

Faster and Splitting-free Vorticity Redistribution

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Abstract

Vortex particle methods have proven to be an excellent tool for the simulation of flows at high Reynolds numbers. Due to their natural treatment of convection, they are essentially free of numerical diffusion and—especially in two dimensions—possess excellent conservation properties [1]. The treatment of *physical* diffusion in vortex methods, however, has long been a subject of debate and by now many different solution approaches have been proposed.

Most available approaches make use of the so-called viscous splitting, in which convection and diffusion are treated separately, one after another. While the viscous splitting facilitates the computer implementation, it effectively limits the method’s accuracy in time to first order. Additionally, most available methods require a more or less frequent ‘remeshing’ of the particles in order to maintain their accuracy, thereby effectively making the method mesh-based again.

Among the few truly mesh-free approaches is the Vorticity Redistribution Method (VRM) by Shankar and van Dommelen [2]. Like the other truly mesh-free approaches, it has often been claimed to be very slow and also makes use of the viscous-splitting. In this work we present an extension to the VRM [3] which avoids the viscous splitting, thereby enabling the use of higher order time-stepping schemes. By only considering a subset of the particles for diffusion, we prevent excessive growth in the total number of particles, while maintaining the spatial consistency order. Stability when combined with Euler’s method is proven. In combination with an efficient implementation of the Simplex method for small-scale problems and a new heuristic for particle insertion, the method’s computational cost could be significantly reduced. When applied to the Lamb–Oseen vortex the method exhibited second order convergence behaviour (Fig. 1).

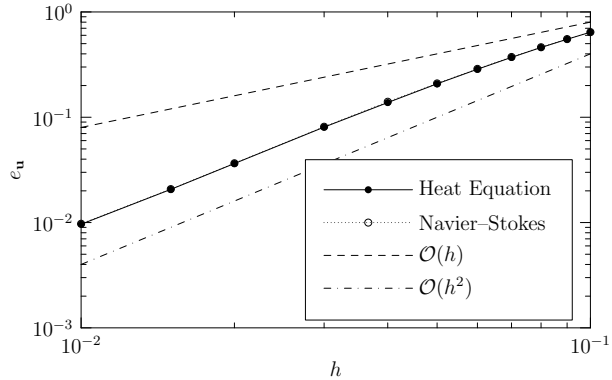


Figure 1: Velocity error vs h .

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Keywords: Mesh-free Methods, Vortex Particle Methods, Vortex Diffusion Scheme, Finite Differences

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