Simple method of approximate calculation of statically indeterminate trusses

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Abstract

The paper presents principles of a simple method, which in two stages makes possible the approximate calculations of statically indeterminate truss systems. The proposed two-stage method applies rules of other methods used for calculations of the statically determinate trusses. In each of the both stages there are considered the statically determinate trusses, patterns of which are obtained as results of suitable taking selected members out from pattern of the basic statically indeterminate truss. These intermediate trusses have the same clear span and construction depth like the basic indeterminate truss but they are loaded by forces of the half values applied to nodes of the same positions like in the basic one. The proposed two-stage method uses theorems and features of calculus of vectors as well as principle of the superposition method. Final values of forces acting in particular members of the basic statically indeterminate truss are resultants of forces calculated in each stage in the counterpart members of the statically determinate trusses. There are presented results of calculations carried out for two cases of loading of a selected type of the plane truss. These results are compared with results of forces determined for the same truss by application of computer calculations carried out by method appropriate for the statically indeterminate systems.

Keywords: Truss system, Calculus of vectors, Cremona's method, Superposition method, Statically indeterminate system, Approximate solution.

Introduction

Methods of calculations of statically indeterminate systems have to make possible the exact computation of the force values acting in members of such systems. Results obtained by application of them are the basis for engineers, who are obliged to design the safe and economic structural systems for very various purposes, like for instance roof structures in the building industry. There are numerous methods commonly used to calculate the statically indeterminate systems starting from e.g. the force method, the displacement method, the iterations methods like the method of successive approximations, the finite elements method etc., which were invented in the past and they are still modified and adapted to requirements of needs of the appropriate computer software [1-4]. The force distribution in area of the structural indeterminate trusses depends among others on the ratios of stiffness of members joining in particular nodes. That is why methods of precise calculations of the force values have to be complex what further implies, that computation procedures and computer calculation software have to be equally complex.

Definition of research problem and proposal of method of its solution

In preliminary structural analysis of the statically indeterminate truss it is usually enough to define only the approximate values of forces acting in its members. For these purposes it is not necessary to use a sophisticated method of calculations that is why one can apply some simple ways of the approximate computations. The proposed two-stage method has been invented during the initial statically analyses of a certain group of the spatial tension-strut structures. It is in detail discussed in papers [5, 6]. These structures consist of cross-braces made in form of struts while other components like vertical members and members of the outer layers are the tension members. Simplified scheme of vertical cross-section of a basic truss system representing this group is shown in Fig. 1a. These types of structural systems have to be suitably pre-stressed. If the tension-strut truss is overloaded by forces F, see Fig. 1b, then certain number of the upper chord members are not able to take the compression forces, because of their big slenderness, what implies that they are excluded from process of the force transmission.

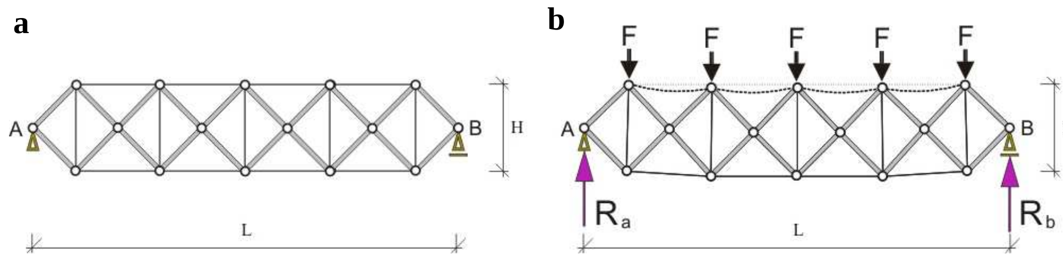


Figure 1. Schemes of plane tension-strut truss systems, a) basic configuration, b) configuration of overloaded structure

It is assumed that number of nodes is defined by symbol “w”, while symbol “p” defines number of members. Condition of the inner statically determinacy of plane truss is determined as:

$$p = 2 \cdot w - 3 \tag{1}$$

The truss system presented in Fig.1a consists of number of nodes w = 16 what implies that the statically determinate truss created by means of this number of nodes has to be built by means of following number of members:

$$29 = 2 \cdot 16 - 3 \tag{2}$$

Truss of the scheme shown in Fig.1a is created by number of members p = 33 what indicates that the structure is the fourfold statically indeterminate system. From analysis of the scheme shown in Fig. 1b follows that number of the excluded members equals 4, what exactly is equal to the degree of statically indeterminacy of the basic truss system. Thus the overloaded basic plane truss can be considered as the statically determinate system, what directly indicates that it can be calculated by application of one of the simple methods like e.g. Cremona’s method, Ritter’s method or other methods suitable for this purpose.

The observation brings to mind a following questions: is it possible to apply for instance Cremona’s method for computation of statically indeterminate plane trusses? If yes, in what way it has to be done? One should be aware that values of forces determined by means of the sought after method will be of approximate values because stiffness of particular members are not taking into account in methods used for calculations of statically determinate trusses. The considered problem refers to the coplanar force system therefore the three basic conditions of equilibrium have to be fulfilled:

$$\sum_{i=1}^{n} F_{ix} = 0 \tag{3}$$

$$\sum_{i=1}^{n} F_{iy} = 0 \tag{4}$$

$$\sum_{i=1}^{n} M_i = 0 \tag{5}$$

Moreover the basic principles of calculus of forces have to be strictly respected. Taking into consideration all indicated requirements it is proposed to introduce the two-stage procedure of calculations, general scheme of which is shown in Fig. 2.

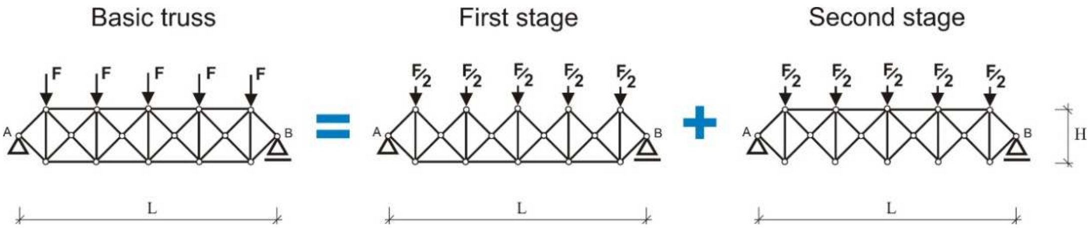


Figure 2. General schemes of two-stage method proposed for approximate calculation of statically indeterminate trusses

The point of proposed method is to carry out static calculations in two independent stages for statically determinate trusses, shapes of which are received through remove the number of members equal to statically indeterminacy from space of the basic truss. The calculated statically determinate truss has in each stage the same geometric parameters like clear span L and construction depth H, but it is loaded by forces of half values applied to the same nodes like in area of the basic truss. Values of the final forces computed in the basic truss will be resultants of forces obtained in each stage for members having the same position in area of considered truss.

Results of calculations and comparative analysis

In order to verify correctness of theoretic assumptions of the two-stage method there were carried out series of computations of simple form of the plane statically indeterminate truss having shape of basic truss shown in Fig. 2, built of steel members, having clear span equals 5.00 meters and of construction depth equal to 1.00 meter. In the basic case the truss is symmetrically loaded in all nodes of the upper chord by concentrated forces, each of value 1.00 kN. In the first stage four members of the upper chord are removed and concentrated forces of value equal to 0.50 kN are applied to all nodes of the upper chord. The own weight of truss is not taken into consideration. After this operation the investigated truss become the statically determinate system what empowers to apply, for instance, the Cremona's method for computation values of forces acting in component members of the truss. Because the basic truss is of symmetric form and it is loaded in the symmetric way that is why the Cremona's method in both the stages can be applied only for half of suitably forms of considered trusses. Results of the first stage of calculations are presented in Fig. 3.

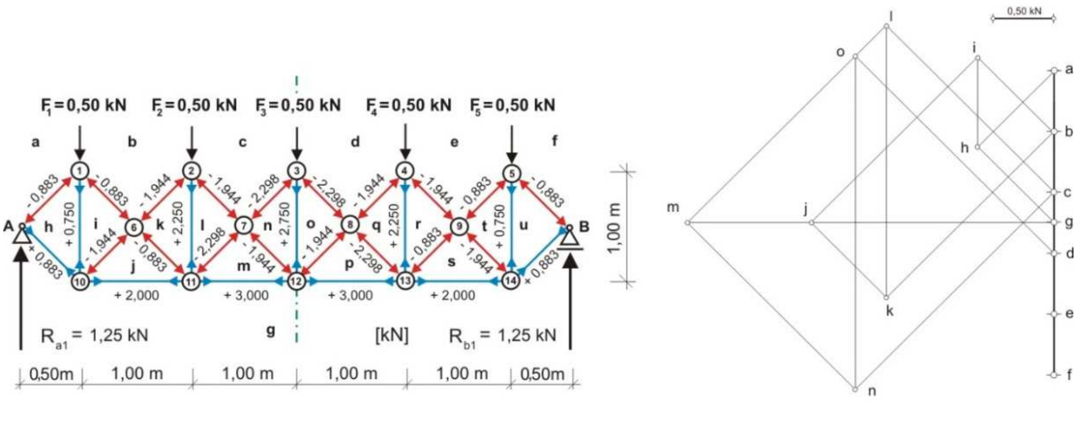


Figure 3. Scheme of distribution of values of forces calculated in the first stage in area of basic truss together with appropriate Cremona's polygon of forces

In the second stage four members, like previously, are rejected but this time from the lower chord of the basic truss and the statically determinate form of truss is loaded by concentrated forces, each of value equal to 0.50 kN and applied to each node of the upper layer. Results of the second stage of calculation are shown in Fig. 4.

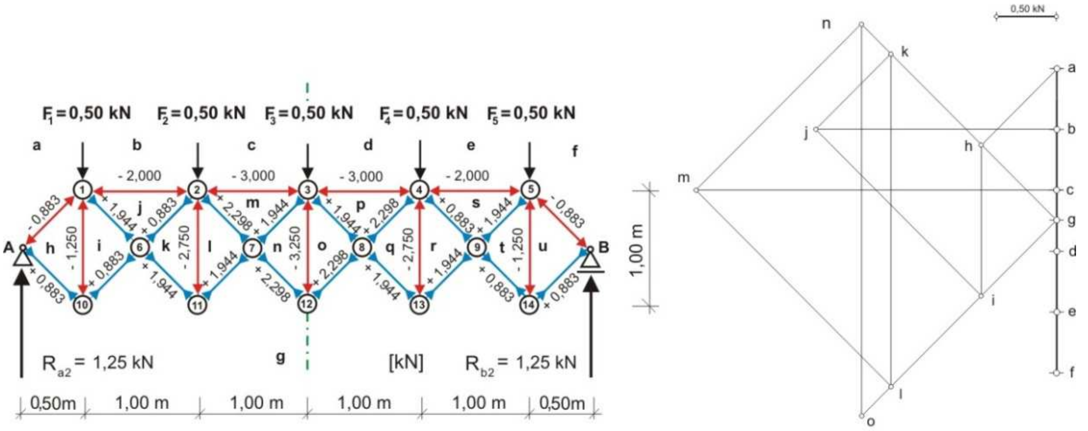
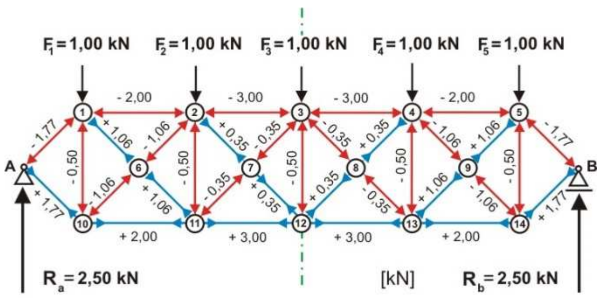


Figure 4. Values of forces defined in the second stage of calculation in area of basic truss together with appropriate Cremona's polygon of forces

Keeping rules of the proposed method the final values of forces acting in particular members are determined as resultants of forces calculated in two independent stages in the counterpart members of trusses considered in each stage, see Fig. 5a. For instance the final force acting in member of the upper chord placed between nodes of numbers 2 and 3 is the resultant of zero value for the not existing member between these nodes in the first stage, see Fig. 3, and the force value equals -3,00 kN acting in the counterpart member, determined in the second stage, see Fig. 4.

a



b

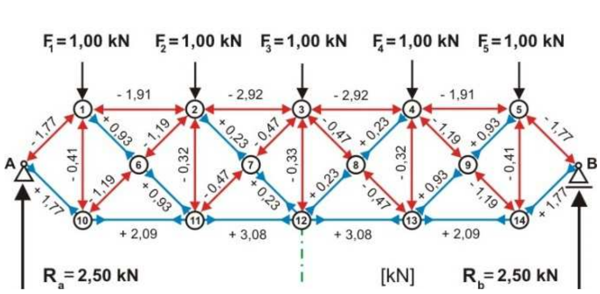


Figure 5. Values of forces calculated in the same members of basic structure by application of, a) proposed two-stage method, b) suitable computer software

The same form of the basic indeterminate truss was calculated under the same conditions by application of Autodesk Robot Structural Analysis Professional 2016, which software takes into consideration all requested mathematic tools necessary for precise computation of the force values in members of the statically indeterminate systems. It was assumed that the investigated truss is built of tubular members having diameter of 30.00 mm, thickness of section equal to 4.00 mm, while their steel material has the Young's modulus equal to 210 GPa. Results received in this way are presented in Fig. 5b. Value of the force in member placed between nodes 2 and 3 defined in the computer calculation equals -2.92 kN, so the difference between outcomes received in the two compared methods is only 0.08 kN what is really small relatively difference because it constitutes only ca. 2.6 % regarding to the bigger force. More differences between particular values one can notice in the force values calculated in these two methods carried out in the cross braces. For instance in member placed between nodes 3 and 7 the force value calculated in the two-stage method equals -0.35 kN, while by application of the suitable computer software it is equal to -0.47 kN, what constitutes the differentiation of around 25 % towards the bigger value. In this place one should to point out that the biggest differences of the force values are observed in members, where are acting the really smallest forces. More precise answer for question about degree of approximation of results obtained in the proposed two-stage method in comparison to results defined in the exact method one can receive due to the static calculation of the same basic truss but conducted now e.g. for an asymmetric way of its load. The demanded computations were carried out for selected case, where two concentrated forces of the same value equal to 1.00 kN are applied to nodes of the upper chord and having numbers 4 and 5. Results of the both intermediate calculations are shown in Fig. 6 and in Fig. 7.

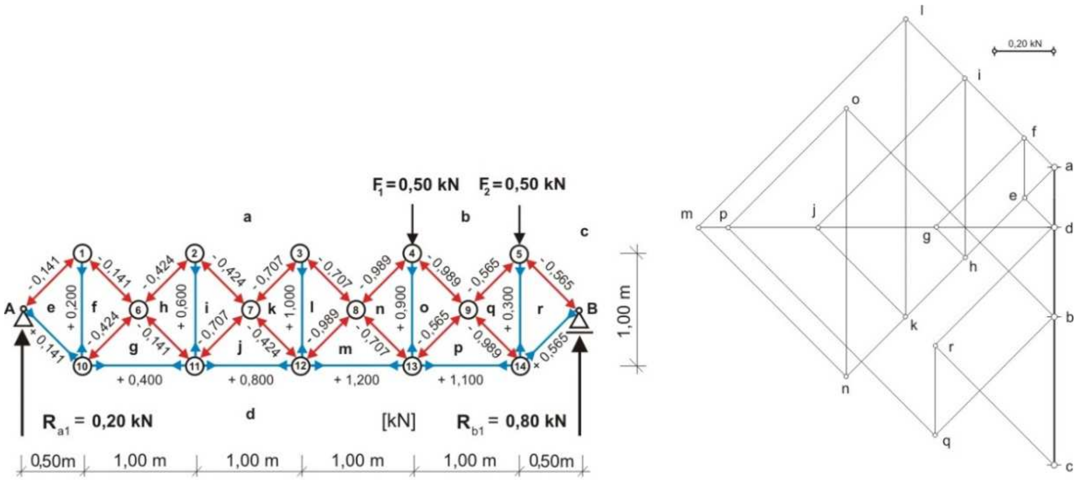


Figure 6. First stage of calculation of basic truss under asymmetric load

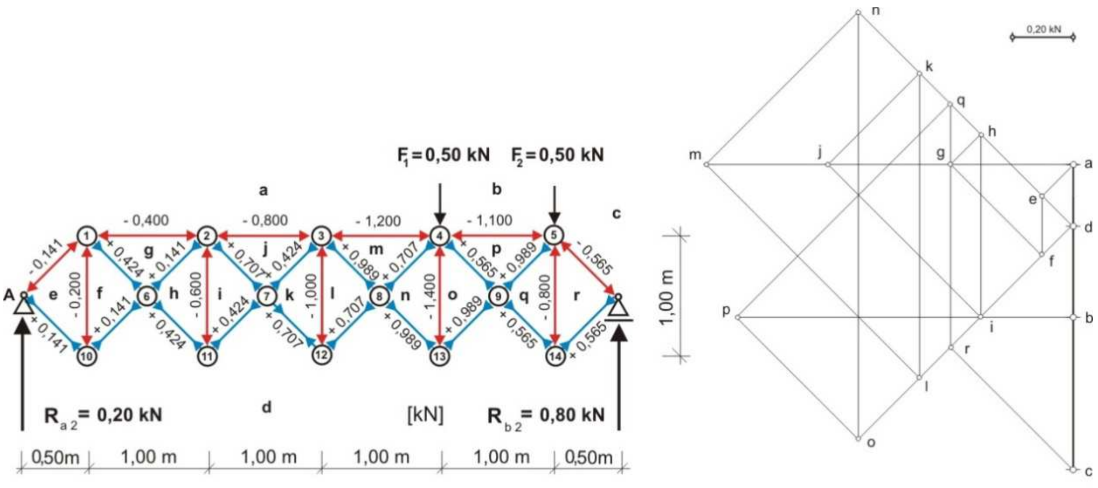


Figure 7. Second stage of calculation of basic truss under asymmetric load

Because of the asymmetric way of application of the load both procedures of computation of the intermediate trusses have to be conducted for the whole structures. From analysis of information overall presented in Fig. 8 follows, that the biggest differences one can notice between the force values calculated by means of both compared methods in certain cross braces. For example the force value in the cross brace located between nodes 5 and 9 defined in the two-stage method equals +0.42 kN, while by means of the computer software its computed value is equal to +0.28 kN. Then the difference is on level of ca. 33 % towards to the bigger value. Significantly smaller differences one can observe between values of forces calculated in both methods in the most strained members of the outer chords. For instance the force value in member placed between nodes 13 and 14 defined in two-stage method equals +1.10 kN but calculated by application of suitable computer software it is equal to +1.20 kN, what constitutes only around 8.3 % towards the bigger value.

a

b

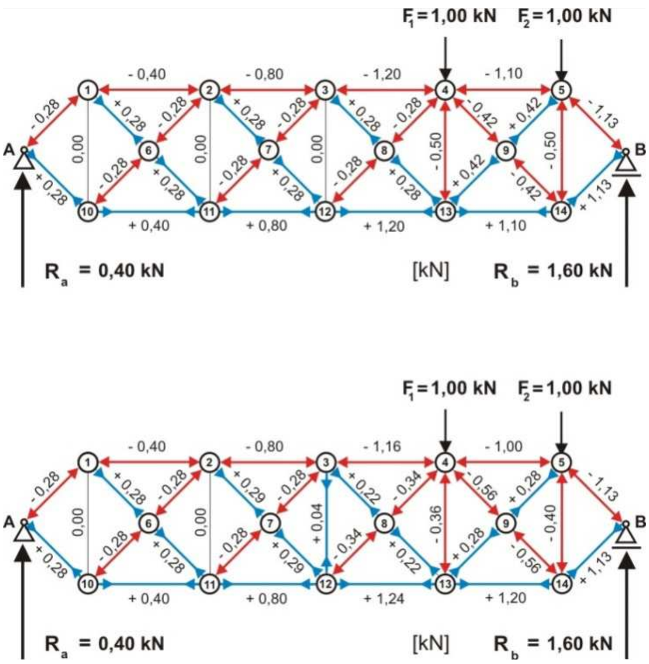


Figure 8. Results of static calculations carried out for the same basic truss by application of a) proposed two-stage method, b) suitable computer software

Conclusions

From comparative analysis of outcomes obtained in the both compared computation ways follows that the two-stage method can be applied as an approximate method of calculation of the plane statically indeterminate trusses. Its accuracy can be improved in the future by taking into consideration the stiffness differences between members connected in particular nodes of the considered structural system.

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