Consistency-driven Pairwise Comparisons Approach to Abandoned Mines Hazard Rating

Waldemar Koczkodaj[1] and Michael Soltys[2]

[1]Department of Mathematics and Computer Science, Laurentian University, Sudbury, Ontario, Canada P3E 2C6
[2]Department of Department of Computer Science, California State University at Channel Islands, One University Drive, Camarillo, CA 93012, USA
*Presenting author: michael.soltys@csuci.edu
†Corresponding author: wkoczkodaj@cs.laurentian.ca

Abstract

The pairwise comparisons method, together with inconsistency analysis, are used to assess the hazard level for abandoned mines. Weights, reflecting the relative importance of the objectives concerned are one of the most commonly used solutions for this type of data. Subjective assessments involve inaccuracy (which is difficult to manage) and inconsistency in assessments (which can be measured and may influence the accuracy). The pairwise comparisons method allows us to define a consistency measure and use it as a validation technique. A consistency-driven knowledge acquisition, supported by a properly designed software, contributes to the improvement of quality of knowledge-based systems.

Keywords: pairwise comparison, knowledge management, multicriteria evaluation, inconsistency, hazard rating.

1 Introduction

The first (somewhat documented but never formally published) use of pairwise comparisons (PC) is attributed to Ramon Llull, a 13th-century mystic and philosopher (see [5]). Thurstone applied pairwise comparisons in the form of "the law of comparative judgment" in [18]. There is a variation of this law known as the BTL (Bradley-Terry-Luce) model (cf. [2]). A number of customized methods of pairwise comparisons followed in numerous (some of them controversial) studies. We do not intend to endorse any such customization here. However, Saaty's seminal work [17] had a considerable impact on the pairwise comparisons (PC) research and should be acknowledged despite serious controversies generated by it.

The technical issues of acquiring this knowledge, representing it, and using it appropriately to construct and explain lines-of-reasoning, are important problems in the design of knowledge-based systems. Knowledge acquisition involves extracting knowledge from human experts, books, documents, sensors, or computer files. In the knowledge validation stage this knowledge is validated and verified until its quality is considered acceptable according to some pre-established standards.

Knowledge acquisition is the extraction of knowledge from sources of expertise and its transfer to the knowledge base. Acquisition is actually done throughout the entire expert system development process. Knowledge is a collection of specialized facts, procedures, and assessment rules and may be collected from many sources. These sources can be divided into two types: documented and undocumented. The latter resides in people's minds. Knowledge can be identified and collected by using any of the human senses. It can also be identified and collected by machines.

The knowledge engineer elicits knowledge from the expert, refines it with the expert, and represents it in the knowledge base. The elicitation of knowledge from the expert can be done manually or with the aid of computers. The main purpose of computerized support to the expert is to reduce or eliminate the potential problems mentioned earlier, especially those of indeterminate bias and ambiguity. These problems dominate the gathering of information for the initial knowledge base and the interactive refinements of this knowledge. A smart knowledge acquisition tool needs to be able to add knowledge incrementally to the knowledge base and refine, or even correct, existing knowledge. Visual modeling techniques are very important in constructing the initial domain model. The objective of the visual modeling approach is to give the user the ability to visualize real-world problems and to manipulate elements of it through the use of graphics.

The expert's knowledge may be, for example, expressed in assessing the number of preferences, relevant criteria or factors, or possible alternatives. When devising methods for formulating and assessing preferences, a knowledge engineer has to take into account the limitations in human capabilities for undertaking such endeavor. One possible technique of extracting the expert's knowledge and preferences is based on the pairwise comparisons method.

2 Pairwise Comparisons Preliminaries

The pairwise comparisons method utilizes statements about expert's preferences and assessments. These statements are expressed by examination of pairs of criteria or objectives. The presented methodology utilizes mapping of inconsistent evaluations by an expert into a numerical scale (see Table 1) that closely approximate his/her assessments. Ordinal numbers are used to express relative preferences. In particular the numbers do not represent "absolute" measure of the mapped criteria, as such may simply not exist (for example, it is hard to define a global measure of public safety but it is still practical to compare it, in relative terms, with the degree of environmental pollution).

Intensity	definition	explanation
1	equal importance	equal contribution
2	weak importance of one over another	slightly favor one criterion over another
3	essential or strong importance	strongly favor one criterion over another
4	demonstrated importance	strong dominance
5	absolute importance	the highest preference
1.2, 2.3, ...,etc.	Intermediate values	when compromise is needed

Table 1: Scale used for pairwise comparisons

The traditional matrix representation of pairwise comparisons (PC) is by using a PC matrix M of the following format:

$$M = \begin{bmatrix} 1 & m_{1,2} & \cdots & m_{1,n} \\ \frac{1}{m_{1,2}} & 1 & \cdots & m_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{m_{1,n}} & \frac{1}{m_{2,n}} & \cdots & 1 \end{bmatrix}.$$

PC matrix elements represent the intensities of an expert's preference between individual pairs of entities (or criteria) expressed as ratios chosen from an assumed scale for subjective data and transformed by the recently published formula in [10]. Note the criteria E_1, E_2,..., E_n (where n is the number of criteria to be compared). The entry m_{ij} in the i-th row and j-th column of the PC matrix M, denotes the relative importance of entity (or criterion) E_i compared with objective E_j, as expressed by an expert. This PC matrix M has all positive elements and has the following reciprocal property:

$$\forall i, j, \quad 1 \leq i, j \leq n, \quad m_{ij} = \frac{1}{m_{ji}}.$$

The PC matrix M is called consistent if $\forall i, j, k, 1 \leq i < j < k \leq n$, it is the case that $m_{ij} * m_{jk} = m_{ik}$. The vectors consisting of the three values $[m_{ij}, m_{ik}, m_{jk}]$ are called "triads." By the reciprocity condition, triads have a mirror image below the diagonal, and so it is sufficient to concentrate on the values above the diagonal.

Let w_i denote the unknown weight of the criterion i. How can the vector $w = [w_1, w_2, ..., w_n]$ be estimated on the basis of the PC matrix M? One possible solution can be the following. If the expert's assessments are completely consistent, one would have:

$$\forall i, j, \quad 1 \leq i, j \leq n, \quad a_{ij} = \frac{w_i}{w_j}.$$

The following heuristic:

$$w_i = (\prod_{j=1}^{n} m_{ij})^{1/n}$$

was proposed in [19] for finding vector w for inconsistent PC matrices. In fact it trivially works also for consistent PC matrices.

A definition of consistency proposed in [9] allows us to locate the most inconsistent assessments and reexamine them. New and more consistent assessments may be expressed in an interactive way. They may contribute to the overall reduction of the inconsistency.

3 Abandoned mines hazard rating

The knowledge engineer usually has to cope with a large number of criteria, factors or alternatives during the data acquisition process. Our model is presented visually[1] in Fig. 1, and is used by a tool called "Concluder."[2]

The model was the result of a team effort involving mining experts from the Ministry of Northern Ontario and Mines, with expertise based on years of experience. One episode that was in everyone's mind was the collapse of a school yard (fortunately, at a time when the children were attending classes in the school building). The yard caved in as it was built on a forgotten abandoned mine. Based on the expertise of the mining professionals, and data from historical reports, pairwise comparisons were gathered into a large matrix. Needless to say, with such a large number of experts and data, the matrix that was created was inconsistent.

4 Inconsistency in pairwise comparisons

For a single triad $[x, y, z]$, the inconsistency indicator is given by the following formula:

[1]The graphic has been produced with Prefuse, a set of software tools for creating rich interactive data visualizations [8].

[2]Which we make available on Sourceforge [3].

$$ii = 1 - \min\left(\frac{y}{x * z}, \frac{x * z}{y}\right).$$

The new definition was proposed in [9], formally generalized to the entire matrix by the use of the $\max$ function for all triads (defined by the consistency condition), and simplified in [14]. Making comparative assessments of intangible criteria (e.g., the degree of an environmental hazard or pollution factors) involves not only imprecise or inexact knowledge but also inconsistency in our own assessments. The improvement of knowledge elicitation by controlling the inconsistency of experts' assessments is not only desirable but absolutely necessary.

Checking the consistency in the pairwise comparisons method could be compared to checking that the divisor is not equal to 0. It does not make sense to divide anything by 0. The proposed solution of the pairwise comparisons method is based on the assumption that the given reciprocal matrix is consistent. However, expecting that all subjective assessments are consistent is not realistic especially if they are subjective. We know that most assessments are subjective, inaccurate, and nearly always contain some kind of bias, and therefore the total consistency is not to be expected.

To have inconsistent assessments we must have at least three criteria to be compared. Consequently we may assume, that all indexes i, j, k must be pairwise different. We may calculate inconsistencies only for triads with indexes holding the property $1 \leq i < j < k \leq n$.

The inconsistency indicator of a PC matrix is the indicator of the quality of the knowledge. The "improvement" process of the quality of the knowledge begins with computing the inconsistency of the assessments. The triad with the largest inconsistency is displayed for the experts to have an opportunity to revise their preferences.

In our case, Concluder highlights the worst triad as illustrated by Fig. 2.

The inconsistency of 0.44 is regarded as too high (the threshold value is assumed 1/3 for similar applications) so experts need to reconsider their assessments. By changing 1.5 in the highlighted triad into 1.3, we can decrease inconsistency indicator to 0.32 which is assumed to be acceptable so weights w (automatically computed and illustrated by Fig. 3) can be used for decision making.

5 Conclusions

The consistency-driven approach presented in this paper was tested in a research project related to the decision process of rehabilitation of abandoned mines in Ontario by the Provincial Ministry of Northern Development and Mines. The implemented system assists middle-level management in making semistructured decisions. The main goal of the system is to provide management with the most comprehensive and most updated information necessary to make responsible decisions (for details see [1]).

The consistency-driven pairwise comparisons refocused the attention from the race of finding better and better approximation of weights for inconsistent matrices to devising heuristics to influencing assessments to be more consistent (but by no means totally consistent). Finding an ideal vector of weights for inconsistent (or very inconsistent) matrices is a mirage. It is a theoretically challenging and exciting task but does not have much practicality. It could be compared to an attempt at finding lengths of objects using a ruler which randomly changes (by, for example, extreme temperature) its length for each of them. The truth is that no "ideal" solution exists and understanding the true source of our problem, that is inconsistency of assessments, is absolutely necessary for decreasing the inaccuracy.

Reducing the inconsistency is not easy unless we know its location (not only its value). The presented definition of inconsistency locates it. The expert is given the feedback and opportunity of reconsideration of his/her assessments by using various approaches (e.g., Delphi method). It may not be advisable to allow the expert the full flexibility since his/her subjective assessment may change due to an unsubstantiated race for consistency of assessments instead of non-biased subjective opinions. We may, for example, allow the referee to change only a fixed number of opinions by a factor of a fixed total. For example, in case of a matrix of order 4 when we have 6 assessments we may allow to modify a maximum of three modifications on condition that the total of all changes does not exceed say 3 (so three assessments may be modified by one up or down, or one assessment may be modified by 3 up or down).

Acknowledgments

This project has been supported in part by the Euro Research grant "Human Capital." The authors are grateful to Grant O. Duncan (Team Lead, Business Intelligence and Software Integration, Health Sciences North, Sudbury, Ontario) for his help with proofreading this text. The authors also acknowledges involvement of William O. Mackasey (the retired expert of abandoned mines, formerly employed by the Ministry of Northern Development and Mines). Numerous researchers on four continents (Australia, Asia, Europe, and North America) have been extremely supportive through this project and we would like to thank all of them.

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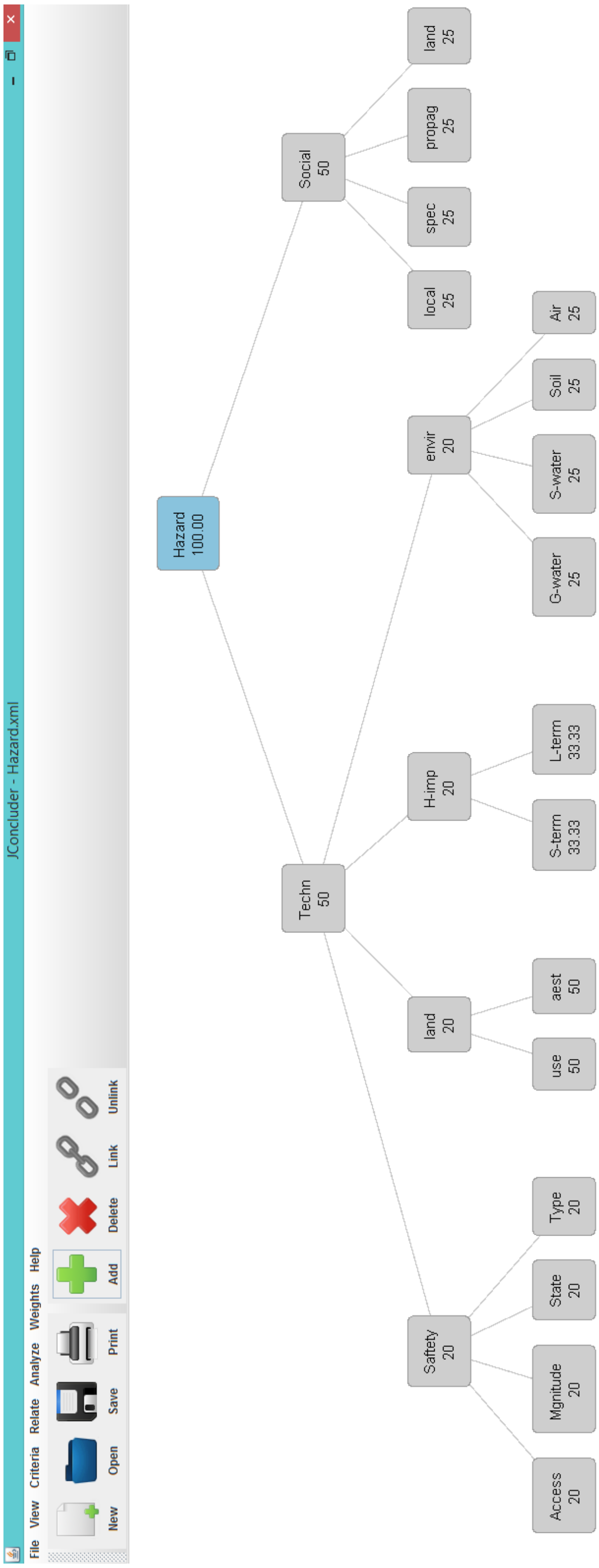

Figure 1: PC model for abandoned mines hazard rating

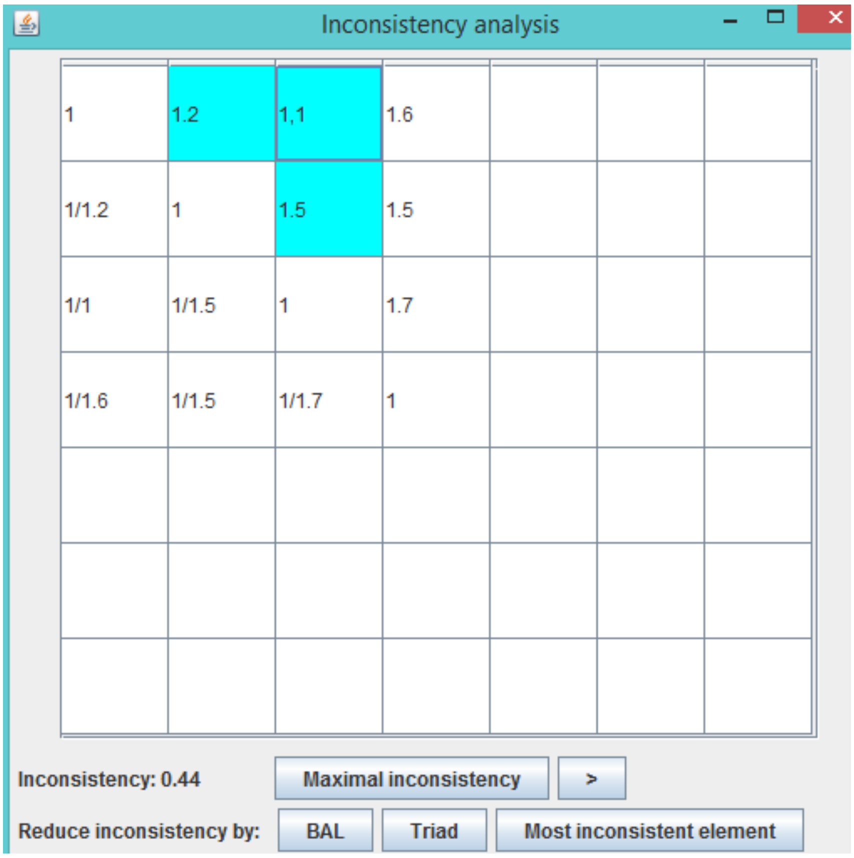

Figure 2: Inconsistency analysis

2.50%	Access
2.50%	State
2.50%	Mgnitude
2.50%	Type
0.83%	S-term
0.83%	L-term
0.21%	G-water
0.21%	S-water
0.21%	Soil
0.21%	Air
5.00%	use
5.00%	aest
12.50%	local
12.50%	spec
12.50%	propag
12.50%	land

Figure 3: The final weights