An Improved Method of Continuum Topology Optimization Subjected to Frequency Constraints Based on Indenpendent Continuous Topological Variables

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Abstract

In this paper, an improved topology optimal model of continuum structures subject to frequency constraints is established based on Independent, Continuous, Mapping (ICM) method. Firstly, two filter functions- Power Function(PF) and Composite Exponential Function(CEF) are selected to recognize the design variables, and to implement the changing process of design variables from "discrete" to "continuous" and back to "discrete". Explicit formulations of frequency constraints are given based on Rayleigh's quotient, filter functions, first -order Taylor series expansion. Then, an improved optimal model is formulated using different filter functions and the explicit frequency constraints. The program based on the dual sequence quadratic programming (DSQP) and global convergent method of moving asymptotes algorithm(GCMMA) for solving the optimal model is developed on the platform of MSC.Patran & Nastran. Finally, numerical examples are given to demonstrate the validity and applicability of the proposed method. By comparison, we find that the results from DSQP method equipped with filter function of composite exponential function are a little better than other methods for the problem of frequency constraints.

Key words: Topological optimization·Continuum·Frequency constraint· Independent Continuous and Mapping(ICM) method·filter function

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Introduction

The essence of topology optimization lies in searching for the optimum path of transferring loads, therefore the computational results of topology optimization are usually more attractive and more challenging than the results of cross-sectional and shape optimization. Although topology optimization is only in conceptual design phase in engineering, the design results significantly impacts the performance of the final structure. Since the landmark paper of Bendsøe and Kikuchi[1], numerical methods for topology optimization of continuum structures have been developed quickly in application[2]. The known are homogenization method[5],6], variable density method(including SIMP and RAMP interpolation model)[7-10], evolutionary structural optimization (ESO) [11-13], level set method [14-16]and so on.

Compared with static topology optimization, the optimization algorithm on dynamic topology optimization is more complicated and the calculation of sensitivity analysis is more enormous. Frequency topology optimization is of great importance in dynamic topology optimization and engineering fields. Topology Optimization with respect to frequencies of structural vibration was first considered by Diaz and Kikuchip[17], who studied the topology optimization of eigenvalues by using the homogenization method where reinforcement of a structure is optimized to maximize eigenvalues. Subsequently, many researches focus on to expand topology optimization in dynamic problems. Ma et al. [18,19], Kosaka and Swan [20]presented different formulations for simultaneous maximization a set of frequencies of free vibration of disk and plate structures. Krog and Olhoff [21], Jensen and Pedersen[22] utilize a variable bound formulation that facilitates proper treatment of multiple frequencies. Pedersen[23]deals with maximum fundamental frequency design of plates and includes a technique to avoid spurious localized modes. Jensen and Pedersen[22] presents a method to maximize the separation of two adjacent frequencies in structures with two material components. Zhu & Zhang[24] emphasize on layout design which is related to optimization of boundary conditions and it is studied to maximize natural frequency of structures. In 2007， Du and Olhoff[25] introduced SIMP method for maximization of first eigenvalue and frequency gaps. In 2009, Niu et al.[26] proposed a two-scale optimization method and found the optimal figurations of macrostructure- microstructure of cellular material with maximum structural fundamental frequency. Huang et al.[27] investigated the maximization of fundamental frequency of beam, plane and three-dimensional block by applying a new bi-directional evolutionary structural optimization (BESO) method, and dealt with localized modes by modifying the traditional penalization function of SIMP method. Qi et al.[28] presented a level set based shape and topology optimization method for maximizing the simple or repeated first eigenvalues of structure vibration. Further development on frequency topology optimization see references[29-33].

Independent, Continuous and Mapping (ICM) method[34], which is proposed by Sui for skeleton and continuum structural topology optimization in 1996. This method generalizes topological variables abstractly independence of the design variables such as sectional sizes, geometrical shape, density or Young’s modulus of material. Filter functions are used to map the changing process of topological design variables from “discrete” to “continuous” and back to “discrete”. The smooth model with minimizing structural weight is established and solved by the traditional algorithms in mathematical programming. This model is beneficial to maintain the consistency of objective and constraint in cross-sectional optimization, shape optimization and topology optimization.

In this paper, we extend our previous research[34-36] primarily about Independent, Continuous and Mapping (ICM) method on static topology optimization issues of continuum structures to dynamic topology optimization field. A model of topology optimization for the lightest structures with frequency constraints is investigated. An improved model of continuum topology optimization with Composite Exponential Function(CEF) as filter function instead of Power function is established. Among the methods of mathematic optimization model solving, mathematical programming (MP) method is popular. Because of the nonlinearity of mathematic optimization model in topology optimization of continuum structure， sequential quadratic programming (SQP) in the MP method are widely used. And the dual theory is used to convert the constrained optimization model to one with reduced number of design variables, and the solving efficiency is greatly improved. Therefore, dual sequential quadratic programming (DSQP) algorithm is employed in this paper, and the results is compared with that of the global convergent method of moving asymptotes algorithm (GCMMA)[37,38].

This paper is organized as follows. In section 2, the optimization formulation and description of filter function are introduced. In section 3, an improved frequency topology optimization model based on ICM method is built. Optimal algorithms to solve the mathematical optimization problem are given in section 4. Numerical simulations are presented in section 5. In section 6, conclusions are given.

1 Problem formulation and description of filter function

1.1 Optimization problem formulation

For structural cross-section and shape optimization, natural frequency of structure is often taken as constraint. We denote f_i as the frequency of *i*-th order, and $\underline{f_i}, \overline{f_i}$ are the low and up bound of *i*-th order frequency respectively. They satisfy the following inequality:

(i) $f_1 \geq \underline{f_1}$;

(ii) $f_i \le \bar{f}_i$ and $f_{i+1} \ge \underline{f_{i+1}}$ in non-frequency band constraints.

For elastic structure, the usual relation between frequency f and eigenvalue is $\lambda = (2\pi f)^2$. Therefore, the frequency constraints can apparently be transformed into eigenvalue constraints using the formula. Here we uniformly use $g(\lambda) \le \overline{\lambda}$ to generalize (i) and (ii) based on $\lambda = (2\pi f)^2$.

Thus, the model of continuum topology optimization with frequency constraints can be formulated as follows

$$\begin{cases} find\ \ \boldsymbol{t} \in E^N \\ make\ W = \sum_{i=1}^{N} w_i \ \to \min \\ s.t. \quad g(\lambda_j) \le \overline{\lambda_j} (j = 1, \cdots, J) \\ \qquad 0 < \underline{t} \le t_i \le 1 \ (i = 1, \cdots, N) \end{cases} \tag{1}$$

where $\boldsymbol{t}$ and W denote the topological design variable vector and the weight of structure. i and j are the i-th element and the j-th order frequency respectively, J and N represent the total number of constraints and elements. And $\underline{t}$ is the lower bound of design variables, e.g. $\underline{t} = 0.001$.

1.2 Description of the filter function

In order to develop the model ICM method, we firstly investigate the essential part of ICM—the filter function. Its definition and choosing determine the establishment and solving of optimization model, and further filter funciton will make great impact on the final performance of topology optimization. In order to map the topological variables from “discrete” to “continuous”, Sui(1996) studied the filter function $f(t_i)$.

Several types of filter function are suggested in ICM method[34]. Among which, Power Function(PF) is used frequently in application[36] and is as follows

$$f(t_i) = t_i^{\alpha}, \ \alpha \ge 1 \tag{2}$$

Here t_i denotes i-th design variable. α is a positive constant.

We introduce a new filter function -Composite Exponential Function(CEF) to take the place of the old one and it is as follows:

$$f(t_i) = \frac{e^{t_i/\gamma} - 1}{e^{1/\gamma} - 1}, \ \gamma > 0 \tag{3}$$

γ is a given positive constant and it is determined by numerical experiments with different problems. In section 5, we compared the performance of the two types of filter function.

Denote $f_w(t_i)$, $f_k(t_i)$ and $f_m(t_i)$ as filter functions for frequency topology optimization and they are given as follows:

$$w_i = f_w(t_i) w_i^0, \ \boldsymbol{k}_i = f_k(t_i) \boldsymbol{k}_i^0, \ \boldsymbol{m}_i = f_m(t_i) \boldsymbol{m}_i^0 \tag{4}$$

Here w_i^0, $\boldsymbol{k}_i^0$ and $\boldsymbol{m}_i^0$ are the element weight, element stiffness matrix and element mass matrix of original structure before the process of topology optimization, respectively. w_i, $\boldsymbol{k}_i$ and $\boldsymbol{m}_i$ are the ones in the process of topology optimization, respectively.

2 Improved model based on ICM method

2.1 Determination of eigenvalue

In the finite element analysis the dynamic behavior of a continuum structure can be represented by the following general eigenvalue problem

$$(\mathbf{K} - \lambda_j \mathbf{M}) \mathbf{u}_j = 0 \tag{5}$$

where, $\boldsymbol{K}$ is the global stiffness matrix and $\boldsymbol{M}$ is the global mass matrix. λ_j is the jth eigenvalue and $\boldsymbol{u}_j$ is the eigenvector corresponding to λ_j. The eigenvalue λ_j and the corresponding eigenvector $\boldsymbol{u}_j$ are related to each other by Rayleigh quotient

$$\lambda_j = \frac{\mathbf{u}_i^T \mathbf{K} \mathbf{u}_i}{\mathbf{u}_i^T \mathbf{M} \mathbf{u}_i} \tag{6}$$

2.2 Sensitivity analysis

Since eigenvalue λ_j is implicitly related with topology variable $\boldsymbol{t}$, we use first-order Taylor series expansion for eigenvalue to express their relationship explicitly. At first, the sensitivity of eigenvalue with respect to design variables should be derived.

Take the reciprocal of stiffness filter function as design variables as follows

$$x_i = \frac{1}{f_k(t_i)} \tag{7}$$

We have

$$t_i = f_k^{-1}(x_i) \tag{8}$$

Therefore, the stiffness matrix filter function, mass matrix filter function and weight filter function are given as follows

$$f_k(t_i) = \frac{1}{x_i} \ ; \ f_m(t_i) = f_m[f_k^{-1}(\frac{1}{x_i})] \ ; \ f_w(t_i) = f_w[f_k^{-1}(\frac{1}{x_i})] \tag{9}$$

In view of (6), we have the derivative of λ_j to design variable as follows:

$$\frac{\partial \lambda_j}{\partial x_i} = \boldsymbol{u}_j^T \frac{\partial \boldsymbol{K}}{\partial x_i} \boldsymbol{u}_j - \lambda_j \boldsymbol{u}_j^T \frac{\partial \boldsymbol{M}}{\partial x_i} \boldsymbol{u}_j \tag{10}$$

Considering Eq.(4) and (9), the global stiffness matrix $\boldsymbol{K}$ and the mass matrix $\boldsymbol{M}$ can be calculated by

$$\boldsymbol{K} = \sum_{i=1}^{N} \boldsymbol{k}_i = \sum_{i=1}^{N} f_k(t_i) \boldsymbol{k}_i^0 = \sum_{i=1}^{N} \frac{1}{x_i} \boldsymbol{k}_i^0 , \qquad \boldsymbol{M} = \sum_{i=1}^{N} \boldsymbol{m}_i = \sum_{i=1}^{N} f_m(t_i) \boldsymbol{m}_i^0 = \sum_{i=1}^{N} f_m \left[f_k^{-1}\left(\frac{1}{x_i}\right)\right] \boldsymbol{m}_i^0 \tag{11}$$

Substituting Eq.(11) to Eq.(10), we have

$$\frac{\partial \lambda_j}{\partial x_i} = -\mathbf{u}_j^{\mathrm{T}} \frac{2\mathbf{k}_i}{2x_i} \mathbf{u}_j + \beta(x_i) \lambda_j \mathbf{u}_j^{\mathrm{T}} \frac{2\mathbf{m}_i}{2x_i} \mathbf{u}_j = -\frac{2}{x_i}(U_{ij} - \beta(x_i) V_{ij}) \tag{12}$$

where, $\beta(x_i) = \dfrac{f_m'[f_k^{-1}(1/x_i)] f_k(1/x_i)}{f_m[f_k^{-1}(1/x_i)] f_k'(1/x_i)} = \dfrac{f_m'(t_i) f_k(t_i)}{f_m(t_i) f_k'(t_i)}$

In (12), $U_{ij} = \frac{1}{2} \boldsymbol{u}_j^{\mathrm{T}} \boldsymbol{k}_i \boldsymbol{u}_j$ and $V_{ij} = \frac{1}{2} \lambda_j \boldsymbol{u}_i^{\mathrm{T}} \boldsymbol{m}_i \boldsymbol{u}_j$ represent the strain energy and the kinetic energy of ith element corresponding to the jth eigenmode, respectively. At this moment, the derivatives of eigenvalue with respect to all design variables can be obtained by subtracting the strain energy and kinetic energy for element mode from the results of modal analyses.

2.3 Explicit expression of eigenvalue

Using the first-order Taylor series expansion, the approximate expression of eigenvalue $\lambda_j(\boldsymbol{t})$ can be obtained

$$\lambda_j(\boldsymbol{t}) = \lambda_j(\boldsymbol{x}^{(\nu)}) + \sum_{i=1}^{N} \frac{\partial \lambda_j}{\partial x_i}(x_i - x_i^{(\nu)}) \tag{13}$$

where superscript ν is the number at the ν-th iteration.

Substitute Eq.(10) into Eq.(13), we get

$$\lambda_j(\boldsymbol{t}) = \lambda_j(\boldsymbol{x}^{(\nu)}) + \sum_{i=1}^{N} 2(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)}) - \sum_{i=1}^{N} \frac{2}{x_i^{(\nu)}}(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)})x_i \tag{14}$$

The constraint $\underline{\lambda}_j \le \lambda_j(\boldsymbol{x})$ can be rewritten as

$$\sum_{i=1}^{N} \frac{2}{x_i^{(\nu)}}(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)})x_i \le -1 \times [\underline{\lambda}_j - \lambda_j(\boldsymbol{x}^{(\nu)}) - \sum_{i=1}^{N} 2(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)})]$$

For constraint $\lambda_j(\boldsymbol{x}) \le \overline{\lambda}_j$, it can be rewritten as

$$-1 \times \sum_{i=1}^{N} \frac{2}{x_i^{(\nu)}}(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)})x_i \le \overline{\lambda}_j - \lambda_j(\boldsymbol{x}^{(\nu)}) - \sum_{i=1}^{N} 2(U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)})$$

If we define D as

$$D = \begin{cases} 1 & for\ \lambda_j \le \overline{\lambda}_j \\ -1 & for\ \lambda_j \ge \underline{\lambda}_j \end{cases} \qquad \tilde{\lambda}_j = \begin{cases} \underline{\lambda}_j & (\lambda_j \ge \underline{\lambda}_j) \\ \overline{\lambda}_j & (\lambda_j \le \overline{\lambda}_j) \end{cases}$$

And further define

$$A_{ij} = U_{ij}^{(\nu)} - \beta(x_i^{(\nu)})V_{ij}^{(\nu)},\ \overline{c}_{ij} = -\frac{2}{x_i^{(\nu)}}A_{ij},\ c_{ij} = -D\overline{c}_{ij}$$

$$\overline{d}_j = -\lambda_j(\mathbf{x}^{(\nu)}) - \sum_{i=1}^{N} 2A_{ij} \quad ,\ d_j = D \times (\tilde{\lambda}_j + \overline{d}_j) \tag{15}$$

Then frequency constraints can be simplified by the following inequality:

$$\sum_{i=1}^{N} c_{ij}x_i \le d_j \tag{16}$$

Thus ends the process of explicitly approximation of the frequency constraints.

2.4 Improved model of frequency topology optimization

Based on the above analysis, the model of topology optimization with frequency constraints by introducing filter function can be transformed as follows:

$$\begin{cases} find\ \ \boldsymbol{t} \in E^N \\ make\ W = \sum_{i=1}^{N} f_w(t_i)w_i^0 \to \min \\ s.t.\quad g(\lambda_j(f_k(t_i), f_m(t_i))) \le \overline{\lambda_j}(j = 1,\cdots,J) \\ \qquad 0 < \underline{t} \le t_i \le 1\ (i = 1,\cdots,N) \end{cases} \tag{17}$$

By using explicitly approximation of the frequency constraints, the model (17) can be written as follows:

$$\begin{cases} \text{find}\quad \boldsymbol{t} \in E^N \\ \text{make}\ \ W = \sum_{i=1}^{N} f_w(t_i)w_i^0 \to \min \\ \text{s.t.}\quad \sum_{i=1}^{N} c_{ij}x_i \le d_j\,(j = 1,\cdots,J) \\ \qquad 1 \le x_i \le \overline{x}_i\,(i = 1,\cdots,N) \end{cases} \tag{18}$$

3. Solution of the improved topology optimization model

3.1 Standardization of objective

Considering model (18) is a programming with nonlinear objective and linear constraints following the explicit process of frequency constraints, the second-order Taylor series expansion is used to approximate the objective function and ignore the constant item. The model is transformed into the following quadratic programming model:

$$\begin{cases} \text{find} \quad \boldsymbol{t} \in E^N \\ \text{make } W = \sum_{i=1}^{N} (a_i x_i^2 + b_i x_i) \to \min \\ \text{s.t.} \quad \sum_{i=1}^{N} c_{ij} x_i \le d_j \, (j = 1, \cdots, J) \\ \qquad 1 \le x_i \le \bar{x}_i \, (i = 1, \cdots, N) \end{cases} \tag{19}$$

As objective function varies with different filter functions, investigation of the different cases following different types of filter functions is necessary. Here we focus on PF and CEF.

When PF is applied as the filter function, it is given as follows:

$$f_w(t_i) = t_i^{\gamma_w}; \ f_k(t_i) = t_i^{\gamma_k}; \ f_m(t_i) = t_i^{\gamma_m} \tag{22}$$

In view of (8), we have $t_i = \dfrac{1}{x_i^{1/\gamma_k}}$, $t_i^{\gamma_w} = \dfrac{1}{x_i^{\gamma_w/\gamma_k}} = \dfrac{1}{x_i^{\alpha}}$, and $\alpha = \dfrac{\gamma_w}{\gamma_k}$.

Therefore the objective function (19) can be rewritten as:

$$W = \sum_{i=1}^{N} t_i^{\gamma_w} w_i^0 = \sum_{i=1}^{N} \frac{w_i^0}{x_i^{\alpha}} \approx \sum_{i=1}^{N} (a_i x^2 + b_i x) \tag{23}$$

Where $a_i = \dfrac{\alpha(\alpha+1) w_i^0}{2(x_i)^{\alpha+2}}$ and $b_i = \dfrac{-\alpha(\alpha+2) w_i^0}{(x_i)^{\alpha+1}}$ are undetermined parameters.

When CEF is applied as the filter function, it is given as follows:

$$f_w(t_i) = \frac{e^{t_i/\gamma_w} - 1}{e^{1/\gamma_w} - 1}; \ f_k(t_i) = \frac{e^{t_i/\gamma_k} - 1}{e^{1/\gamma_k} - 1}; \ f_m(t_i) = \frac{e^{t_i/\gamma_m} - 1}{e^{1/\gamma_m} - 1} \tag{24}$$

We have $x_i = \dfrac{1}{f_k(t_i)} = \dfrac{e^{1/\gamma_k} - 1}{e^{t_i/\gamma_k} - 1}$, and therefore

$$f_w(t_i) = \frac{\left(\dfrac{e^{1/\gamma_k} - 1}{x_i} + 1\right)^{\frac{\gamma_k}{\gamma_w}} - 1}{e^{1/\gamma_w} - 1}; \quad f_m(t_i) = \frac{\left(\dfrac{e^{1/\gamma_k} - 1}{x_i} + 1\right)^{\frac{\gamma_k}{\gamma_m}} - 1}{e^{1/\gamma_m} - 1} \tag{25}$$

Similarly, the objective function in (20) can be expressed as

$$\sum_{i=1}^{N} \frac{\left(\dfrac{e^{1/\gamma_k} - 1}{x_i} + 1\right)^{\frac{\gamma_k}{\gamma_w}} - 1}{e^{1/\gamma_w} - 1} w_i^0 \approx \sum_{i=1}^{N} (a_i x_i^2 + b_i x_i) \tag{26}$$

Where $a_i = \dfrac{1}{2} \dfrac{\gamma_k}{\gamma_w} \dfrac{w_i^0}{\left(x_i^{(v)}\right)^4} \dfrac{e^{1/\gamma_k} - 1}{e^{1/\gamma_w} - 1} \left(\dfrac{e^{1/\gamma_k} - 1}{x_i^{(v)}} + 1\right)^{\frac{\gamma_k}{\gamma_w} - 2} \left[\left(\dfrac{\gamma_k}{\gamma_w} + 1\right)\left(e^{1/\gamma_k} - 1\right) + 2x_i^{(v)}\right]$,

$$b_i = -\frac{\gamma_k}{\gamma_w} \frac{w_i^0}{\left(x_i^{(v)}\right)^3} \frac{e^{1/\gamma_k} - 1}{e^{1/\gamma_w} - 1} \left(\frac{e^{1/\gamma_k} - 1}{x_i^{(v)}} + 1\right)^{\frac{\gamma_k}{\gamma_w} - 2} \left[\left(\frac{\gamma_k}{\gamma_w} + 2\right)\left(e^{1/\gamma_k} - 1\right) + 3x_i^{(v)}\right].$$

They are two undetermined parameters.

3.2 Solving algorithms of optimization model

With the above analysis and solving of (19), DSQP and GCMMA are employed. The optimal topology structure with continuous design variables is obtained. The iterating computation will end until following condition is satisfied

$$\frac{\left\| \boldsymbol{x}^{(\nu+1)} - \boldsymbol{x}^{(\nu)} \right\|}{\left\| \boldsymbol{x}^{(\nu)} \right\|} \le \varepsilon$$

x* obtained at this moment is just the optimal solution of Eq. (19) however. Then t* can be calculated based on Eq. (8). Let $\boldsymbol{t}^{(k+1)} = \boldsymbol{t}^*$ and modify the last structure via immediate iteration optimizing, and again enter next iteration. Similarly, iterating in this way until the following condition is satisfied

$$\Delta W = \left| \frac{(W^{(\nu+1)}\text{-}W^{(\nu)})}{W^{(\nu+1)}} \right| \le \varepsilon \tag{20}$$

Where $W^{(\nu)}$ and $W^{(\nu+1)}$ is the structural weight of previous iteration and current iteration, respectively. ε is a precision of convergence, which is prescribed to be 0.001 herein.

3.3 Discretization of continuous design variables

To map design variables from "continuous" to "discrete" back, filter threshold value is needed. We denote filter threshold value as T_0 to determine whether the element is deleted or not. In order to measure the discreteness degree of topology variables, we use M_{nd}[39] as a criterion and it is given (21).

$$M_{nd} = \frac{\sum_{i=1}^{n} 4T_i\left(1-T_i\right)}{n} \times 100\% \tag{21}$$

where T_i is the topological variable value for the i-th element and n is the total number of the elements. Following (21) , if all the topological values are 0 or 1, M_{nd} is 0; if the topological values are 0.5, M_{nd} is 1; the more closer of the topological values to 0 or 1, the more smaller value of M_{nd} and the better of the optimal result.

4. Numerical examples

In this section, we illustrate the proposed method with three examples for the topology optimization with frequency constraints. The first one is a rectangular beam with two frequency constraints. We address the ability of schemes to obtain discrete solutions and compare the solutions obtained using two different filter function. We show how it is possible to formulate and solve optimal problems. The second one is a cylindrical shell structure by second frequency constraint. We aims to compare with the results by using two algorithms combined with two filter functions. For the computation, the initial values of topology variables are all given as unit (t=1), the lowest bounds of topology variables and the convergence precision values are 0.01 and 0.001, respectively.

Example 1 Rectangular beam with two frequency constraints

It is a rectangular beam with two ends clamped and the thickness of beam is assumed as 1mm shown as Fig.1. The design domain is 140mm×20mm, and a concentrated mass (Mc = 50g) is attached at the center of base structure and it has inertia only in Y direction. The Young's modulus E = 100GPa, Poisson's ratio μ=0.3 and mass density ρ = 1000kg/m^3. The structure is divided into 140×20 four-node rectangular elements. We set frequency constraints for the design problem is $f_1 \geqslant$ 8000Hz, $f_2 \geqslant$ 60000Hz. The topology optimization equation was formulated combine PF and CEF filter functions, respectively.

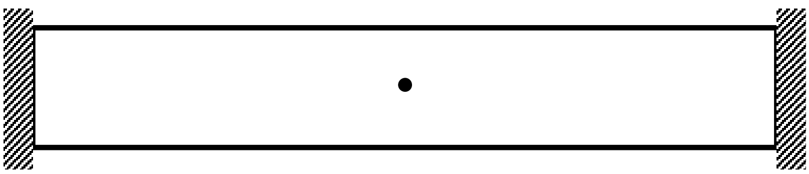


Fig.1 Geometry model of clamped beam

(a)Optimal topology configurations with PF

(b)Optimal topology configurations with CEF

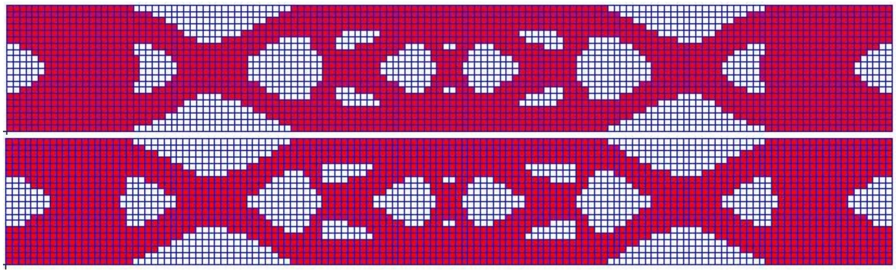


Fig.3 Optimal topology configurations with PF different filter functions

The solving topology configuration of the beam with different filter functions is given in Fig.3. The iterative curve of computation with different filter functions are described in Fig.5-8. To describe the dynamics of optimal structure, the first three modal shapes of optimal structure with two filter functions are computed and displayed in Fig.4-6. The frequency and structural weight changing with time in the optimization process are presented in Fig.7 and Fig.8 with different filter functions. The optimal results with different filter function are shown in Table.1 and the computed distribution of topological design variable values is listed in Table 2.

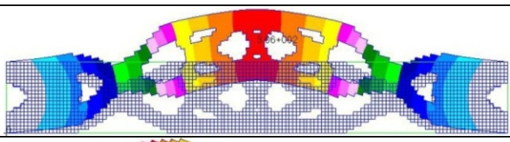
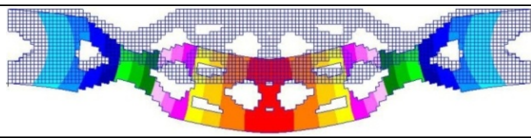
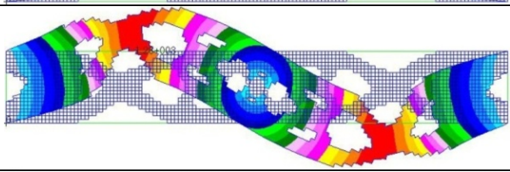
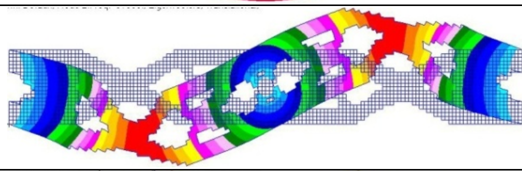
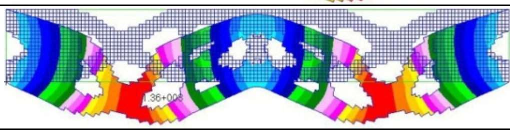
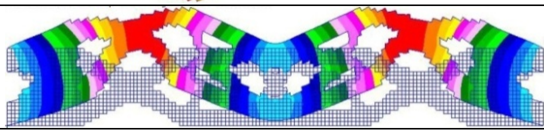
Order	PF	CEF
1		
2		
3		

Fig.4 Modal shapes of optimal structure with different filter functions

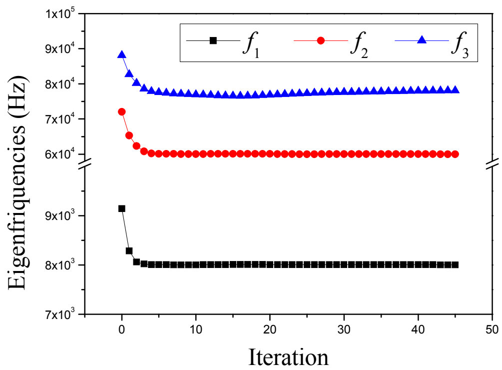


Fig.5 Iteration curves of frequency with PF

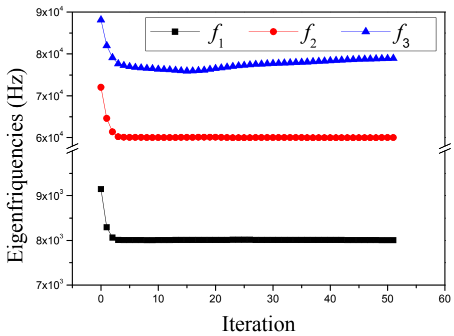


Fig.6 Iteration curves of frequency with CEF

Table1 Optimal results with different filter functions

Filter function	PF	CEF
Iteration	45	51
Mass (g)	2.067093018	1.9778114014
f_1 (Hz)	8003.934082	8003.0073242
f_2 (Hz)	59968.550781	60027.289063

Table2 Distribution of topological value with different filter functions

Distribution of topology value	PF	CEF
(0,0.1]	240	472
(0.1,0.2]	72	60
(0.2,0.3]	52	56
(0.3,0.4]	24	64
(0.4,0.5]	52	48
(0.5,0.6]	108	44
(0.6,0.7]	160	28
(0.7,0.8]	232	84
(0.8,0.9]	216	188
(0.9,1]	1644	1756
Total number of element	2800	2800
M_{nd}	26.74%	16.36%

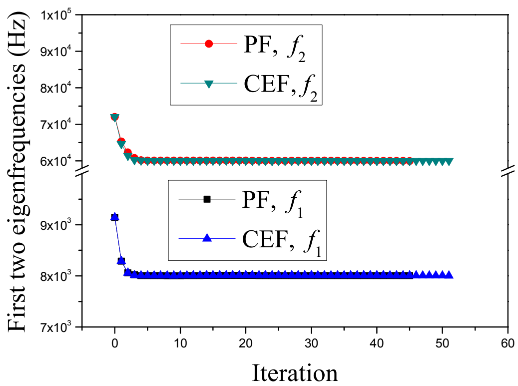


Fig.7 Iteration curves of constrainted frequencies with different filter functions

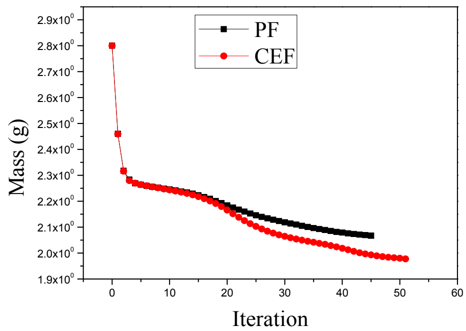


Fig.8 Iteration curves of mass with different filter functions

Example2 Cylindrical shell with the second frequency constraint

A cylindrical shell structure with thickness is 1m, bus-bar a= 20m, arc b=20m, central angle α =0.25 and radius R=80m was shown in Fig.10. In addition, a concentrate mass M=312000kg was attached on the center of cylindrical shell. The Young's modulus E = 100GPa, Poisson's ratio μ=0.3 and mass density ρ = 7800kg/m^3. The structure was divided into 30×30 four-node rectangular elements. The constraint frequency for the design problem is $f_2 \geqslant 28$ Hz. The topology optimization equation was formulated combine PF and CEF filter functions, respectively.

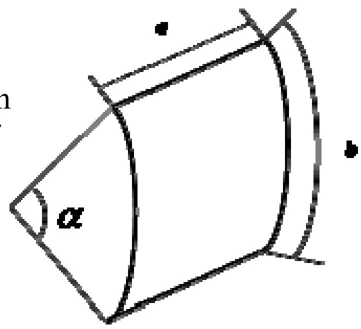
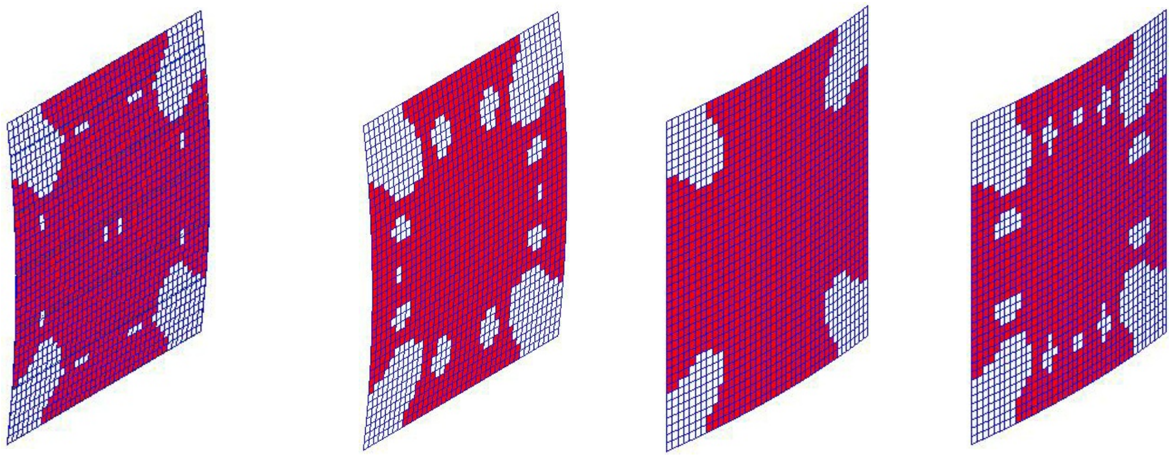


Fig.9 Geometry model of cylindrical shell

Optimal topology configurations after optimization are shown in Fig.10. Iteration curves of first three frequencies with different algorithms and filter functions are given in Figure11. From Fig.12 and Fig.13, we can get the iteration curves of second frequencies and the iteration curves of structural mass for different algorithms and filter functions. Table3 lists the results of optimization for cylinder shell.

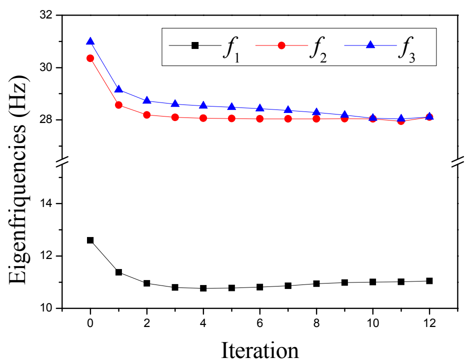
Table3 Results of optimization for cylinder shell

Algorithm and filter function	GCMMA& PF	GCMMA&CEF	DSQP&PF	DSQP&CEF
Iteration	12	29	12	46
Mass (kg)	2634298.85	2309522.20	2633713.28	2146202.33
f_2 (Hz)	28.104261398	28.017398834	28.0874	28.0153
f_3 (Hz)	28.114189148	28.737268448	28.1656	28.7392

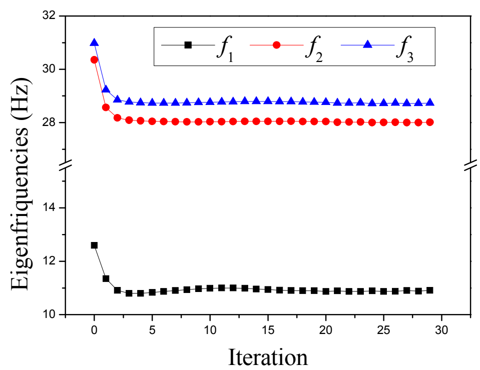


(a) GCMMA& PF (b) GCMMA&CEF (c) DSQP& PF (d) DSQP& CEF

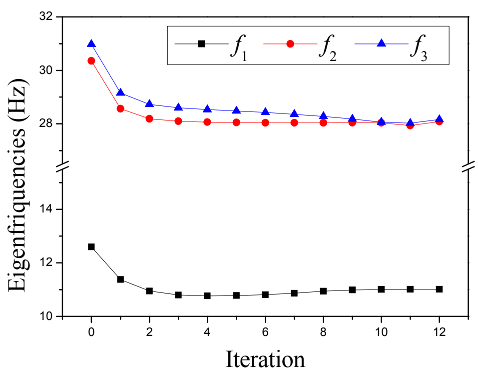
Fig.1 Optimal topology configuration with different algorithms and filter functions.



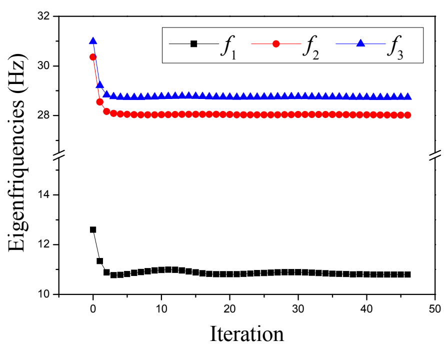
(a) GCMMA& PF



(b) GCMMA&CEF



(c) DSQP& PF



(d) DSQP& CEF

Fig.11 Iteration curves of first three frequencies with different algorithms and filter functions

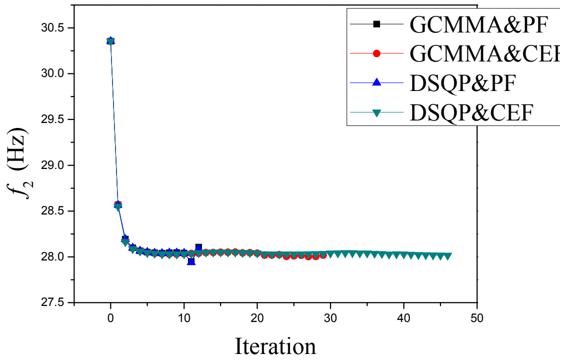


Fig.12 Iteration curves of second frequency

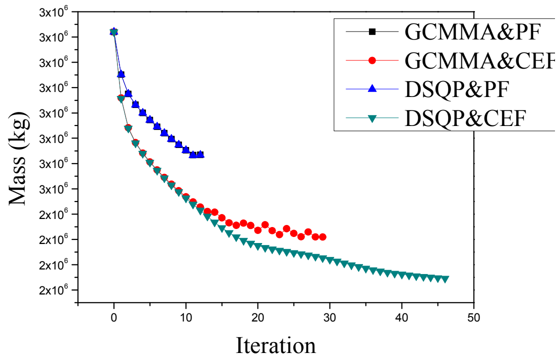


Fig.13 Iteration curves of mass

Conclusion

In this paper, an improved frequency topology optimization model of continuum structure is developed based on ICM method. CEF- a new filter function is selected to recognize the design variables, as well as to implement much better the changing process of design variables from “discrete” to “continuous” and back to “discrete”. Explicit formulations of frequency constraints are given by extracting structural strain and structural kinetic energy from the results of structural modal analysis. An improved optimal model is formulated using CEF and the explicit frequency constraints. The program based on DSQP and GCMMA for solving the optimal model is developed on the platform of MSC.Patran & Nastran. Finally, two examples of continuum structure are computed to demonstrate the feasibility of the proposed method.

The performance of the developed program are given in Fig.3,Table1, Table2, Table3, Fig.7, Fig.8, Fig.10, Fig.12, Fig.13. The results from Fig.3 and Fig.10 show that clear and stable configurations can be obtained using different algorithms and filter functions, and we find that configurations computed with DSQP combined PF and DSQP combined CEF, GCMMA combined PF and GCMMA combined CEF are similar between one and the other in Fig.10. From Table 1, we can see that the objective (weight)with CEF is apparent lower than that of PF. However, the iterative step numbers of CEF is larger than that of PF for the convergence. We can also find that DSQP combined CEF has the best performance for the optimization example from the point of view of optimal objective in Fig.13. From the point of the discrete degree, Table2 for the distribution of optimal topological values show that the M_{nd} with PF and CEF are 26.74% and 16.36%, the difference is apparent . CEF has the better performance in the process of optimization.

Although the comparison of DSQP with GCMMA from the recent reference are done, and we have better results coupled with two different filter function, we just give compared results based on ICM method. To improve continuum structure optimal algorithms , it is necessary to investigate the algorithm based on other methods.

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