Smoothed Particle Hydrodynamics (SPH) Applications in Some Sediment Dispersion Problems

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Abstract

We present results from an on-going research effort which aims at applying the Smoothed Particle Hydrodynamics (SPH) method to the large-scale particle transport as well as complex flows involving particle-suspension/structure interaction. The main application to this modelling work is related to the estimation of the seabed disturbance created by a moving harvesting device near the seabed for deep-sea applications. Current results are presented for a lab-scale model of sediment disturbance in the vicinity of the harvester as well as ocean-scale sediment transport. The latter includes new developments on SPH formulations of anisotropic diffusion.

Keywords: Smoothed Particle Hydrodynamics, turbulent sediment transport, anisotropic diffusion, sediment/equipment interaction.

Introduction

In view of the environmental impact assessment required prior to harvesting operations in deep-sea environment, predictions of the extent of sediment disturbance and transport need to be provided. The proposed method relies on the description of particle suspensions via a mixture model [1] which was adapted to the Lagrangian framework of SPH [2-3]. Particle transport is modelled through the convection-diffusion of the sediment volume fraction. This accounts for particle sedimentation and particle turbulent diffusion which can be obtained from standard models relating the diffusion coefficient to the turbulent viscosity. In the case of complex flows such as equipment/seabed interactions, the latter can be extracted from the solution of standard turbulence models which are coupled to the momentum equation via Boussinesq's concept of turbulent viscosity. For ocean-scale sediment transport however, the turbulent diffusion is usually specified as directionally dependent and an improved SPH formulation for anisotropic diffusion is presented. The proposed implementation offers the possibility to account for the non-Newtonian nature of the seabed rheology which is mainly composed of clay material. Cohesive particle suspensions may be modelled through volume fraction dependent yield stress fluid models such as Herschel-Bulkley's (for classical viscoplastic fluid models, see [4]) or Papanastasiou's [5] models. The rheological properties of the sediment suspensions may be regarded as functions or functionals of local particle concentration and shear rate. With this perspective in mind, the formulation is applied to a near-field and a far-filed sediment dispersion problem. In the near-field problem, a laboratory-scale setup was designed and comprises a horizontally translating inclined blade partially immersed into a layer of clay sediment. The behavior of the sediment layer was characterised by means of rheological measurements and is modelled as a yield stress fluid. The induced sediment dispersion is investigated for various cases and the SPH predictions are compared with visual observations obtained from the experiments, highlighting the capture of various flow and sediment transport characteristics. In the far-field problem, the spread of the sediment due to a logarithmic steady current from a distributed source is investigated. With a sediment released rate of 3.6 tons per hour for 5 hours (total of 18 tons), the simulation results show that about 93% of the released sediment has been deposited on the floor up to about 11km from the source location, after 3.5 days.

Methodology

The model used is based on the mixture model [5] in which the continuity, momentum conservation and transport of the sediment concentration ϕ are given respectively by

$$\frac{D\rho}{Dt} + \rho\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\boldsymbol{u} + \nabla \cdot \rho \boldsymbol{u}\boldsymbol{u} = -\nabla P + \nabla \cdot \left(\mathbf{T}^{\eta} + \mathbf{T}^{\mathcal{T}}\right) + \rho \boldsymbol{g}, \tag{2}$$

$$\frac{D\phi}{Dt} = -\phi \nabla \cdot \boldsymbol{u} - \nabla \cdot \left(\phi\left(1 - \frac{\phi\rho_s}{\rho}\right)\boldsymbol{u}_s\right) + \nabla \cdot \left(D^{\mathcal{T}}\nabla\phi\right). \tag{3}$$

Here, $\boldsymbol{u}$ is the barycentric velocity of a volume of mixture, D/Dt is the material derivative, and $\rho = \phi\rho_s + (1-\phi)\rho_f$ is the mixture density, $\mathbf{T}^{\eta}$ and $\mathbf{T}^{\mathcal{T}}$ are the viscous and turbulent diffusion stresses, $\boldsymbol{u}_s$ is the sediment settling velocity and $D^{\mathcal{T}}$ is the diffusivity. In addition, the standard $k-\varepsilon$ and $k-\omega$ SST models have been considered to model turbulent viscosity and particle diffusivity. These equations have been transformed in a Lagrangian framework and discretized using the standard SPH derivatives. The turbulent modelling aspects are not in focus here – rather, the diffusion of the sediment is of concern.

Anisotropic diffusion of sediment

Due to stratifications of the water column in oceans, disturbed sediment diffuses mainly in the horizontal directions and is quite limited in the vertical direction. In other words, the sediment diffusion process near the ocean bottom is basically anisotropic. In the transport equation, diffusion coefficient thus is not a scalar but a tensor,

$$\boldsymbol{D} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{bmatrix}, \tag{4}$$

which is a symmetric and positive definite tensor. Currently, there is a lack of an appropriate SPH formulation for an anisotropic diffusion operator. Thus, a new SPH expression for a general diffusion operator is derived in this study and it is named ASPHAD (Anisotropic SPH approximation for Anisotropic Diffusion). ASPHAD has the form

$$\left(\boldsymbol{\nabla} \cdot (\boldsymbol{D}\boldsymbol{\nabla} c)\right)_i = 2\int_{\Omega_i} \frac{c(\boldsymbol{r}) - c(\boldsymbol{r}')}{|\boldsymbol{r}'-\boldsymbol{r}||\boldsymbol{L}^{-1}\boldsymbol{e}_{r'r}|^2} \frac{\partial W(|\boldsymbol{r}'-\boldsymbol{r}|, h)}{\partial|\boldsymbol{r}'-\boldsymbol{r}|} d\boldsymbol{r}' + \mathcal{O}(h^2) \tag{5}$$

in which $\boldsymbol{e}_{\boldsymbol{r}'\boldsymbol{r}} = (\boldsymbol{r}'-\boldsymbol{r})/|\boldsymbol{r}'-\boldsymbol{r}|$ and $\boldsymbol{L}^{-1}$ is inverse matrix of $\boldsymbol{L}$, which is given by $\boldsymbol{D} = \boldsymbol{L}\boldsymbol{L}^T$. The tensor decomposition could be either singular value decomposition or Cholesky decomposition. Particle, or SPH, discretization form of ASPHAD is

$$\left(\boldsymbol{\nabla} \cdot (\boldsymbol{D}\boldsymbol{\nabla} c)\right)_i = 2\sum_j \frac{V_j c_{ij}}{r_{ij}\left|\boldsymbol{L}^{-1}\boldsymbol{e}_{ij}\right|^2} \frac{\partial W_{ij}}{\partial r_{ij}} \tag{6}$$

For illustration purposes, ASPHAD is used to simulate anisotropic diffusion of a contaminant source in fluid with diffusion coefficients given by

$$\boldsymbol{D} = \begin{bmatrix} 0.12 & 0 \\ 0 & 0.02 \end{bmatrix} \ (m^2/s) \tag{7}$$

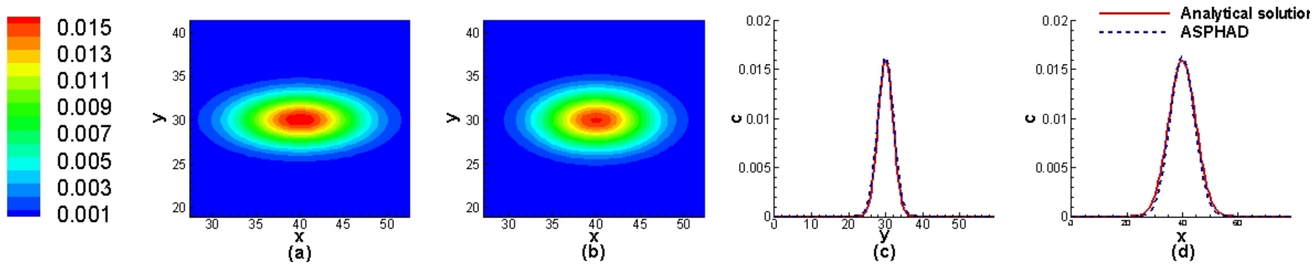


Figure 1: Anisotropic diffusion with diffusing rates of $0.12\ m^2/s$ in x-direction and $0.02\ m^2/s$ in y-direction. (a) Analytical solution, (b) ASPHAD, (c) and (d) Vertical and horizontal concentration distribution through source location $(x_0, y_0) = (40m, 30m)$

Sediment/Equipment Interaction Problems

In order to generate relevant experimental data to validate the proposed model for sediment disturbance induced by a harvester in operation on the seabed, a lab-scale experimental setup was designed in which an inclined blade moves horizontally through a layer of clay suspension mimicking the behavior of the seabed. The experiments take place in a $2m \times 0.5m \times 0.5m$ tank with a blade velocity $O(0.1m.s^{-1})$ and the scale of the experiment is only about one order of magnitude smaller than the real operation scales. Seabed samples recovered from the operation region revealed a behavior similar to suspensions of bentonite clay. Hence, the latter was used as a model for the seabed and the rheological properties of suspensions of various concentrations were measured and fitted to a Papanastasiou model [4] for yield stress fluids. The concentration dependent parameters obtained were fed into our numerical model. The induced seabed disturbance is then compared with SPH numerical simulations and a typical example is shown in Fig. 2, where it can be observed that the numerical simulation captures several of the flow features and sediment disturbance, notably the development of vortex trails in the wake of the blade.

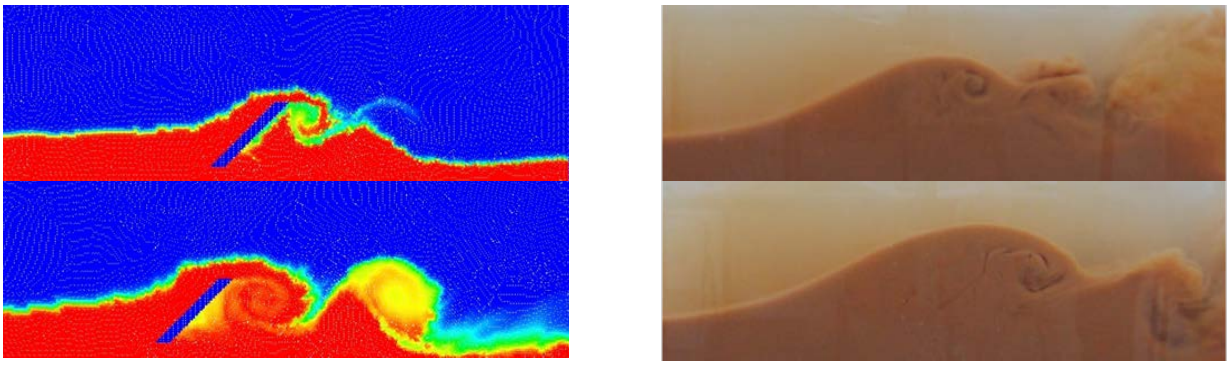
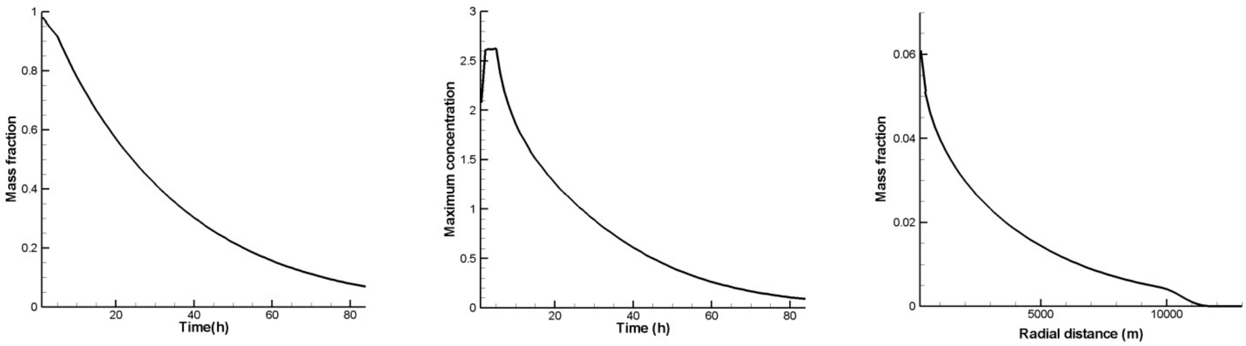


Figure 2. Comparison between SPH simulation results (left) and the experiments (right)

Far-field sediment dispersion simulation



(a) (b) (c)

Figure 3: (a) mass fraction of still suspended sediment (to the total amount of released sediment), (b) maximum sediment concentration (kg/m^3) during the simulation period, (c) mass fraction of sediment deposited on the floor versus radial distance from the source location.

Sediment at seabed disturbed by technical activities has a spectrum of sizes, from a few microns to hundreds of microns. These sediment particles are heavier than water and therefore in the presence of a subsea current, they can be persistent in ocean water and indeed can be carried away to relatively long distances away from the disturbance site. Together with current-related convections, sediment particles also settle downward and deposit on the ocean floor. These deposited sediments may be re-suspended if the shear stress on the floor is high enough. The distributions of suspended and deposited sediments are the main concern in an environmental impact assessment.

A large-scale sediment dispersion simulation is carried out with SPH in a 150m(H) ×15km(W) domain. Mathematical model basically follows equations (1)-(3). Boundary condition for the sediment concentration at the ocean bottom is generally given by $w_s c_b f_d + M_e f_e$, in which f_d and f_e are probabilities of sediment deposition and bottom erosion, w_s and c_b are sediment settling velocity and sediment concentration right above the bottom. Current flows from left to right and is assumed to follow logarithmic profile with a height-averaged velocity of 0.05m/s. Eddy viscosity and sediment-fluid mixing coefficients are calculated from a mixing length model. Average settling velocity of sediment is $w_s = 10^{-4}\, m/s$ (corresponding to a sediment size of about 10 μm). Sediment source, locating at 2km from the left boundary, follows Gaussian distribution in horizontal direction and exponential distribution in vertical direction and continually releases sediments at a rate of 3.6 tons per hour during 5 hours. The simulation results for suspended and re-deposited sediment after 3.5 days are shown in the figure 3. After 3.5 days, about 93% of the released sediment deposited along the floor up to about 11km from the source location. Maximum concentration reaches to $2.6 g/l$ (or kg/m^3) at 5 hours and gradually reduces to $0.09 g/l$ after 3.5 days.

Conclusions

The motivation behind these works is to provide an assessment of the sediment dispersion problem due to a sediment source disturbance at a certain location. To this end, we find that SPH is a good numerical method that tracks fluid particles and interfaces, including the complexity of the constitutive equations for the fluids and flows with an immersed moving structure. The experimental setup is currently being equipped with sensors in order to measure the force exerted on the blade and to compare it with the one extracted from simulations. The prediction and minimization of this force is also useful in the design of the harvesting equipment. Further work includes the extension to 3D and harvester-scale simulations in order to provide of sediment source estimates for the ocean-scale simulations.

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