An Interpolative Particle Level Set Method

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Abstract

There exist a wide range of applications for solutions to multiphase flow problems with moving interfacial dynamics. These include engineering, fluid mechanics, melting metals, geophysical, medical, computer graphics and image processing. Over the years there have been a large effort in the numerical method community to solve these types of problems. Capturing topological changes with physical accuracy remains a challenge. The two main computational approaches for simulating moving interfaces can be categorized as interface capturing (most notably volume of fluid (VOF) and the level set method) and interface tracking methods. The advantages of both kinds of methods can be combined using hybrid methods, such as the particle level set method [1]. In this paper we propose a new particle level set method which uses an interpolation scheme to update the radii of the interface particles. Preliminary results show that this method can outperform the original particle level set method using fewer particles.

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INTRODUCTION

Advantages of the level set method include natural merging and pinch-off behavior as well as straightforward calculation for the interface normal vector and the radius of curvature. However, mass conservation due to numerical diffusion is a problem that plagues this approach. Reinitialization of the signed distance function is typically necessary for the level set to retain its signed distance property and to limit mass loss. Reinitialization procedures are also prone to numerical diffusion and without careful implementation have the tendency to move the zero level set interface, which is not desired. Another downside of the level set method is that it is limited by the grid size - finer features of the interface or regions of high curvature cannot be resolved if they are thinner than the local grid width.

Lagrangian particle methods conserve mass by nature and are excellent at resolving fine scales and curvatures of the interface in flow regimes that do not cause major deformation or stretching of the interface. The downside is that a large number of points are needed to create the interface and a special approach must be in place to back out the surface geometry (e.g. the surface normal and curvature) since there is no connectivity between particles. These methods fail the shrinking square test [5] and cases with merging fronts, but this is due to how the velocity gets interpolated from a background mesh [2]. Reseeding is also necessary as the interface gets stretched, since the particles can get spread out and fine scale resolution gets lost. A self organizing particle method [6] has been developed, where particles move to adapt to local resolution requirements. As holes and particle clustering form, particles get essentially remeshed using pseudo-forces and dynamic insertion and removal. In addition, it is worth noting that topological changes must be specially handled in Lagrangian particle methods, again since there is no measure of connectivity between particles. A Lagrangian particle level set method was developed by Hieber *et al.* [4] using techniques from vortex methods and particles as essentially quadrature points. This paper develops an approach to cutting and reconnecting the interface.

The hybrid particle level set method was developed by Enright *et al.* [1]. Lagrangian particles are placed near the interface and are used to correct the level set function for mass loss (in addition to a traditional reinitialization approach) when "escaped" particles are detected. Adjacent to both sides of the interface defined by the level set equation, massless marker particles of randomly varying size are initially placed. They are given a sign (positive or negative) and move with same velocity field used for advection of the signed distance function. When these particles end up on the wrong side of the interface due to numerical error, the particles are used to correct the signed distance field using the radius of the marker particle as a measure of the local level set. In this method, a 5th order WENO scheme for the computation of the spatial term $\nabla\phi$ is combined with a 3rd order TVD Runge Kutta procedure for time integration [3]. In a following paper [2], they show that this correction procedure makes high order integration schemes for the level set function unnecessary. Instead, a semi-Lagrangian method [8] coupled with a first order fast marching method [7] for reinitialization is used as a faster alternative (and the resulting numerical diffusion is effectively mitigated with the incorporation of the particle correction procedure.

Most particle-level set hybrid methods methods use a large number of particles to preform their calculations (64 per cell in 2D in [1]), and most of these particles do not even contribute to the correction procedure since only the escaped particles contribute to updating the level set function. In this work we suggest a different approach, using all particles adjacent to the level set and within one grid spacing, and are able to get a smooth and accurate method with only 12 particles per cell close to the interface. We are able to accomplish this by instead using an interpolation scheme to update grid points near the interface using the distances of nearby particles (escaped or not). Using this approach we do not have to check the escaped status of a particle or calculate the projection of the distance between particle and grid point to see if it is normal or tangent to the interface. In our approach, the radius of an interface particle is the signed distance from the zero level set and we use bilinear interpolation at each grid point to up date the "coarse" grid level set function with the information from the "finer" set of particles near the interface. Our Lagrangian particles do not get reinitialized since they reside near the zero level set (which, within Δx, remains fixed during a reininitialization event).

Results

In order to compare methods, we test this method against the original particle level set method [1]. We look at a pseudo one dimensional test case in which the particles and the level set defined on the (2D) grid were given a linear profile ($\phi(\mathbf{x}) = x - 0.5$). Then the level set field was given a constant error by shifting it by $\Delta x/2$, so the particles and the level set field differ by $\Delta x/2$. We assume that the particles are "correct" and that there is error in the level set field, and use each of the three methods to attempt to correct the zero level set. The results are shown in Figures 1 and 2. Other particle correction methods require a large number of particles per cell since only the escaped particles get used to update the level set. This new approach alternatively uses all neighboring interfacial particle information.

References

[1] Enright, D., Fedkiw, R., Ferziger J., Mitchell, I. (2002) A hybrid particle level set method for improved interface capturing Journal of Computational Physics **183**, 83–116.

[2] Enright, D., Losasso, F. and Fedkiw, R. (2005) A fast and accurate semi-Lagrangian particle level set method Computers & structures **83**(6), 479–490.

[3] Jiang, G. S. and Peng, D. (2000) Weighted ENO schemes for Hamilton–Jacobi equations SIAM Journal on Scientific Computing **21**(6), 2126–2143.

[4] Hieber, S. E. and Koumoutsakos, P. (2005) A Lagrangian particle level set method **210**, 342–367.

[5] Osher, S. and Fedkiw, R. (2007) Level Set Methods and Dynamic Implicit Surfaces

[6] Reboux, Sylvain and Schrader, Birte and Sbalzarini, Ivo F (2012) Journal of Computational Physics **231**(9), 3623–3646.

[7] Sethian, J. A. (1996) A fast marching level set method for monotonically advancing fronts Proceedings of the National Academy of Sciences **93**(4), 1591–1595.

[8] Strain, J. (1999) Semi-Lagrangian methods for level set equations Journal of Computational Physics **151**(2), 498–533.

[9] Wang, Z., Yang, J. and Stern, F. (2009) An improved particle correction procedure for the particle level set method, Journal of Computational Physics **228**(16), 5819–5837.

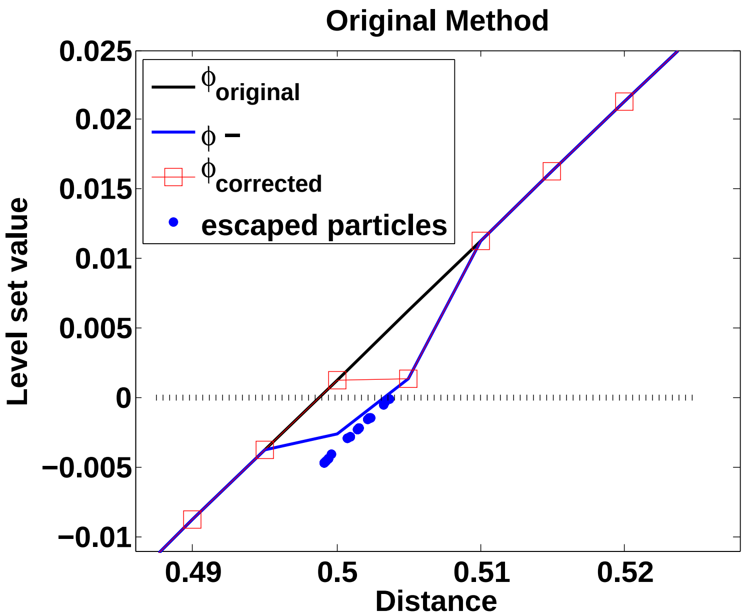

Figure 1: **Illustrating particle correction for a psuedo 1D problem in which the level set field is off by $\Delta x/2$ and there is no error in the particle positions and radii. In the original method [1] the location of the zero level set does not get updated.**

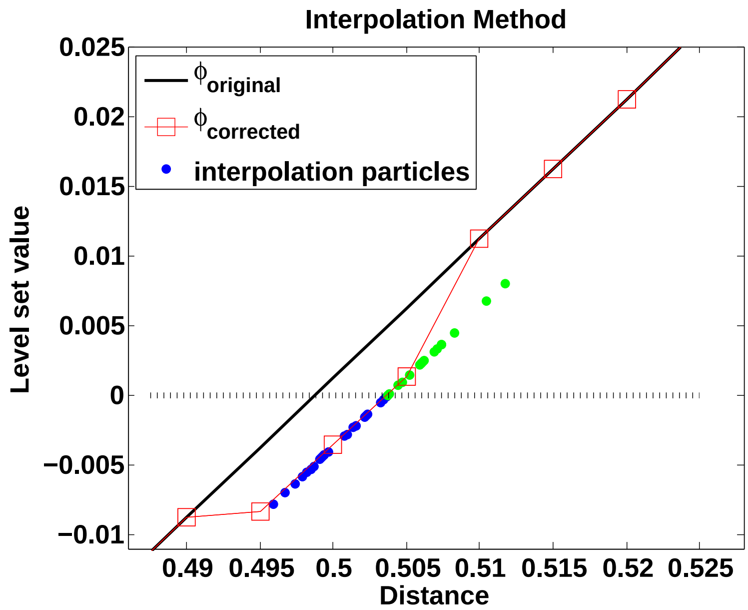

Figure 2: **Illustrating particle correction for a psuedo 1D problem in which the level set field is off by $\Delta x/2$ and there is no error in the particle positions and radii for the new interpolation method proposed in this paper.**