Optimal sensors/actuators placement in smart structure using island model parallel genetic algorithm

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Abstract

Determination of optimal placements of sensors/actuators in large structures is a difficult job as large number of possible combinations leads to a very high computational time and storage. Therefore this kind of optimization problem demands a parallel implementation of the optimization schemes. Island model genetic algorithm (GA) being inherently parallel has been used for searching optimal placements of collocated sensors/actuators. Numerical simulations have been done for determination of optimal placements of collocated PZT sensors and actuators in smart fiber reinforced shell structures using island model parallel GA (IMPGA) in conjunction with electro-mechanical finite element analysis with an objective of maximizing the controllability index. It has been observed that the present IMPGA based formulation not only makes it possible to determine optimal sensors/actuators locations for large structures but also leads to a better solution at a much reduced and achievable computational time.

Keywords: Optimal placement, Sensors/Actuators, Island Model Parallel Genetic Algorithms, Smart Structures.

Introduction

Optimal placement of sensors and actuators plays an important role in deciding the efficacy of smart structures in suppressing undesirable disturbances. For active vibration control of large structures requiring a large number of sensors/actuators a very large number of possibilities exist from which the optimal locations of the sensors/actuators need to be chosen to achieve the maximum actuation. Therefore, determination of optimal placements of sensors and actuators has been an important area of research and a number of works have already been reported. Some of the important works are described here. Kang et al [1] has worked on optimal placement of piezoelectric sensor/actuator for active vibration control of laminated beams. Kim and Kim [2] presented optimal distribution of an active damping layer consuming minimum control energy on a flexible plate. Since optimal placement of sensors and actuators is a discrete optimization problem, genetic algorithm (GA) ideally suits as an optimization tool for this kind of problems. Rao et al [3] used GAs to obtain the optimal actuators placement in an actively controlled two-bay truss. Dhuri and Seshu [4] used GA for active vibration control of flexible structure. Roy and Chakraborty [5] presented an improved GA for optimal vibration control of smart fiber reinforced polymer (FRP) composite structure. Multi-objective optimization of hybrid composite laminates using serial genetic algorithm (SGA) and finite element method (FEM) has also been reported by Rahul et al. [6]. Agarwal et al [7] proposed a gene manipulation, multi-objective genetic algorithm to optimize the placement of active devices and sensors in frame structures. Roy and Chakraborty [8] used GA based linear-quadratic regulator (LQR) control scheme for designing an optimal controller to maximize the closed loop.

It has already been reported that GA based placements leads to superior results compared to commonly used mode shape based placement [5]. However necessary requirements of large population size and a large number of generations for convergence to the optimal solution put constraints on computational time and storage. Moreover, for structural applications, the fitness is calculated using FEM whose accuracy is again decided by spatial and time discretizations. This is more important for large structures where the number of combinations to be searched for converging to the optimal solution is very large. Therefore, IMPGA could be advantageously used to search optimal sensors/actuators placements in such smart structures. Even though there are few works [9] where IMPGA has been used for design of optimal stacking sequence of composite structures, to the best of author's knowledge, no work has been reported in literature to obtain optimal sensors and actuators placement using IMPGA. Therefore the present paper aims at developing an island model parallel GA based methodology to search for optimal placements of collocated sensors and actuators leading to a better solution compared to SGA and at a reduced and achievable computational time.

Problem Formulation

Figure 1 shows the schematics of a smart laminated structure having patches of piezoelectric material bonded on the top and bottom surfaces of the base structure, one as sensor and the other as actuator. Signal from the sensor is used as a feedback in a closed–loop feedback control system. An appropriate control law determines the feedback signal to be given to the actuator. In Fig. 1, F_t is the excited force, ϕ_s is the voltage generated by the sensor and ϕ_a is the voltage input to the actuator in order to control the displacement.

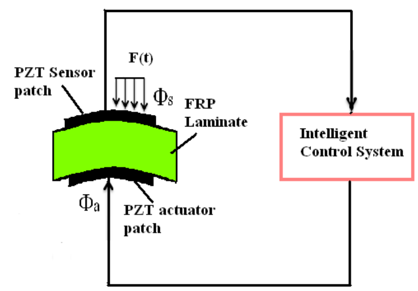

Figure 1. Smart structure

Finite Element Formulation for Controllability Index

An eight noded isoparametric shell elements have been used for finite element electromechanical analysis of the smart FRP shells [10]. The direct and converse piezoelectric equations are given by equations (1) and (2) respectively as

$$\{\boldsymbol{D}\} = [\boldsymbol{e}]\{\varepsilon\} + [\boldsymbol{\in}]\{\boldsymbol{E}\} \tag{1}$$

$$\{\boldsymbol{\sigma}\} = [\boldsymbol{C}]\{\varepsilon\} - [\boldsymbol{e}]^T\{\boldsymbol{E}\} \tag{2}$$

where, $\{\boldsymbol{D}\}$ denotes the electric displacement vector, $\{\boldsymbol{\sigma}\}$ denotes the stress vector, $\{\varepsilon\}$ denotes the strain vector and $\{\boldsymbol{E}\}$ denotes the electric field vector. Further $[\boldsymbol{e}] = [\boldsymbol{d}][\boldsymbol{C}]$, where $[\boldsymbol{e}]$ comprises the piezoelectric coupling constants, $[\boldsymbol{d}]$ denotes the piezoelectric constant matrix and $[\boldsymbol{\in}]$ denotes the dielectric constant matrix. Electrical potential has been assumed to only vary in the thickness direction linearly and the electric field strengths of an element in terms of the electrical potential for the actuators and the sensors patches respectively are expressed as

$$\{-\boldsymbol{E}_a^e\} = [\boldsymbol{B}_a^e]\{\boldsymbol{\phi}_a^e\} = \begin{bmatrix} 0 \\ 0 \\ 1/h_a \end{bmatrix}\{\boldsymbol{\phi}_a^e\} \text{ and } \{-\boldsymbol{E}_s^e\} = [\boldsymbol{B}_s^e]\{\boldsymbol{\phi}_s^e\} = \begin{bmatrix} 0 \\ 0 \\ 1/h_s \end{bmatrix}\{\boldsymbol{\phi}_s^e\} \tag{3}$$

where subscripts *a* and *s* refer to the actuator patch and the sensor patch respectively. $[\boldsymbol{B}_a^e]$ and $[\boldsymbol{B}_s^e]$ are the electric field gradient matrices of the actuator and the sensor elements respectively. The dynamic finite element equations of a piezo-laminated composite shell can be derived from the Hamilton principle and for one-element it is

$$\begin{pmatrix} [M_{uu}^e] & [0] & [0] \\ [0] & [0] & [0] \\ [0] & [0] & [0] \end{pmatrix} \begin{Bmatrix} \{\ddot{d}\} \\ \{\ddot{\phi}_a\} \\ \{\ddot{\phi}_s\} \end{Bmatrix} + \begin{pmatrix} [K_{uu}^e] & [K_{ua}^e] & [K_{us}^e] \\ [K_{au}^e] & [K_{aa}^e] & [0] \\ [K_{su}^e] & [0] & [K_{ss}^e] \end{pmatrix} \begin{Bmatrix} \{d\} \\ \{\phi_a\} \\ \{\phi_s\} \end{Bmatrix} = \begin{Bmatrix} \{F^e\} \\ \{G^e\} \\ \{0\} \end{Bmatrix} \tag{4}$$

where $\left[M_{uu}^e\right]$ is the global mass matrix, $\left[K_{uu}^e\right]$ is the global elastic stiffness matrix, $\left[K_{ua}^e\right]$ and $\left[K_{us}^e\right]$ are the global piezoelectric coupling matrices of actuator and sensor patches respectively. $[K_{aa}]$ and $\left[K_{ss}^e\right]$ are the global dielectric stiffness matrices of actuator and sensor patches respectively. $\{d\}$ is displacement vector, $\{F^e\}$ is the element external mechanical force vector and $\{G^e\}$ is the element external electrical force vector. After assembling the overall dynamic finite element equation is

$$[M_{uu}]\{\ddot{d}\} + \left[[K_{uu}] - [K_{ua}][K_{aa}]^{-1}[K_{au}] - [K_{us}][K_{ss}]^{-1}[K_{su}]\right]\{d\} = \{F\} - [K_{ua}]\{\phi_a\} \tag{5}$$

The decoupled dynamic equations considering modal damping can be written as

$$\left\{\ddot{\eta}_i(t)\right\} + 2\xi_{di}\omega_i\left\{\dot{\eta}_i(t)\right\} + \omega_i^2\{\eta_i(t)\} = [\psi]^T\{F\} - [\psi]^T[K_{ua}]\{\phi_a\} \tag{6}$$

where ω_i the i^{th} natural frequency and ξ_{di} is the damping ratio, $[\psi] = [\psi_1 \psi_2 \dots \psi_r]$ is the truncated modal matrix which transforms the generalized coordinates $d(t)$ to the modal coordinates $\eta(t)$ as $\{d(t)\} = [\psi]\{\eta(t)\}$. In state-space form

$$\{\dot{X}\} = [A]\{X\} + [B]\{\phi_a\} + [\hat{B}]\{u_d\} \tag{7}$$

where $[A] = \begin{bmatrix} [0] & [I] \\ -[\omega_i^2] & [2\xi_{di}\omega_i] \end{bmatrix}$ is the system matrix, $[B] = \begin{bmatrix} [0] \\ -[\psi]^T[K_{ua}] \end{bmatrix}$ is the control matrix, $[\hat{B}] = \begin{bmatrix} [0] \\ [\psi]^T\{F\} \end{bmatrix}$ is the disturbance matrix, $\{u_d\}$ is the disturbance input vector, $\{\phi_a\}$ is the control input, and $\{\dot{X}\} = \begin{Bmatrix} \dot{\eta} \\ \ddot{\eta} \end{Bmatrix}$ and $\{X\} = \begin{Bmatrix} \eta \\ \dot{\eta} \end{Bmatrix}$. The sensor output equation can be written as

$$\{y\} = [C_0]\{X\} \tag{8}$$

where $[C_0]$ depends on the modal matrix $[\psi]$ and the sensor coupling matrix $[K_{us}]$. The modal control force f_c applied to the system can be written as

$$\{f_c\} = [B]\{\phi_a\} \tag{9}$$

It follows from Eq. (9) that

$$\{f_c\}^T\{f_c\}=\{\phi_a\}^T[B]^T[B]\{\phi_a\} \tag{10}$$

Using the singular value analysis, $[B]=[M][S][N]^T$ where $[M]^T[M]=[I]$,$[N]^T[N]=[I]$ and $[S]=\begin{bmatrix}\sigma_1 & \cdots & 0\\ 0 & \ddots & \vdots\\ \vdots & \cdots & \sigma_{n_a}\\ 0 & \cdots & 0\end{bmatrix}$ where n_a is the number of actuator. Eq. (10) can be rewritten as

$$\|\{f_c\}\|^2=\|\{\phi_a\}\|^2\|S\|^2 \tag{11}$$

Thus, maximizing this norm independently on the input voltage $\{\phi_a\}$ induces maximizing $\|S\|^2$. The magnitude of $\boldsymbol{\sigma}_i$ is a function of location and the size of piezoelectric actuators. Wang and Wang [11] proposed maximizing the controllability index as

$$Maximize\ \Omega=\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i \tag{12}$$

Island Model Parallel Genetic Algorithm for Optimum Sensor/Actuator

In the present problem, the design variables are the positions of the actuators, and are represented in a string of integers specifying the locations of actuators. Referring to Eq. (12), the higher the controllability index, the smaller will be the electrical potential required for control. In modal control, however, one of the important issues is to decide the number of control modes where actuations need to be done. Providing actuations to higher modes (which are residual modes actually not excited) might lead to instability known as control spill over. In the present work therefore the fitness/objective function which needs to be maximized in the GA ensuring optimal actuators locations has been proposed as follows

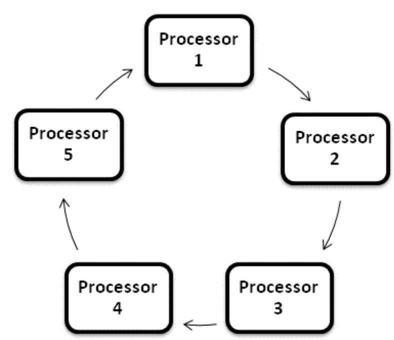

Figure 2. A 5 processor IMPGA

$$\boldsymbol{\Omega}=\begin{Bmatrix}\left(\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i-\gamma'\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i^R\right)\ if\ \left(\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i\right)>\left(\gamma'\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i^R\right)\\ \left(\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i-\gamma'\prod_{i=1}^{n_a}\boldsymbol{\sigma}_i^R\right)\times10^{-12},\ otherwise\end{Bmatrix} \tag{13}$$

where $\boldsymbol{\sigma}_i^R$ are the components of $[\boldsymbol{S}^R]$ corresponding to residual modes and γ' is a weight constant. In this objective function, if the contribution of residual modes dominates, fitness of that population is forced to a very low value thereby eliminating the chances of such populations to grow in successive generations.

In IMPGA approach (Fig.2), first the population size is decided as a multiple of number of processors so that the total population of chromosomes is divided into a number of sub-populations. String length of each population is decided by the number of actuators. For example referring to Fig. 3 if the population size is 60 and there are 5 processors (islands), each processor will handle a sub population size of 12. In each of the processor, for sub populations, the fitness value for each chromosome is obtained using the FEA independently and new sets of chromosomes are generated by applying genetic operators after each generation. After a certain number of generations, the best population of one processor is allowed to migrate only to its neighboring processor, replacing the worst population. For example, the best candidate from processor 1 will replace the worst candidate of processor 2, the best candidate of processor 2 will replace the worst candidate of processor 3 and so on. Thus, migration does not change the size of population. At the end of each generation, a better population results, and is used in successive generations to achieve populations with even better fitness. This is repeated until the solution converges and the optimal locations of actuators are selected corresponding to the chromosome with best controllability index.

Results and Discussions

A parallel code has been developed using MPI libraries as well as migration routines *(Island Model)* for optimization. The parallel code has been run on parallel cluster at IIT Guwahati. The cluster has 5 nodes and each node consists of 8 (1.5 GHz) processors. On one of the nodes of the cluster, the code has been run using SGA corresponding to same genetic parameters and population. The results obtained from IMPGA as well as SGA model for optimal placement of sensors and actuators have been compared to study the efficacy of the present approach.

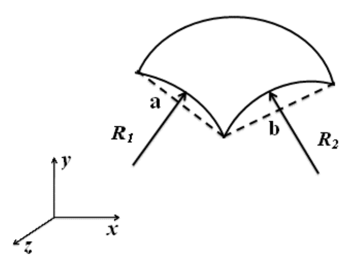

Figure 3. Curved shell

Problem Definition

In this study, a [p/[0/90]$_s$/p] graphite/epoxy (GR/E) doubly curved shell with the four edges simply supported, having a=b=0.02m, R_1=2R_2=R=0.06m R/a =3, a/h=10 (Fig. 3) has been considered. Here 'p' stands for piezo-patches one for sensing and the other for actuation. Thickness of each piezoelectric patch has been considered as 0.5 mm and that of each GR/E lamina has been considered as 0.25 mm. A 10×10 finite element mesh has been used to model the shell panel and optimal actuators placements have been calculated considering the first eight modes with first four modes as control modes and others as residual modes. The material properties have been listed in the Table 1.

Table 1. Material properties

Property	Gr/E	PZT
E_1 (GPa)	172.5	63.0
E_2=E_3 (GPa)	6.9	63.0
G_{12}=G_{13} (GPa)	3.45	24.6
G_{23} (GPa)	1.38	24.6
$\upsilon_{12}=\upsilon_{13}=\upsilon_{23}$	0.25	0.28
ρ (kg m^{-3})	1600	7600
e_{31}=e_{32} (C m^{-2})	0.0	10.62
$\in_{11}=\in_{22}=\in_{33}$ (F m^{-1})	0.0	0.15 x10^{-7}

Table 2. Input parameters for GA

Initial population	60
Maximum generation	100
Number of actuators/sensors	6
Mutation rate	20%
Crossover rate	90%

Table 2 shows various input parameters considered for SGA as well as IMPGA. The stated genetic parameters used in IMPGA are finalized from the values obtained from multiple runs in the SGA. Thus, an optimized value of 100 generation with an initial population 60 is used to obtain comparative results regarding fitness and time between IMPGA and SGA. Six numbers of actuators are considered leading to a string length of 6.

Table 3. Controllability index for SGA

Initial population	Maximum generation	Fitness	Time (sec)
5	100	0.237	210993
30	100	0.248	1264376
60	100	0.255	2526150

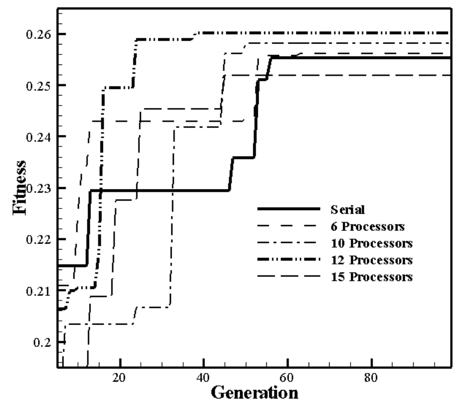

Figure 4. Convergence of fitness

Effect of Population Size on Controllability Index and CPU time

The code has been run up to 100 generations in one processor as SGA with increasing population size and table 3 shows the effect of population size on the controllability index. It is clear from the table that as the population size increases, controllability index increases but as expected the computational time also increases. It is therefore necessary that the optimal placement of sensors and actuators are searched from a larger population. However computational time requirement puts a restriction on the upper limit of the population size when such a problem is run on a serial platform. Therefore a parallel GA provides a feasible solution for such problems and island model GA being inherently parallel has been advantageously used in the present study.

Optimal Placement using Island Model Parallel GA

Five different schemes were used to study the effect of parallelization. The schemes are decided based on two factors viz. a maximum population size which could be run in a serial GA and with different number of processors such that in each case the number of processors is an integer factor of the population size. Different schemes considered here are:

- Scheme **1**:- SGA with one processor having population size of 60.
- Scheme **2**:- IMPGA with 6 processors having a sub population size of 10 in each
- Scheme **3**:- IMPGA with 10 processors having a sub population size of 6 in each
- Scheme **4**:- IMPGA with 12 processors having a sub population size of 5 in each
- Scheme **5**:- IMPGA with 15 processors having a sub population size of 4 in each

Table 4 shows the comparative performances of these 5 schemes up to 100 generations. It could be observed that increasing number of processors leads to increase in fitness up to 12 processors but beyond that the fitness decreases. This indicates that for this particular problem, the maximum number of processors that could be used for a population of 60 is 12. This is due to the fact that depending upon the number of populations increasing the number of processors leads to smaller sub population size in each processor and more communication overheads. Thus the minimum population size for this problem is 5. Figure 4 shows the convergence of fitness for the optimal placement problem of smart shell structure for these 5 schemes. It is clear that with the increasing number of processors, it is not only that the fitness is higher compared to that in SGA but this fitness is achieved at a much less number of generations. This is due to the fact that the better solutions evolve independently in different processors and those processors (islands) interact (migration) after certain number of generations thereby passing on populations with better fitness only in each processor. Therefore even though the population size is larger, increasing number of processors still reduces the number of generation required for convergence.

Table 4. Controllability index for different schemes

Scheme	Initial population	Maximum generation	Number of processor	Fitness	Time (Sec)
1	60	100	1	0.255	2526150
2	60	100	6	0.256	443612
3	60	100	10	0.258	270812
4	60	100	12	0.260	227651
5	60	100	15	0.252	183234

Comparison Serial GA and IMPGA

Figure 5 shows the variation of computational time with the increasing number of processors

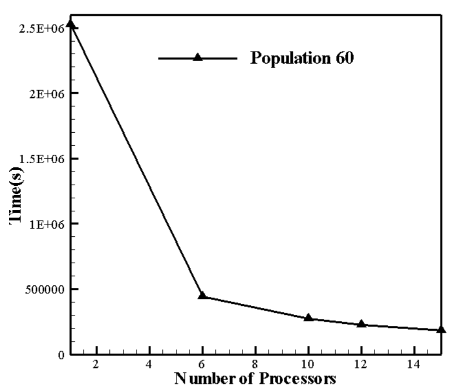

Figure 5. Time Vs no. of processors

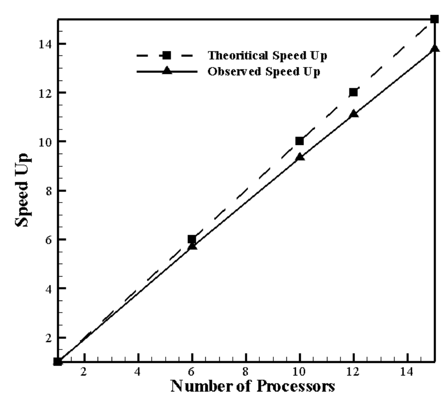

Figure 6. Speedup Vs no. of processors

while using IMPGA. Here, the use of one processor implies SGA executed using a single processor on the parallel platform. It could be observed from Fig. 5 that for a fixed number of initial population and generation, increase in number of processors leads to significant decrease in computational time. In the present problem of optimal placement of collocated actuators/ sensors using IMPGA with 12 numbers of processors takes 2,27,651 seconds while SGA takes 25,26,150 seconds under the same condition. The better computational performance of IMPGA is only because of better mixing of population due to migration which leads to faster convergence to optimal solution. The performance of a parallel code is evaluated in terms of factors such as speedup, efficiency and scalability. The speedup, $S_N = T_S / T_{par}$ where, T_S and T_{par} represent time taken with a single processor multiple processors respectively. The efficiency of a parallel algorithm, $E_N = S_N / N$ where N is the number of processors. In the present study, a comparison has been made between SGA and IMPGA based on these factors. Table 5 shows the speed up obtained with increasing number of processors for a fixed population size of 60 up to 100 generations. Figure 6 shows the speedup comparison between SGA and IMPGA. It could be observed that with the increase in number of processors, speedup increases effectively. This is also clearly observed that there is a decrease in efficiency with the increase in processors (Fig.7). This is due to the fact that in the IMPGA, overhead increases due to increase in migration as the number of processors increases.

Further, to understand the behavior of the present IMPGA application, with increasing number of population, scalability analysis has been carried out keeping the number of processors fixed and the same is compared with SGA. In the present study, computational time has been noted for 100 generations for SGA and 15 processors IMPGA. Figure 8 shows the CPU time versus population size for both the cases. It could be clearly observed that in the

Table 5: Speedup and efficiency for 100 generation with fixed population size of 60

No. of Processors	Time (Sec)	Speedup Actual	Speedup Theoretical	Efficiency Actual	Efficiency Theoretical
1(Serial)	2526150	1	1	--	--
6	443612	5.69	6	94.9%	100%
10	270812	9.32	10	93.2%	100%
12	227651	11.09	12	92.5%	100%
15	183234	13.78	20	91.9%	100%

case of SGA the magnification in CPU time is equal to the magnification in population size. However, in the case of 15 processors IMPGA, increase in CPU time is much less compared to the magnification in population size. This shows that the proposed IMPGA based model in determination of optimal sensors/actuators location will be more efficient for larger population size and hence for larger structures.

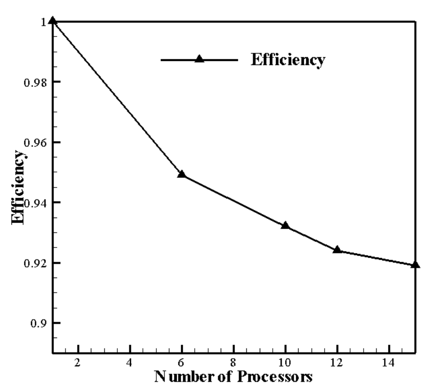

Figure 8 Time Vs no of processors

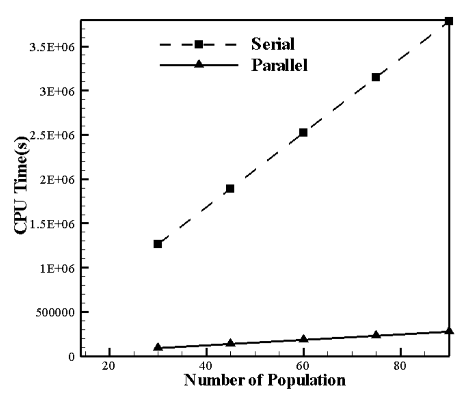

Figure 7. Efficiency Vs no of processors

Conclusions

In the present work an island model parallel genetic algorithm in conjunction with FEA has been developed for evaluation of optimal placements of collocated actuators/ sensors on a smart FRP shell structure. Controllability index determined from finite element analysis has been used as the measure of fitness with the actuators location as the variables. The present method not only leads to a better solution, but also finds that at a much reduced computational time. This method will be especially suitable for large structures where large number of sensor and actuators need to be used requiring larger population size and sequential GA fails due to limitation in population size. It has been observed from the present study that the present IMPGA based method is far superior compared to the sequential GA method in determining the optimal placements of actuators/sensors.

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