method should meet the target reliability of system, as shown in Eq. (3).

$$\begin{cases} R_S = F_S(R_1, R_2, \cdots, R_m) \\ R_S \geq R_S^* \end{cases} \tag{3}$$

where, R_S is the predicted reliability of system, R_i is the allocated reliability of *ith* unit, $F_S(\cdot)$ is the reliability of output signal flow of system by using allocated reliability of unit based on GO method, R_S^* is the target reliability of system, $i = 1, 2, \cdots, m$.

Objective function of reliability optimization allocation

The cost of system is greatly concerned, so the objective function of reliability optimization allocation in this paper is to minimize the cost, as follows:

$$\min C_S(R) = \sum_{i=1}^{m} c_i\left(P_i, R_i, R_{i,\min}, R_{i,\max}\right) \tag{4}$$

where, $C_S(\cdot)$ is the cost function of system; $c_i\left(P_i, R_i, R_{i,\min}, R_{i,\max}\right)$ is the cost function of design unit, i.e. $c_i\left(P_i, R_i, R_{i,\min}\right) = P_i e^{\left(\frac{R_i - R_{i,\max}}{R_{i,\min} - R_{i,\max}}\right)}$, P_i is the basic cost of *ith* unit, R_i is allocated reliability of *ith* unit, $R_{i,\min}$ is the lower limit value of reliability of *ith* unit.

Mathematic model of reliability optimization allocation

Combining above the objective function and constraints, the reliability optimization allocation problem with differentiation of functions can be given by

$$\begin{cases} \min C_S(R) = \sum_{i=1}^{m} P_i e^{\left(\frac{R_i}{R_{i,\min}} - 1\right)} \\ s.t. \\ \qquad R_{i,\min} \le R_i \le R_{i,\max} \qquad i = 1, 2, \cdots, m \\ g(R_{Gw}) = g(F_w(R_{w1}, R_{w2}, \cdots, R_{wj})) \ge g(R_{Gw}^*) \quad w = 1, 2, \cdots, n \\ R_S = F_S(R_1, R_2, \cdots, R_m) \ge R_S^* \end{cases} \tag{5}$$

Improved AOC for reliability optimization allocation problem

In order to obtain the satisfactory results effectively, the basic AOC is improved for solving the reliability optimization allocation problem with differentiation of functions. Its steps are as follows:

(1) Establishing ant colony path diagram

All of the directed paths allowed the ant individual to walk constitute the ant colony path diagram. Each directed path corresponds to an optimization allocated results, and each node value corresponds to an allocated reliability of design unit. The ant colony path diagram is shown in Figure 1.

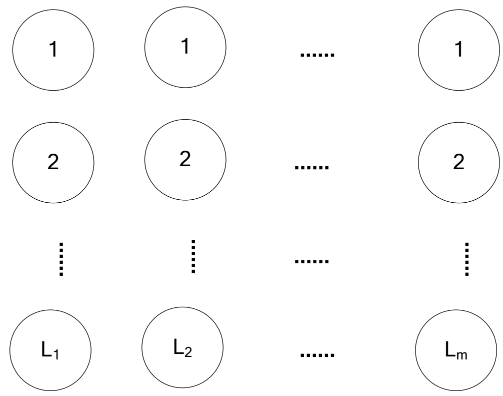

Figure 1. Ant colony path diagram

In Figure 1, the node values of each column represent the selectable allocated results of corresponding design unit. And the number of node can be obtained by Eq. (6).

$$N_i = \frac{R_{i,\max} - R_{i,\min}}{L} \tag{6}$$

where, N_i is the number of node of *i-th* column, L is the step length of node interval.

The ant colony path diagram can be represented by the cell array, as follows:

$$R = \begin{Bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,\mathrm{m}} \\ R_{2,1} & R_{2,2} & \cdots & R_{2,m} \\ \vdots & \vdots & \cdots & \vdots \\ R_{L1,1} & R_{L2,2} & \cdots & R_{Lm,m} \end{Bmatrix} \tag{7}$$

where, $R_{j,i}$ is the *j-th* selectable allocated result of *i-th* unit, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, L_i$.

(2) Initializing pheromone path diagram

According to ant colony path diagram, the corresponding pheromone path diagram can be established. The carrier of pheromone is the moving path of ant individual. And the pheromone concentration of the moving path for ant individual correspond the quality of objective function value for such moving path. The pheromone diagram will update with the change of the number of iterations, and the pheromone path diagram can be represented by the cell array, as follows:

$$\tau(Loop) = \begin{Bmatrix} \tau_{1,1} & \tau_{1,2} & \cdots & \tau_{1,\mathrm{m}} \\ \tau_{2,1} & \tau_{2,2} & \cdots & \tau_{2,m} \\ \vdots & \vdots & \cdots & \vdots \\ \tau_{L1,1} & \tau_{L2,2} & \cdots & \tau_{Lm,m} \end{Bmatrix} \tag{8}$$

where, $\tau_{j,i}$ is the pheromone element of the *j-th* selectable allocated result of *i-th* unit; Loop is the iterations times, when $Loop = 1$, the pheromone path diagram is the initialization pheromone path diagram, and $\tau_{j,i} = 1$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, l_i$.

(3) Ant colony moving

The process of formation path for each ant is defined as ant colony moving. Each path correspond a reliability allocated result. And the reliability allocated result is determined by the pheromone path diagram and the cost of each node. The reliability allocated result is obtained as follows:

(i) To establish the node probability diagram in order to represent the selected probability for ant individual in the node. The node probability diagram can be represented by the cell array, as follows:

$$P = \left\{ \begin{matrix} P_{1,1} & P_{1,2} & \cdots & P_{1,m} \\ P_{2,1} & P_{2,2} & \cdots & P_{2,m} \\ \vdots & \vdots & \cdots & \vdots \\ P_{L1,1} & P_{L2,2} & \cdots & P_{Lm,m} \end{matrix} \right\} \tag{9}$$

where, $P_{i,j}$ is the selected probability of the *j-th* selectable allocated result of *i-th* unit, $P_{i,j} = \frac{\tau_{i,j} \cdot \frac{1}{C_{i,j}}}{\sum_{j=1}^{L_i} \tau_{i,j} \cdot \frac{1}{C_{i,j}}}$, $C_{i,j}$ is the cost of *j-th* selectable allocated result of *i-th* unit.

(ii) To obtain the reliability allocated result by using the roulette wheel method to select node of each column in ant colony path diagram.

(4) Constraint IF and solving the objective function

After the ant colony moving, the reliability allocated result obtained by each ant individual needs to judge if it meets the constraints based on GO method. If it meets the constraints, setting constrain value is 1, i.e., $constrain = 1$; otherwise, setting $constrain = 0$. Then, the ant individual corresponding to the minimum value of objective function is determined among the ant individuals Satisfied the constraint.

(5) Updating pheromone path diagram

When making the next iteration computation, the pheromone path diagram needs to update in order to improve the convergence efficiency and obtain the satisfactory results. The approach of updating pheromone path diagram is as follows:

(i) For the ant individual corresponding to the minimum value of objective function in the previous iteration, the formula of updating pheromone is given by

$$\tau_{i,j}(Loop+1) = constrain \cdot \tau_{i,j}(Loop) + constrain \cdot (\frac{X}{C}) \tag{10}$$

where, X is the convergence operator, C is the cost of such ant individual.

(ii) For other ant individuals, the formula of updating pheromone is given by

$$\tau_{i,j}(Loop+1) = constrain \cdot \tau_{i,j}(Loop) \tag{11}$$

(6) Judging the termination condition

The iteration times as the termination condition of the improve ACO. If it meets the termination condition, the optimal allocation results and system cost will be output. And if it does not meet the termination condition, it will operate the algorithm from Step (3): Ant colony moving.

Above all, the operation process of improved ACO is shown in Figure 2.

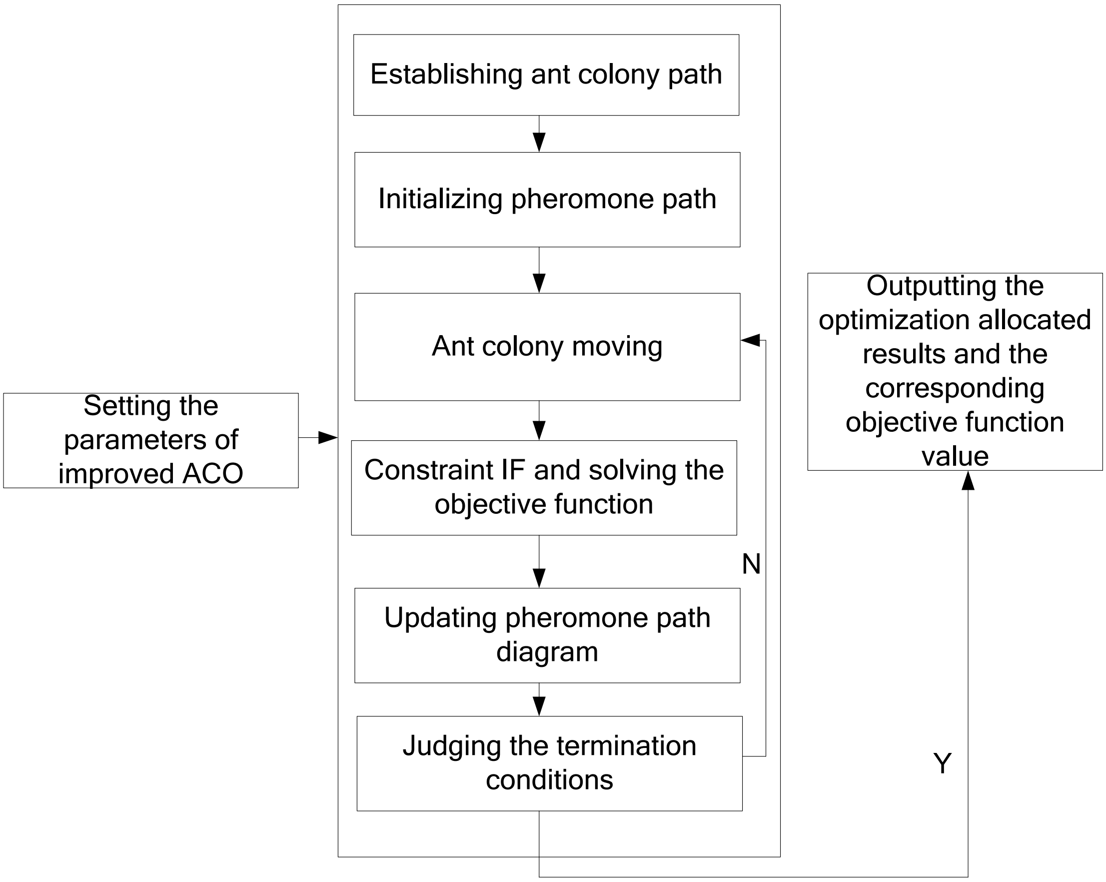

Figure 2. Operation process of improved ACO.

Reliability optimization allocation process considering differentiation of functions based on GO method

For systems with differentiation of functions, the process of reliability optimization allocation under the goal of minimizing the system cost, based on GO method, are formulated, as shown in Figure 3.

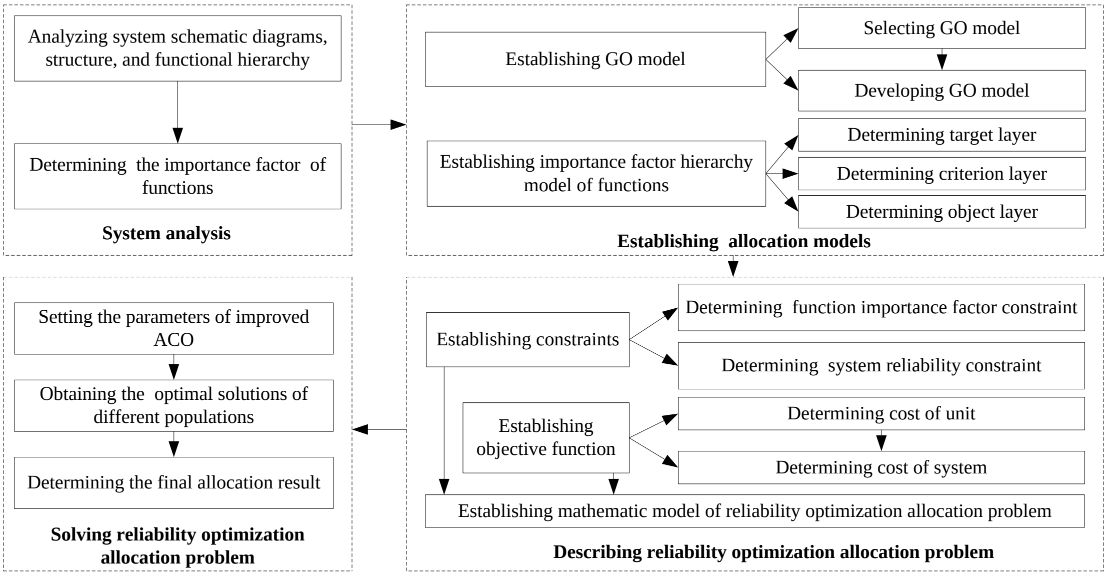

Figure 3. Reliability optimization allocation process for systems with differentiation of functions under the goal of minimizing the system cost based on GO method.

Example

The reliability optimization allocation of PSST considering differentiations of functions under the goal of minimize the system cost is conduct by the new method proposed in this paper in order to illustrate its feasibility and advantage. In order to show conveniently and compare with other results, we assume that:

(1) The PSST is very complex system, which is consist of hundreds of units in 16 subsystems, so the system reliability is allocated to units of the oil supple systems, and other subsystems in this paper. In addition, the tube and interface of system is not considered in the oil supple systems.

(2) The four main functions of PSST is considering in this paper. They are straight driving function, steering function, braking function, and fan cooling function, respectively.

(3) The basic costs of unit are set 15.

System analysis of PSST

(1) Analyzing system working principle and function

The PSST is consist of 16 subsystems, as shown in Figure 4.

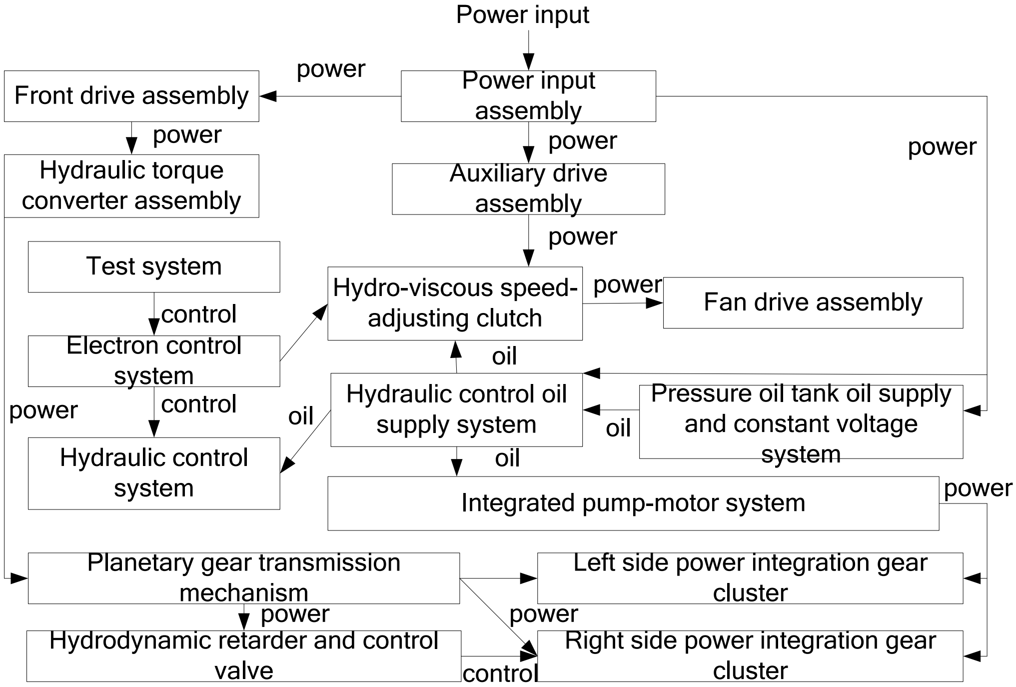

Figure 4. Working principle diagram of PSST.

The oil supply systems contain pressure oil tank oil supply and constant voltage system, and hydraulic control oil supply system. And the structure diagram of oil supply systems is shown in Figure 5.

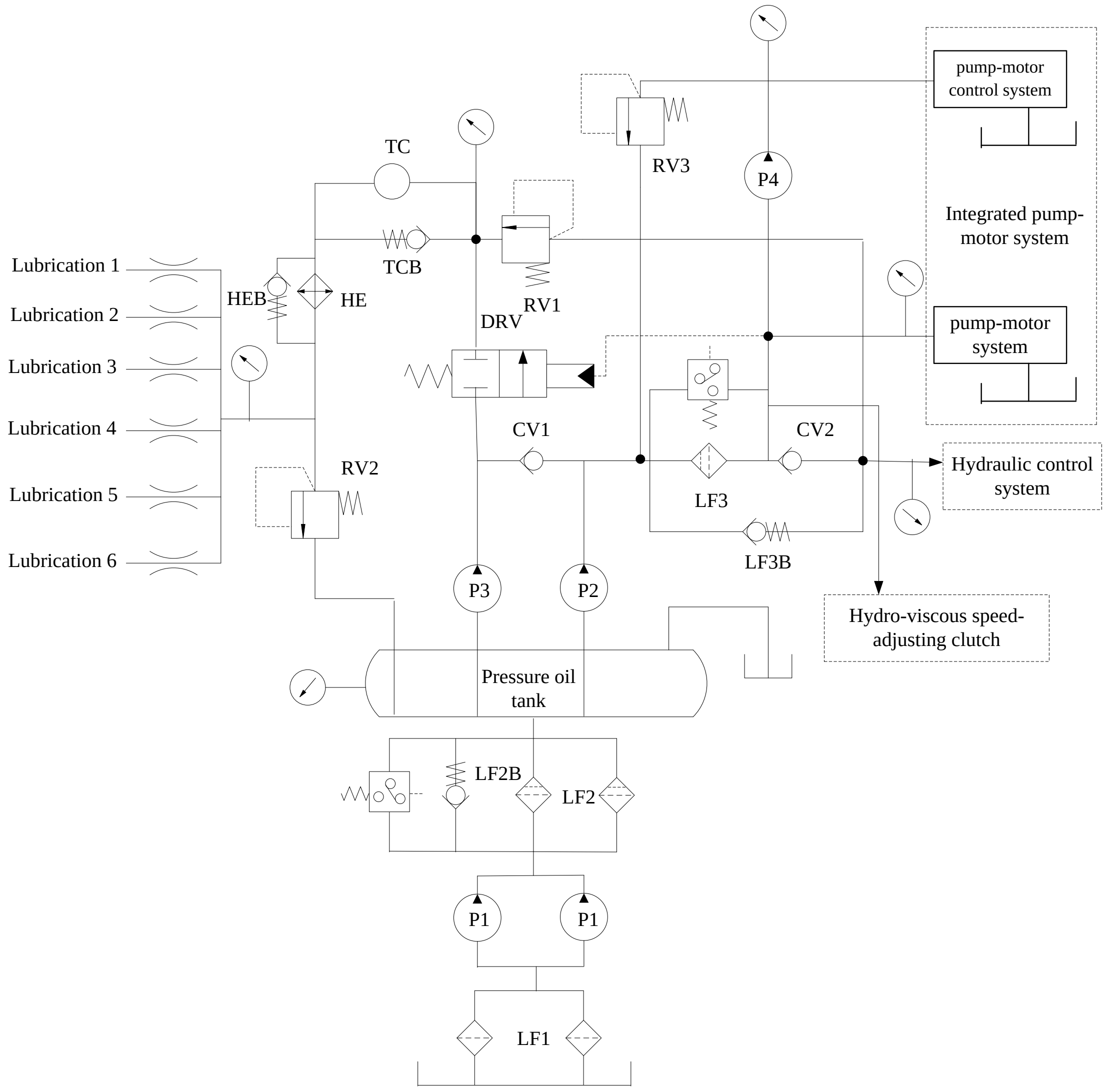

Figure 5. Structure diagram of oil supply systems.

(2) Determining the important factor of function

Only the four functions of PSST all meet the requirements in terms of importance factor, the system can be denoted as success. The reliability of function is affected by working time, functional property, and the design level. Thus, the target layer A corresponds to the system reliability index, the criterion layer C corresponds to the various factors, and the object layer P corresponds to the functions of system.

Establishing allocation models of PSST

(1) Establishing GO model of PSST

According to the analysis result of PSST, the GO operator types are corresponding function description are presented in Tab. 1. In Tab. 1, there are 6 basic GO operators, Type 5’ represents virtual input signal, whose success probability is 1, Type 5 represents input unit, Type 1 represents two-state unit, Type 6 represents unit controlled by two signals, Type 2 represents logical relation of OR, Type 10 represents logical relation of AND: [Shen et al.

(2002)]. In Tab. 1, Type 22 represents multiple-Input and multi-function unit, Type 15B represents multi-conditions control signal of multiple-Input and multi-function unit: [Yi et al. (2015b)].

Table 1. GO operator type in GO model

NO. (operator)	NO. (unit)	Component	Type	NO. (operator)	NO. (unit)	Component	Type
1	1	Power input	5'	25	22	HE	1
2	2	Power input assembly	1	26	23	HEB	1
3	3	Front drive assembly	1	27	—	OR gate	2
4	4	Hydraulic torque converter assembly	6	28	24	RV2	1
5	5	Planetary gear transmission mechanism	6	29	—	AND gate	10
6	6	Auxiliary drive assembly	1	30	25	LF3	1
7	7	Hydro-viscous speed-adjusting clutch	22	31	26	LF3B	1
8	8	Fan drive assembly	1	32	27	CV2	1
9	9	Oil pan	5	33	—	OR gate	2
10	10	LF1	1	34	28	RV1	1
11	11	LF1	1	35	29	P4	6
12	—	OR gate	2	36	30	RV3	1
13	12	P1	6	37	31	Integrated pump-motor system	6
14	13	P1	6	38	32	Hydraulic control system	6
15	—	OR gate	2	39	33	Test system	5
16	14	LF2	1	40	34	Electron control system	1
17	15	LF2	1	41	—	Auxiliary operator	15B
18	16	LF2B	1	42	35	Hydrodynamic retarder and control valve	1
19	—	OR gate	2	43	—	Auxiliary operator	15B
20	17	Oil tank	1	44	36	Left side power integration gear	22

						cluster	
21	18	P3	6	45	37	Right side power integration gear cluster	22
22	19	P2	6	46	—	AND gate	10
23	20	DRV	1	47	—	AND gate	10
24	21	TCB	1	48	—	AND gate	10

According to Figure 4, Figure 5, and Tab.1, the GO model of PSST is developed from system input to system output, as shown in Figure 6. In operators of the GO model, the former number is type of operator, and the latter number is a serial number. The number on a signal flow is serial number of signal flow. Signal flows 8, 40, 46, 47, and 48 represent output of fan cooling, output of breaking, output of steering function, output of straight driving function, and system output, respectively.

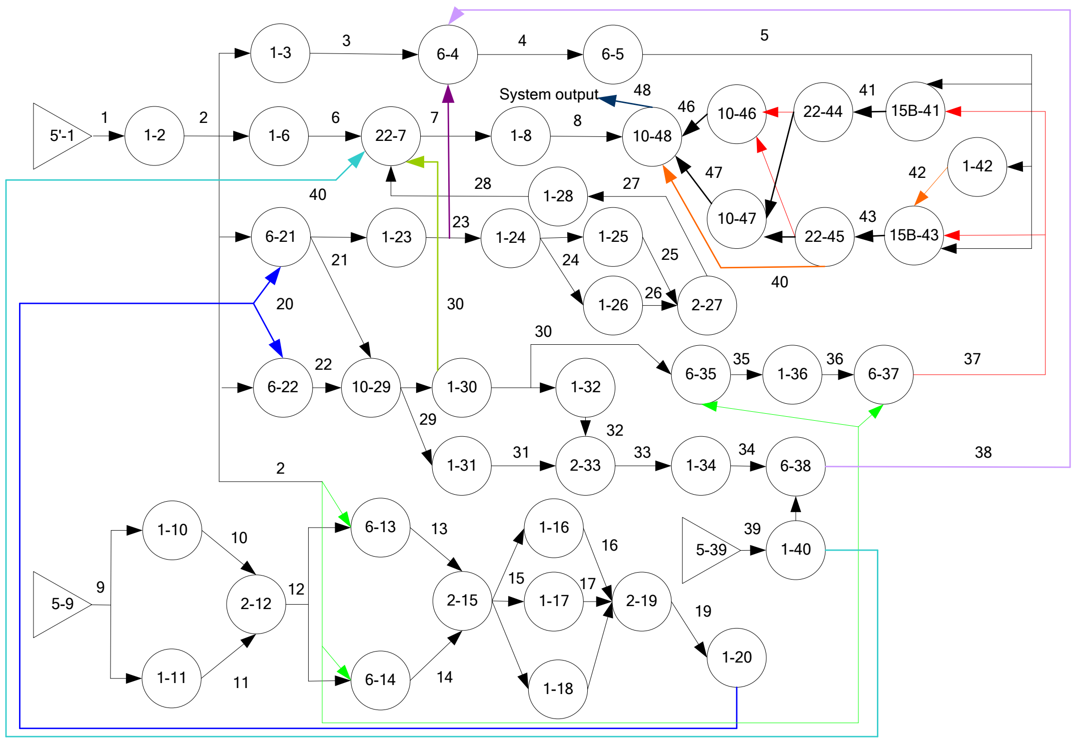

Figure 6. Structure diagram of oil supply systems.

(2) Establishing importance factor hierarchy model of functions

According to important factor analysis of function, the corresponding importance factor hierarchy model of functions is shown as Figure 7.

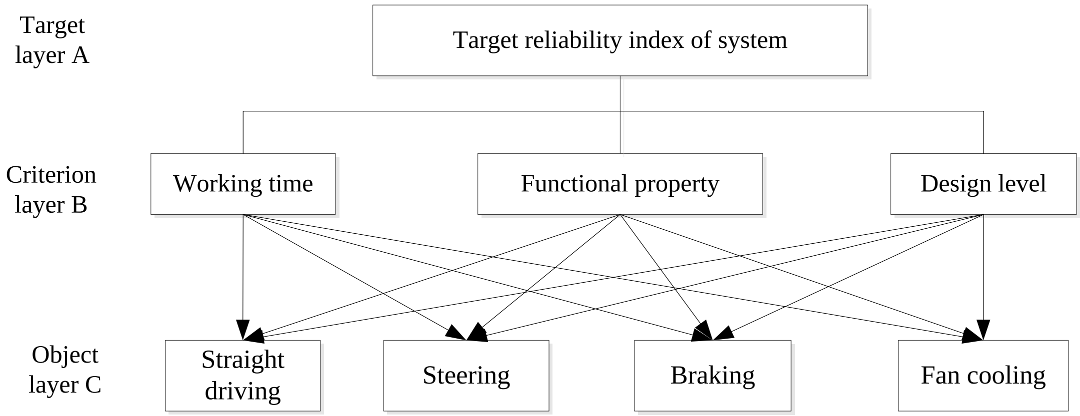

Figure 7. Importance factor hierarchy model of functions.

Describing reliability optimization allocation problem

(1) Establishing constraints

According to Eq. (2), Figure 6 and Figure 7, the function importance factor constraint based on GO method is obtained by Eq. (12).

$$\begin{cases} g(R_{G1}) = e^{(P_{S8}-1)} \geq g(R_{G1}^*) = 0.993 \\ g(R_{G2}) = e^{(P_{S40}-1)} \geq g(R_{G2}^*) = 0.9905 \\ g(R_{G3}) = e^{(P_{S46}-1)} \geq g(R_{G3}^*) = 0.9819 \\ g(R_{G4}) = e^{(P_{S47}-1)} \geq g(R_{G4}^*) = 0.9818 \end{cases} \tag{12}$$

where, $g(R_{G1})$, $g(R_{G2})$, $g(R_{G3})$, and $g(R_{G4})$ are the predicted function importance factors of fan cooling function, breaking function, steering function, and straight driving function, respectively; P_{S8}, P_{S40}, P_{S46}, and P_{S47} are reliability of signal flow 8, 40, 46, 47 in GO model, respectively; $g(R_{G1}^*)$, $g(R_{G2}^*)$, $g(R_{G3}^*)$, and $g(R_{G4}^*)$ are the allocated function importance factors by the estimation of function importance factors [Yi et al. (2016a)].

According to Eq. (3), the system reliability constraint based on GO method is obtained by Eq. (13).

$$R_S = P_{S48} \geq R_S^* = 0.951 \tag{13}$$

where, R_S is the predicted reliability of system based on GO method, i.e. P_{S48}, R_S^* is the target reliability of system.

(2) Establishing objective function

The objective function of reliability optimization allocation in this paper is to minimize the cost, as follows:

$$\min C_S = \sum_{i=1}^{37} c_i \tag{14}$$

where, C_S and C_i are the cost of system and unit, respectively.

(3) Establishing mathematic model of reliability optimization allocation problem

According to Eq. (12), (13), and (14), the reliability optimization allocation problem with differentiation of functions is described by Eq. (15).

$$\begin{cases} \min C_S = \sum_{i=1}^{37} c_i \\ s.t. \\ g(R_{G1}) = e^{(P_{S8}-1)} \geq g(R_{G1}^*) = 0.993 \\ g(R_{G2}) = e^{(P_{S40}-1)} \geq g(R_{G2}^*) = 0.9905 \\ g(R_{G3}) = e^{(P_{S46}-1)} \geq g(R_{G3}^*) = 0.9919 \\ g(R_{G4}) = e^{(P_{S47}-1)} \geq g(R_{G4}^*) = 0.9918 \\ R_S = P_{S48} \geq R_S^* = 0.957 \\ 0.999 \leq R_i \leq 0.99999, i = 1, 2, \cdots, 37 \end{cases} \tag{15}$$

Solving reliability optimization allocation problem

According to the improved AOC proposed in this paper, the parameters of improved AOC are presented in Tab. 2, and the system cost of different iterative times are shown in Figure 8.

Table 2. The parameters of improved AOC for solving Eq. (9)

Parameter	Node	Iterative times	Ant individuals	convergence operator
Value	20	500	150	500

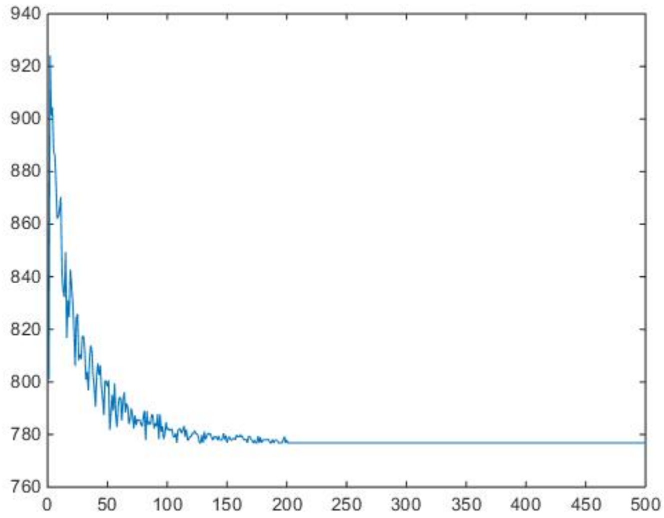

Figure 8. The system cost of different iterative times by improved ACO.

According to Figure 8, the solution convergence is at the 200th convergence time, and the system cost is 7.768215814845887e+02. The allocated reliabilities of corresponding design units are presented in Tab. 3.

Table 3. The optimization allocated reliabilities of design units by improved ACO

NO. (unit)	Reliability	NO. (unit)	Reliability	NO. (unit)	Reliability
1	0.999885789	14	0.999104211	27	0.999260526
2	0.999312632	15	0.999052105	28	0.999521053
3	0.999468947	16	0.999104211	29	0.999100000
4	0.999208421	17	0.999521053	30	0.999729474
5	0.999156316	18	0.999364737	31	0.999052105
6	0.999208421	19	0.999729000	32	0.999312632
7	0.999000000	20	0.999261000	33	0.999208421
8	0.999625263	21	0.999417000	34	0.999260526
9	0.999364737	22	0.999000000	35	0.999208421
10	0.999260526	23	0.999000000	36	0.999364737
11	0.999468947	24	0.999469000	37	0.999260526
12	0.999000000	25	0.999208000		
13	0.999521053	26	0.999469000		

4 Result Analysis

In order to illustrate feasibility and advantage of the new method, the result the results by the new method is compared with the results by using basic AOC [Nahas N. et al. (2005)]. First, setting the parameters of the node, iterative times, and ant individuals are 20, 500, and 150, respectively. Second, the reliability allocated results for each ant individual are obtained by operating algorithm according to the node transition rule. Furthermore, the feasible solution is improved according to the heuristic rule, and the non-feasible solution is improved by local search to let it become the feasible solution as much as possible. The optimal allocated results are obtained in once iterative. Then, the pheromones are updated. If it meets the termination condition, the optimal allocation results and system cost will be output. And if it does not meet the termination condition, it will operate the algorithm again. The system cost of different iterative times is shown in Figure 9.

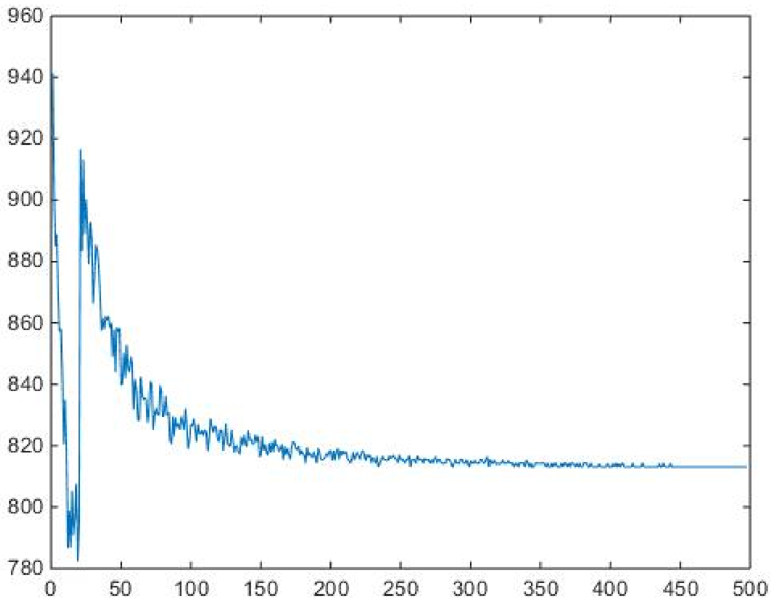

Figure 9. The system cost of different iterative times by basic ACO.

According to Figure 9, the solution convergence is at the 450th convergence time, the system cost is 8.130765482215386e+02. The allocated reliabilities of corresponding design units are presented in Tab. 4.

Table 4. The optimization allocated reliabilities of design units by improved ACO

NO. (unit)	Reliability	NO. (unit)	Reliability	NO. (unit)	Reliability
1	0.999468947	14	0.999104211	27	0.999104211
2	0.999208421	15	0.999910000	28	0.999521053
3	0.999416842	16	0.999364737	29	0.999156316
4	0.999416842	17	0.999312632	30	0.999312632
5	0.999260526	18	0.999521053	31	0.999625263
6	0.999500000	19	0.999416842	32	0.999915632
7	0.999952105	20	0.999625263	33	0.999468947
8	0.999416842	21	0.999677368	34	0.999941684
9	0.999573158	22	0.999416842	35	0.999208421
10	0.999312632	23	0.999416842	36	0.999156316
11	0.999625263	24	0.999416842	37	0.999573158
12	0.999915632	25	0.999364737		
13	0.999312632	26	0.999931263		

Tab. 3, Tab. 4, Figure 8, and Figure 9 show that:

(1) The system cost by using the basic ACO is larger than the system cost by using the improved ACO proposed in this paper. In addition, the solution convergence of improved ACO is faster than that of basic ACO. Thus, it is illustrated that the improved ACO is more effective, and more reasonable for solving the reliability optimization allocation problem with differentiation of functions.

(2) The reliability allocated results of 7^{th} unit, 15^{th} unit, 26^{th} unit, 32^{th} unit, 12^{th} unit, and 34^{th} unit by using basic ACO exceed 0.9999, which is hard to design in engineering. While, the reliability allocated results of all units by using improved ACO are less than 0.9999. Thus, it is illustrated that the improve ACO can obtain more satisfactory results, which meet the engineering practice.

(3) The analysis process of this new method shows that the new reliability optimization allocated method proposed in this paper can overcome the aforementioned disadvantages of the existing reliability optimization allocation methods efficiently so that it can quickly, efficiently, and directly allocate the system reliability index to design units for complex systems.

Conclusion

This study proposes a new reliability optimization allocation for multifunction systems considering differentiation of functions based on GO method. First, the description of reliability optimization allocation problem is proposed in aspects of constraints considering differentiations of functions based on GO method, the objective function of optimization allocation problem whose goal is minimize the system cost, and the mathematic model of reliability optimization allocation problem. Then, an improved ACO is proposed to solve above mathematic model. Furthermore, the process of the new method is formulated. Finally, the new method is applied in reliability optimization allocation of PSST whose goal is to minimize the system is cost. In order to verify the advantages and engineering applicability of the new method, the results by using improved ACO are compared with the results by using basic ACO. And the comparison results show that the new method is reasonable, advantageous, and feasible for the reliability optimization allocation problem with differentiation of functions. Clearly, this study solves the aforementioned disadvantages of the existing reliability optimization allocation methods efficiently so that it can quickly, efficiently, and directly allocate the system reliability index to design units for complex systems.

All in all, this paper not only provides a new approach to conduct reliability optimization allocation for multifunction systems considering differentiation of functions, but also improves the theory and widens the application of GO methodology. In addition, this paper can also provide guidance for the similar reliability optimization problem.

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