Newtonian Gravitational Force for predicting Distribution Centre Location of a Supply Chain Network

***A.A.G. Akanmu[1] and F.Z. Wang[2]**

[1]School of Computing, University of Kent at Medway, UK
[2]School of Computing, University of Kent at Canterbury, UK

*†Presenting/corresponding author: aagakanmu@gmail.com

Abstract

Occasions do arise when researchers and industrialists alike are faced with the decision of where to cite new structures (shops, stores, distribution centers etc) in order to benefit the consumers and the business entity as well. Such decisions might take the importance of vertices and/or edges of a network (e.g. Supply Chain Network) into consideration. In particular, the strength of the vertices and those of the edges play an important role in arriving at such decisions. In this paper, as against the most common and traditional measures of centralities, that is - Degree, Closeness, Betweenness and Eigen-Vector centralities, a new centrality measure, Top Eigen-Vector Weighted Centrality (TEVWC) which takes into consideration the clique structure of a network and the strengths attached to the vertices/edges of the network, was used to predict the location of a distribution center in a supply chain management. The accuracy of prediction on a sample dataset of supply chain network, using the TEVWC was found to be 94.6%, which is 10.6% higher than the result outcome from the method of Newtonian Gravitational Force when driving distances are considered, but with the earth distances the accuracy obtained is 99%.

Keywords: Cliques, Centre of Mass, Link-weights, Node-weights, Network Centrality, Supply Chain Network

Introduction

Any network consists of nodes and links; the nodes are also severally referred to as vertices, actors and points, while the links are also often referred to as edges, arcs, and ties. Different meanings have been adduced to the weighted-ness of a network, so many literature have at instances made references to link-weights as the weights of the entire network, even though any network as described above would at least consist of node(s) and link(s) as the case may be. This therefore implies that there has to be node-weights as separated from link-weights and the combination of the two would thereby emerge as weights of any typical network.
In his work on identifying cohesive subgroups [1] laid emphasis on the link of a graph thus "Further, the definitions based on path length are restrictive in that they specify the nature of the relationship between each pair of actors within a subgroup instead of a general relationship between each actor and all others in the subgroup", thereby leaving out the actors/nodes' strength. According to the definition of the Topological Centrality (TC) of an

edge, the weights of edges are the sum of the weights of its two end nodes [2]. Here, the definitions of the weights of edges and weights of nodes are somehow fuzzy, as it is not clear cut what made up the weights of the end nodes.

[3] defined a weighted network as that in which ties are not just either present or absent, but have some form of weight attached to them, hence the emphasis of his paper on the trade-off between the weight on the tie and the number of ties. This was however silent on the attributes of the node (which in most cases form the weights on the nodes). This viewpoint was partly shared by [8] when he said "Second category of measures (i.e., h-Degree, a-Degree and g-Degree) takes into account the links' weights of a node in a weighted network. Third category of measures (i.e., Hw-Degree, Aw-Degree and Gw-Degree) combines both neighbors' degree and their links' weight."

[4] [5] [6] have also attempted to generalize the traditional network centrality measures (degree, betweenness and closeness) to weighted networks, but they were only able to implement their generalisations as the link-weighted network, thus not putting the node-weights into consideration.

Another emphasis on link-weighted-ness in terms of duration is that by [7] whereby they introduced a time-variant approach to the degree centrality measure, that is, the time scale degree centrality (TSDC), whereby the presence and duration of links between actors are considered while leaving out the node attributes. On hybrid centrality measures, [9] reported having considered a network as having the centrality measures of each node as the attribute of the node, while [10] in their analysis of results for scholars performance and social capital measures also buttressed this view point by submitting that repeated co-authorships are merged by increasing more weight(tie strength) to their link(tie) for each relation, so also [11] whereby they referred to weight of undirected graph as the link-weight. However, all these arguments are again centred on link-weights as against the weights of the network that could have considered a combination or mergers of node-weights and link-weights.

In their new method of constructing co-authorship, [12] used the times of co-authorship to calculate the distance between each pair of authors, and to also evaluate the importance of their cooperation to each other with the law of gravity. This relies again on the use of link weights.

The mixed-mean centrality measure of [13] took into consideration, the number of links, link-weights and node-weights in their study of co-authorship network, while [14] used the clique structure and node-weighted centrality to predict the distribution centre location in a supply chain management, thus clarifying what the link-weights and node-weights actually represent in a weighted network.

It is still largely unknown how newtonian gravitational force of attraction and the top eigen-vector weighted centrality can be applied to predict location of structures in a network. Thus, it is important to still find out whether the attributes of the nodes in any network is of importance or not; one might also want to know how accurate the mergers of node-weights and link-weights can be in terms of prediction of where to cite structures (for example, where to cite a distribution centre); and finally how accurate would the prediction of the location for a DC become, given a new centrality measure, which takes into consideration, the clique structure of a network combined with the node-weights and link-weights of the network.

The nodes of the clique for each of the cities considered are ranked in line with their eigen-vectors, and the representative node (the highest ranking node) for that clique becomes the representative node of that city. The centre of mass for the emergent nodes is thereafter taken into consideration. This method is important in that it only takes the node-weights and link-weights into consideration while trying to achieve the results, thereby saving other resources.

Section II discusses the link-weighted centrality and node-weighted centrality and the third section discusses methods employed in this paper and their implementation, while the fourth lays out the output results from the methodology and the last forms the conclusion.

Weighted Centralities

Link-Weighted Centrality

The equation (1) below represents the weighted degree centrality with respect to the edges or links.

$$S_p = C_D^W(p) = \frac{\sum_q^N w_{pq}}{n-1} \qquad (1)$$

Where C_D^W represents the weighted degree centrality; p is the focal node ; q= adjacent node ; w= weight attached to the edge ; and n= total number of nodes in the graph. This reasoning can be extended to the weighted centrality of the Closeness, Betweenness and the Eigenvector. As an example, the weighted eigenvector centrality could be seen as

$$\boldsymbol{\lambda x = A^w x} \qquad (2)$$

where $\mathbf{A^w}$ is a square matrix of the weights on the edges of **A** and **x** is an eigenvector of **A** .

A tuning parameter α was introduced to determine the relative importance of the number of ties compared to the weights on the ties by [3]. Equation (3) below thereby represents the product of degree of a focal node and the average weight to these nodes as adjusted by the introduced tuning parameter. So, for weighted degree centrality at α we have:

$$c_d^{w\alpha}(p) = k_p \times \left(\frac{s_p}{k_p}\right)^{\alpha} = k_p^{(1-\alpha)} \times s_p^{\alpha} \qquad (3)$$

where k_p = degree of nodes

S_p = c_D^w(p) as defined in (1) above , and α is ≥ 0

This argument could also equally be applied to the closeness centrality; betweenness centrality and eigenvector centrality.

Node-Weighted Centrality

As an extension to equation (3), a tuning parameter β was introduced by [13] to include the weightedness on the nodes, therefore, for weighted degree centrality at α *and* β we shall now have

$$c_d^{wab}(i) = k_i \times \left(\frac{s_i}{k_i}\right)^{a} = k_i^{(1-a)} \times s_i^{a} \times z_i^{b} \qquad (4)$$

where k_i = degree of nodes

s_i = $c_D^w(s)$ as defined in (1)

z_i = weight of nodes, where $\alpha \geq 0$; $\{\beta \in Z : -1 \leq \beta \leq 1\}$

The value β depends on whether the weight is having positive or negative effect on the centrality measure, if for instance the weight is having a positive effect (e.g. profit) β is +1 else it is -1 (i.e. loss). Values of *α ranging from ¼ to 1¾ is used in order to vary the effect of α*, i.e. values less than 1 and those greater than 1.

Top-Eigen Weighted Vector Centrality and Newtonian Gravitational Force

The node-weights of the sample used for this study is the sales value while the edges are the driving distances between the shops in the sampled area. The sampled shops here are maximally connected as all of them have road links. Hence, we take the advantage of the clique structure by making the most central node (the one with highest centrality) from each clique to be a representative of that clique. By that, we have a representative node each from the two cliques considered for the purpose of the prediction of a proposed DC (see figure 1 below).

In the county of Scotland, two major cities with higher concentration of shops were chosen for our sample, the city of Glasgow and Edinburgh. In each of the cities, the ranking of the nodes(i.e. shops) based on eigen-vector centrality were considered, tested for all the four centralities (degree, closeness, betweeness and eigen-vector), thereafter, the highest ranking node called the top eigen-vector weight was made to be representative of that city (see Table I). The driving distances apart of each of the representative cliques for Glasgow and Edinburgh were obtained from google MAPI. UCINET , tnet and Excel software are used for obtaining the centralities and doing the final calculations (see Figure2).

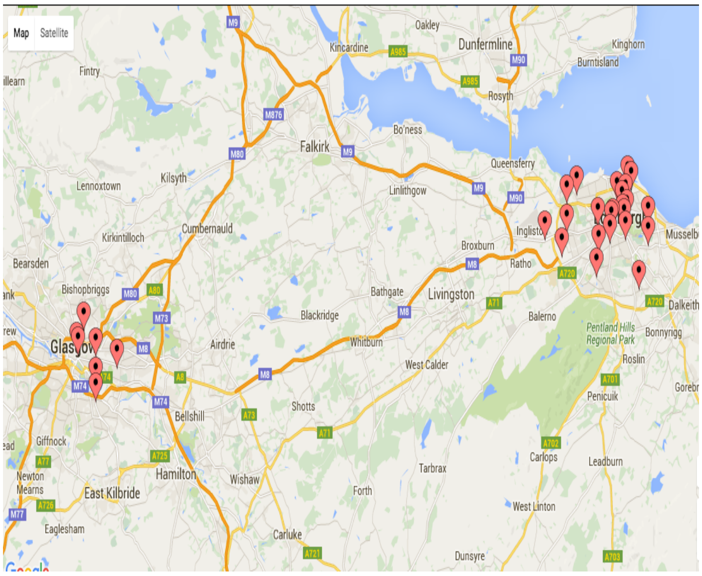

Figure.1. Figure showing the two cliques of Scotland shops (Glasgow on the left and Edinburgh on the right)

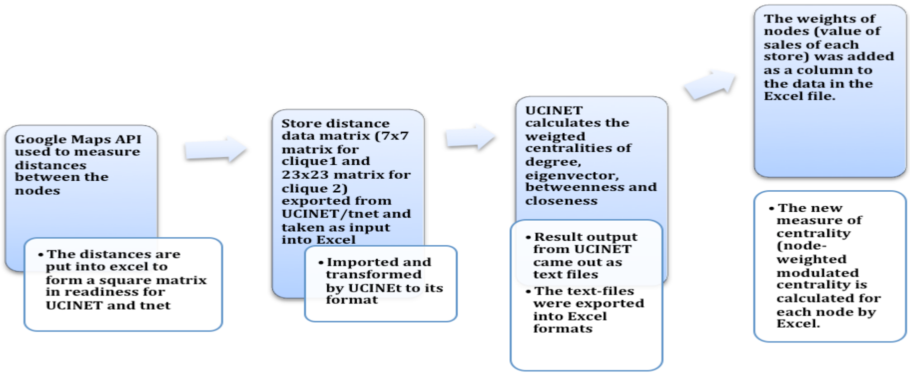

Figure 2. Figure showing the implementation of top-eigen vector weighted centrality measure to the cliques of Scotland

The newton gravitational force was later introduced after the implementation of the Top eigen-vector weighted centrality, and it is later explained with equation (5).

Top-Eigen Weighted Vector Centrality

Node 22 with postcode EH12 7UQ being the highest ranking always, was chosen as the representative of the clique from Edinburgh when the Top eigen-vector weighted centrality is used. Similar procedure was carried out for Glasgow clique and Node 5 with postcode G21 1YL came out being the representative of that clique.(Figure 3 and Figure 4)

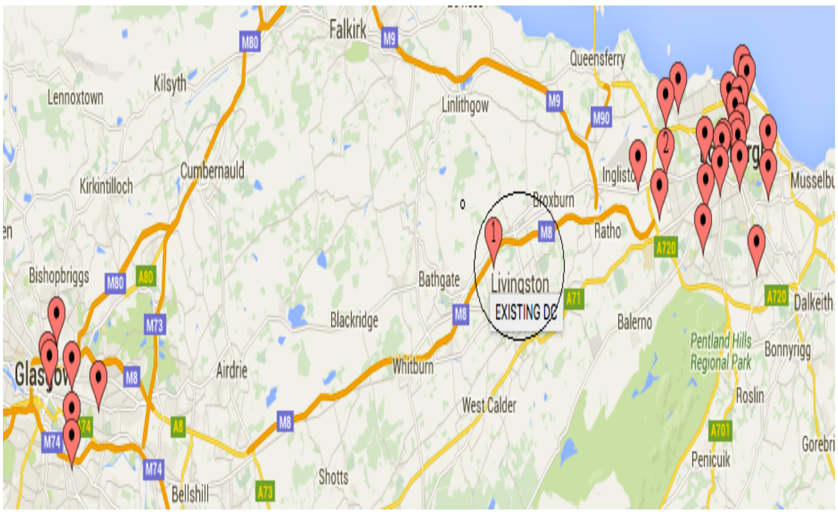

Figure.3 Figure showing the Existing DC at Livingston(encircled) and the clique representative node at Edinburgh marked "2".

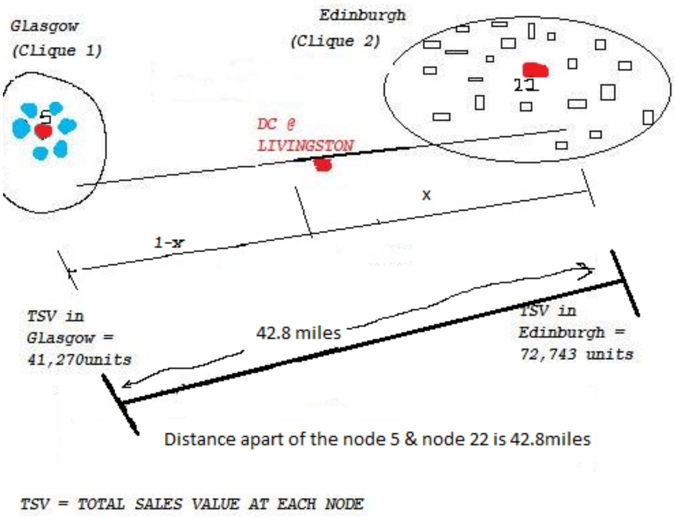

Figure 4. Figure showing the representative cliques at Scotland cities of Glasgowand Edinburgh

From Figure.4 above, let *x* be the proportional distance to the predicted Distribution Centre, and since the driving distance between node 5 (representing Glasgow clique) and node 22 (representing Edinburgh clique) is 42.8miles, by proportion

$$1\text{-}x / x = 72743/41270,$$

then $x = 0.36$ (i.e. 36% of 42.8) which is 15.4miles

If x is some 15.4miles away from the Edinburgh clique representative, and the existing DC is 13.1 miles away from node 22, the difference of the predicted DC will be 2.3miles away from the existing DC, hence,
the error rate of the predicted DC = (2.3/42.80) x 100 = 5.37% i.e. the percentage accuracy of the prediction = 94.63%

Newtonian Gravitational Force

This method is fashioned after the Newton's gravitational law which ascerts that every object's mass will ascertain some amount of force on any neighboring object, no matter the distance between them. The formula is:

$$F = k * \frac{(m * M)}{R^2} \qquad (5)$$

where
F = Gravitational Force
k = constant
m =the mass of the first object
M=the mass of the second object
R=Distance between the two objects (it can be driving distance or the earth distance)

Table 1. Table showing the clique result according to Top Eigen-Vector Weighted Centrality of selection from Edinburgh and Glasgow

	NODE-WEIGHTED EIGEN-VECTOR CENTRALITY FOR EDINBURGH						NODE-WEIGHTED EIGEN-VECTOR CENTRALITY FOR GLASGOW						
ODE	α= ¼	α= ½	α= ¾	α= 1¼	α= 1½	α= 1¾	NODE	α= ¼	α= ½	α= ¾	α= 1¼	α= 1½	α= 1¾
22	453097.5	510928.4	576140.5	732596.9	826101.5	931540.6	1	51495.5	83776.9	136294.6	360734.7	586870.9	954766.6
23	33870.3	33585.7	33303.6	32746.4	32471.3	32198.5	3	111193.7	183949.4	304310.2	832822.6	1377750.4	2279232.3
24	63165.7	70052.7	77690.6	95555.4	105973.8	117528.2	4	73621.3	130901.3	232747.0	735808.2	1308292.6	2326189.8
25	78587.1	77797.5	77015.8	75475.8	74717.4	73966.6	5	145086.8	264307.3	481493.6	1597913.3	2910948.7	5302930.0
30	152807.8	154952.0	157126.2	161566.5	163833.5	166132.3	11	41543.2	75860.3	138525.5	461912.0	843478.6	1540241.6
31	328540.4	360246.8	395013.1	474935.1	520769.5	571027.4	13	4465.1	7280.7	11871.6	31563.2	51465.8	83918.1
32	297308.6	346383.0	403557.8	547777.5	638195.0	743537.0	15	3662.5	6052.2	10001.1	27309.7	45128.5	74573.5
33	165476.9	173857.7	182662.9	201633.7	211845.6	222574.7							
35	3412.6	3404.2	3395.9	3379.3	3371.0	3362.7							
36	21128.7	21025.6	20923.1	20719.5	20618.5	20518.0							
37	5285.0	6115.7	7076.9	9476.3	10965.7	12689.2							
38	8971.3	9984.7	11112.6	13764.9	15319.7	17050.2							
39	7943.9	7919.5	7895.2	7846.8	7822.7	7798.7							
40	4531.7	5270.7	6130.4	8293.1	9645.7	11218.9							
41	28334.9	28259.3	28183.9	28033.6	27958.7	27884.1							
42	11302.1	11272.0	11242.1	11182.4	11152.7	11123.0							
43	2201.3	2631.5	3145.8	4495.6	5374.3	6424.6							
44	9406.4	9964.9	10556.6	11847.4	12550.8	13296.0							
45	4698.1	5294.3	5966.3	7576.8	8538.3	9622.0							
46	17504.4	17952.4	18411.8	19366.2	19861.8	20370.1							
47	4229.0	4130.8	4034.8	3849.6	3760.1	3672.8							
48	22823.0	23793.4	24805.0	26959.2	28105.4	29300.4							
49	17663.5	17373.2	17087.6	16530.6	16258.9	15991.7							

In case of the objects, which in this case are the 30 shops of Scotland (consisting of seven shops from Glasgow and 23 shops from Edinburgh) as shown in Figure 3 above. The shops have pull effects on the DC at Livingston, as such the vectorial resultant force F of each node(shop) is calculated using the earth distances apart and the driving distances apart.

Earth Distance with 30shops/nodes
When the representative clique (EH12 7UQ i.e. Node 22) was used as origin (leaving 29 shops for consideration) as shown in Figure 5 below, the total force is 314.53units but when the actual DC for Scotland (EH54 8QW) was used as origin (as in Figure 3) for all 30shops the total force was 12.28units.

Figure 5. Figure showing the representative clique of Edinburgh herein marked "1" with other shops in Scotland

In the Figure 5. above, the point marked "1" is the representative clique (node 22) EH12 7UQ. This node is used as the origin for the other 29nodes in the region of Glasgow and Edinburgh, which is, excluding the existing DC (EH54 8QW) at Livingston.

To make things clearer, the figure below shows the existing DC – EH54 8QW (at Livingston) as "1" while "2" represents the predicted DC – EH12 7UQ (at Edinburgh)

Figure 6. Figure showing the Existing DC at Livingston and the representative clique at Edinburgh

Driving Distance with 30shops/nodes

For the driving distance, the total force for the DC as origin is 1,394,170.15 while the representative clique as origin yielded 29,690,905.18 . The table 2 below summarises the findings of the resultant forces when each of the driving distances and earth distances is used in the calculations.

Table2. Table showing the total force for Earth and Driving forces for Scotland shops

S/No	TYPE OF DISTANCE	EXISTING DC	PREDICTED DC
1	EARTH DISTANCE	1.23 E01	6.0 E01
2	DRIVING DISTANCE	1.39 E06	4.76 E06

Scotland with 7 shops/nodes at Glasgow, 23shops/nodes at Edinburgh and three additional shops

We consider three additional shops which are outliers , that is, not within Glasgow and Edinburgh but within an increased coverage radius of 36miles against the previous 30miles radius. This means we now consider 33 shops as our sample instead of the previous 30 shops, these newly added shops are at South Queenferry, Hardington and Bathgate. With these additional three shops added from within Scotland but outside Glasgow and Edinburgh, we have the results in Figure 7 below:

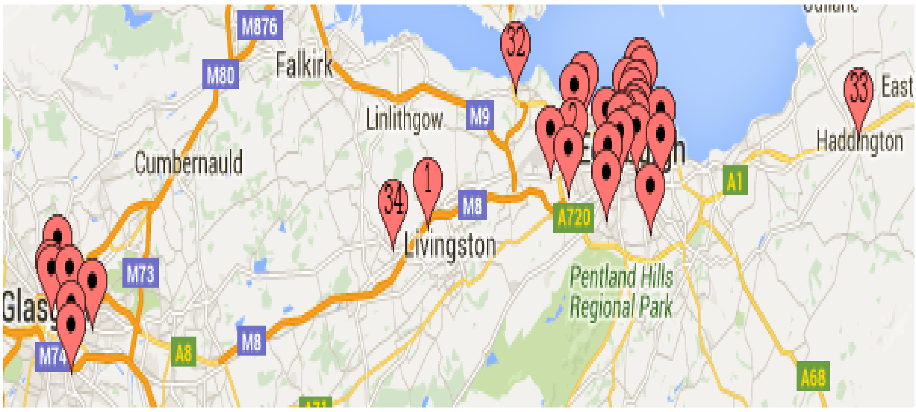

Figure 7. Figure showing newly added nodes 32, 33 & 34 outside Glasgow and Edinburgh

The details of the new shops/nodes are as shown in the table 3 below:

Table 3. Table showing details of the three new nodes added to the existing 30 nodes/shops

S/ N	Node	Post Code	Dist to Existing DC	City	Sales Values	Lat	Long
1	32	EH30 9QZ	11.9	SOUTH QUEENSFERRY	7948	55.9828	3.3990
2	33	EH41 3LZ	36.4	HADDINGTON	9358	55.9571	2.7777
3	34	EH48 2ES	3.8	BATHGATE	13746	55.8936	3.6215

With the addition of the three new shops and using each one as the origin to the remaining 32 shops, the table 4 below compares the results with the existing DC and former representative clique node using centrality measures.

Table 4. Table showing the total force for Earth and Driving forces for Scotland with additional three shops as new origins

S/No	TYPE OF DISTANCE	EXISTING DC	PREDICTED DC	New Shop1 (EH30 9QZ) as Origin	New Shop2 (EH41 3LZ) as Origin	New Shop3 (EH48 2ES) as Origin
1	EARTH DISTANCE	1.98 E01	1.3 E02	2.8 E01	6.6 E01	4.4 E01
2	DRIVING DISTANCE	6.1 E06	2.3 E07	1.1 E07	1.3 E07	5.9 E06

Figure 8. Figure shows the newly predicted DC as against the earlier predicted node labeled 2

Newtonian Gravitational Force with 30 shops of Glasgow and Edinburgh

Using the Earth distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 5 below:

Table 5. Table showing the forces exerted by highest/lowest valued nodes while considering earth distance

	Glasgow			Edinburgh			
	Node	Post Code	Value of Force	Node	Post Code	Value of Force	Distance Apart of Nodes
Highest Value Nodes	Node5	G21 1YL	1.5890	Node22	EH12 7UQ	1.7703	42.2
Lowest Value Nodes	Node15	G1 1EJ	0.0447	Node43	EH12 9BH	0.0080	41.3

Using the driving distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 6 below:

Table 6. Table showing the forces exerted by highest/lowest valued nodes while considering driving distance

	Glasgow			Edinburgh			
	Node	Post Code	Value of Force	Node	Post Code	Value of Force	Distance Apart of Nodes
Highest Value Nodes	Node5	G21 1YL	57,189.34	Node8	EH12 7UQ	404,474.18	2.2
Lowest Value Nodes	Node15	G1 1EJ	1,549.52	Node45	EH8 7NG	1,081.94	53.7

Newtonian Centrifugal Force with 33 shops of Glasgow and Edinburgh
Using the driving distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 7 below:

Table 7. Table showing the forces exerted by highest/lowest valued nodes while considering driving distance

	Glasgow			Edinburgh			
	Node	Post Code	Value of Force	Node	Post Code	Value of Force	Distance Apart of Nodes
Highest Value Nodes	Node 5	G21 1YL	66,150.20	Node 52A	EH48 2ES	4,184,638.00	27.7
Lowest Value Nodes	Node 15	G1 1EJ	1,792.31	Node 45	EH8 7NG	1,251.47	53.7

Using the earth distance between the shops and the Existing Distribution Centre (EDC) as origin, we have the results in Table 8 below:

Table 8. Table showing the forces exerted by highest/lowest valued nodes while considering earth distance

	Glasgow			Edinburgh			
	Node	Post Code	Value of Force	Node	Post Code	Value of Force	Distance Apart of Nodes
Highest Value Nodes	Node5	G21 1YL	2.0218	Node22	EH12 7UQ	2.2525	42.2
Lowest Value Nodes	Node15	G1 1EJ	0.0568	Node43	EH12 9BH	0.0102	41.3

SUMMARY OF ACCURACY WITH THE SALES VALUES USED AS NODE-WEIGHTS

Table 9. Accuracy of results obtained for both earth/driving distances for 30 shops and 33 shops

	PERCENTAGE ACCURACY OF THE HIGHEST FORCE NODES FROM GLASGOW TO EDINBURGH	PERCENTAGE ACCURACY OF THE LOWEST FORCE NODES FROM GLASGOW TO EDINBURGH
EARTH DISTANCE WITH 30 SHOPS	64.9%	63.2%
EARTH DISTANCE WITH 33 SHOPS	99.1%	99%
DRIVING DISTANCE WITH 30 SHOPS	64.9%	79.9%
DRIVING DISTANCE WITH 33 SHOPS	63.5%	84%

Conclusions

The Newtonian Gravitational force provides a more accurate percentage of 4.4% more than when the TEVW centrality was applied. The set of input resources for this method are the node-weights and link-weights, even though there are other factors to consider in the citing of a distribution centre, this makes this method a cheaper one with high accuracy of prediction. The assumptions in this study is that the driving distances are taken to be a straight line in the model figures in this paper, whereas in reality this might not necessarily be so.

In future, the range of values for α might transcend the range of ¼ and 1¾ as some interesting outcomes might surface, also, the domain of application could still be further expanded to cover area such as bioinformatics whereby the visualisation and understanding of biology networks will make one to be able to predict the reaction of cells to pharmaceutical drugs due to their positioning in such a network. Healthcare is another area of consideration, as the study of the connections between hospitals, patients, doctors and healthworkers can help a lot in the prediction of where to cite new hospitals and even how to arrest or prevent epidemics. In terms of network security, a more central node is protected and given more attention in order to prevent or repel attacks from any form of intrusion.

It is clear that the node-weights (node attributes) actually count in any network as confirmed in this research whereby it forms the basis of prediction of a distribution centre with a higher accuracy while making use of the newtonian gravitational force as compared with the centrality measure – Top Eigen-Vector Weighted Centrality (TEVWC).

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