Optimization of stiffened composite plate using adjusted different evolution algorithm

†Thuan Lam-Phat[1,*], Son Nguyen-Hoai[1], Vinh Ho-Huu[2], Trung Nguyen-Thoi[2]

[1] GACES, HCMC University of Technology and Education, Vietnam
[2] Division of Computational Mathematics and Engineering (CME), Institute of Computational Science (INCOS), Ton Duc Thang University, Hochiminh City, Viet Nam

*Presenting author: lamphatthuan@gmail.com
†Corresponding author(*): *lamphatthuan@gmail.com*

Abstract

Stiffened composite plate has been widely used in many braches of engineering area and the demand of optimizing the cost of manufacturing is also very high. One of many approaches to minimize the cost is to optimize the weight of the structure. In this paper, an improved version of the Differential Evolution (DE) algorithm is adopted to solve for suitable values of the fiber angle and the thickness of the stiffened composite plate to achieve the structure with minimum weight. For computing the constrained conditions of stress and strain in the optimization process, the finite element analysis using the CS-DSG3 element is used. To verify the accuracy and the effectiveness of the algorithm, the numerical solutions obtained from the proposed method are compared with those of other available approaches.

Keywords: *Stiffened composite plate, Differential Evolution (DE), Cell-based smoothed discrete shear gap method (CS-DSG3), Optimization analysis.*

1. Introduction

Nowadays, stiffened composite plates have been widely used in many branches of structural engineering such as aircraft, ships, bridges, buildings, etc. For its advantages in both bending stiffness and the amount of material in comparison with common bending plate structures, stiffened composite plate usually has higher economic efficiency in practical applications. However, choosing the best design that satisfies the working requirement is difficult. In addition, the complex mechanical behavior of composite materials also increases the difficulty of the problems related to their design [1]. In this case, the design optimization tools combined with numerical methods must be utilized. Design optimization is one of the most interesting research directions that brings a lots of profits in both life and industry. And so, methods for design optimization are also quickly developed. The optimization methods can be classified into two main groups: gradient-based and popular-based approach. Methods based on gradient information is fast but usually stuck in local solution and depended too much on a good initial point to obtain global optimal solution. T. Nguyen-Thoi et al [2] used SQP to find the optimal fiber orientations for stiffened composite plate, but the results still depend on the initial point to get the exact solution. To deal with such disadvantages, population-based global optimization methods are utilized alternatively. Marin et al. [3] used the genetic algorithm, including the application of elitism, which preserved the use of the Pareto front to optimize the design of a composite material-stiffened panel. Falzon and Faggiani [4] applied the genetic algorithm to improve the post-buckling strength of stiffened composite panels. And among many proposed global optimization algorithms, Differential Evolution (DE) firstly introduced by Storn and Price in 1997 [5] was one of the most potential algorithms. The DE has demonstrated excellently performance in solving many different engineering problems. Wang et al. [6] applied the DE for designing optimal truss structures with continuous and discrete variables. Wu and Tseng [7] applied a multi-population differential evolution with a penalty-based, self-adaptive strategy to solve the COP of the truss structures. Le-Anh et al. [8] using an adjusted Differential Evolution algorithm and a smoothed triangular plate element for static and frequency optimization of folded laminated composite plates. Ho-Huu et al. [9] proposed a new version of the DE to optimize the shape and size of truss with discrete variables. However, using the method in finding the global optimum solution still gets highly computational cost. Hence, many approaches have been proposed to increase the effectiveness of the algorithm. Most recently, Ho-Huu et al. [10] also introduced two new improvement steps to increase the convergence of DE algorithm based on roulette wheel selection (ReDE). The new modified DE algorithm is applied for solving shape-and-size optimization problem of truss structure with frequency constraints and show its high effectiveness.

In this paper, this new improved version of the Differential Evolution (ReDE) algorithm is adopted to solve for suitable values of the fiber angle and the thickness of the stiffened composite plate to achieve the structure with minimum weight. For computing the constrained conditions of stress and train in the optimization process, the finite element analysis using the cell-based smoothed discrete shear gap technique with triangular elements (CS-DSG3) proposed by T. Nguyen-Thoi et al [11,12] is used. The numerical solutions obtained from the method are compared with references to show the effectiveness and the accuracy of the algorithm.

2. Theory Fundamental

An optimization problem can be expressed as follows:

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.} \quad \begin{cases} h_i(\mathbf{x}) = 0 & i = 1,\ldots,l \\ g_j(\mathbf{x}) \le 0 & j = 1,\ldots,m \end{cases} \tag{1}$$

where $\mathbf{x}$ is the vector of design variables; $h_i(\mathbf{x}) = 0$ and $g_j(\mathbf{x}) \le 0$ are inequality and equality constraints; *l, m* are the number of inequality and equality constraints, respectively; $f(\mathbf{x})$ is the objective function which can be the function of weight, cost, etc.

Design optimization of a structure is to find optimal values of design variables in design space such that the objective function is minimum [2]. Dealing with such problems, many optimization methods are used including gradient-based and population-based approach to find the solution. In this paper, the Differential Evolution is utilized to solve the problem of finding optimal fiber orientations and thickness of the stiffened composite plate.

2.1 Brief on the differential evolution algorithm [10,9]

The original differential evolution algorithm firstly proposed by Storn and Price [5] has been widely used to solve many kinds of optimization problems. The scheme of this algorithm consists of four main phases as follows:

Phase 1: Initialization
 Create an initial population by randomly sampling from the search space
Phase 2: Mutation
 Generate a new mutant vector v_i from each current individual x_i based on mutation operations.
Phase 3: Crossover
 Create a trial vector u_i by replacing some elements of the mutant vector v_i via crossover operation.
Phase 4: Selection
 Compare the trial vector u_i with the target vector x_i. One with lower objective function value will survive in the next generation

To improve the effectiveness of the algorithm, the *Mutation phase* and the *Selection phase* are modified to increase the convergence rate as follow:

In the *mutation phase,* parent vectors are chosen randomly from the current population. This may make the DE be slow at exploitation of the solution. Therefore, the individuals participating in mutation should be chosen following a priority based on their fitness. By doing this, good information of parents in offspring will be stored for later use, and hence will help to increase the convergence speed. To store good information in offspring populations, the individuals is chosen based on Roulette wheel selection via acceptant stochastic proposed by Lipowski and Lipowska [13] instead of the random selection.

In the *selection phase*, the elitist operator introduced by Padhye et al. [14] is used for the selection progress instead of basic selection as in the conventional DE. In the elitist process, the children population C consisting of trial vectors is combined with parent population P of target vectors to create a combined population Q. Then, best individuals are chosen from the combined population Q to construct the population for the next generation. By doing so, the best individuals of the whole population are always saved for the next generation.
The modified algorithm Roulette-wheel-Elitist Differential Evolution is then expressed as below:

```
Generate the initial population
Evaluate the fitness for each individual in the population
while <the stop criterion is not met> do
    Calculate the selection probability for each individual
    for i =1 to NP do {NP: Size of population}
        Do mutation phase based on Roulette wheel selection
        j_rand = randi(1,D) {D: number of design variables}
        for j =1 to D do
            if rand[0,1] < CR or j == j_rand then {CR: crossover control parameter}
                    u_i,j = x_r1,j + Fx(x_r2,j - x_r3,j) {F:randomly chosen within [0,1] interval}
            else
                    u_i,j = x_i,j
            end if
        end for
        Evaluate the trial vector u_i
end for
Do selection phase based on Elitist selection operator
end while
```

2.2 Brief on the behavior equation of stiffened composite plate [2]

Stiffened composite plate can be seen as the combination between composite plate elements and the stiffening Timoshenko composite beam elements, as illustrated in Figure 1. The stiffening composite beam is set parallel with the axes in the surface of plate and the centroid of beam has a distance *e* from the middle plane of the plate. The plate-beam system is discretized by a set of node. The degree of freedom (DOF) of each node of the plate is $\mathbf{d}=[u,v,w,\beta_x,\beta_y]^T$, in which u,v,w are the displacements at the middle of the plate and β_x,β_y are the rotations around the *y*-axis and *x*-axis. The DOF of each node of the beam is $\mathbf{d}_{st}=[u_r,u_s,u_z,\beta_r,\beta_s]^T$. The centroid displacements of beam are expressed as

$$u=u_r(r)+z\beta_r(r) \quad ; \quad v=z\beta_s(r) \quad ; \quad w=u_z(r) \tag{2}$$

where u_r,u_s,u_z are respectively centroid displacements of beam and β_r,β_s are the rotations of beam around *r*-axis and *s*-axis.

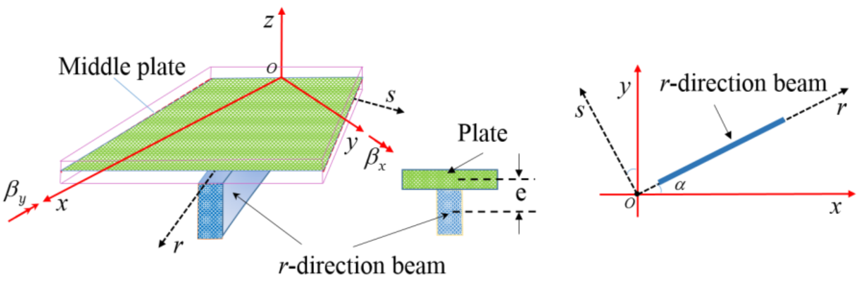

Figure 1. A plate composite stiffened by an *r*-direction stiffener

* Energy equation of stiffened composite plates

The strain energy of composite plate is given by

$$U_P = \frac{1}{2}\iint_A \left(\boldsymbol{\varepsilon}_0^T \mathbf{D}^m \boldsymbol{\varepsilon}_0 + \boldsymbol{\varepsilon}_0^T \mathbf{D}^{mb} \boldsymbol{\kappa}_b + \boldsymbol{\kappa}_b^T \mathbf{D}^{mb} \boldsymbol{\varepsilon}_0 + \boldsymbol{\kappa}_b^T \mathbf{D}^b \boldsymbol{\kappa}_b + \boldsymbol{\gamma}^T \mathbf{D}^s \boldsymbol{\gamma}\right) \mathrm{d}A \tag{3}$$

where $\boldsymbol{\varepsilon}_0, \boldsymbol{\kappa}_b, \boldsymbol{\gamma}$ are respectively membrane, bending and shear strains of composite plate and are expressed as follows

$$\boldsymbol{\varepsilon}_0 = [u_{,x}, v_{,y}, u_{,y} + v_{,x}]^T ; \boldsymbol{\kappa}_b = [\beta_{x,x}, \beta_{y,y}, \beta_{x,y} + \beta_{y,x}]^T ; \boldsymbol{\gamma} = [w_{,x} + \beta_x, w_{,y} + \beta_y]^T . \tag{4}$$

$\mathbf{D}^m, \mathbf{D}^{mb}, \mathbf{D}^b, \mathbf{D}^s$ are material matrices of plate

The strain energy of composite stiffener is given by

$$U_{st} = \tfrac{1}{2}\int_l \left((\boldsymbol{\varepsilon}_{st}^b)^T \mathbf{D}_{st}^b \boldsymbol{\varepsilon}_{st}^b + (\boldsymbol{\varepsilon}_{st}^s)^T \mathbf{D}_{st}^s \boldsymbol{\varepsilon}_{st}^s\right) \mathrm{d}x \tag{5}$$

where $\boldsymbol{\varepsilon}_{st}^b, \boldsymbol{\varepsilon}_{st}^s$ are respectively bending, shear strain of beam and are expressed as follows

$$\boldsymbol{\varepsilon}_{st}^b = [u_{r,r} + z_0 \beta_{r,r}, \beta_{r,r}, \beta_{s,r}]^T ; \boldsymbol{\varepsilon}_{st}^s = [u_{z,r} + \beta_r]^T \tag{6}$$

$\mathbf{D}_{st}^b, \mathbf{D}_{st}^s$ are material matrices of composite beam

Using the superposition principle, total energy strain of stiffened composite plate is obtained by

$$U = U_P + \sum_{i=1}^{N_{si}} U_{st} \tag{7}$$

where N_{st} is the number of stiffeners.

For static analysis, the global equations for the stiffened composite plate $[\mathbf{K}]\{\Delta\} = \{\mathbf{F}\}$ can found in [16] for detail.

3. Numerical Results

3.1 Unconstrained problem for fiber angle optimization

Consider an optimization analysis of a composite plate stiffened by a composite beam according to *x*-direction as in Figure 2 under simply-supported condition. The parameters of the problem are given by a = 254 mm, h = 12.7 mm, c_x = 6.35 mm and d_x = 25.4 mm. The analysis is carried out with two cases of square (b = 254 mm) and rectangular (b = 508 mm) plate.

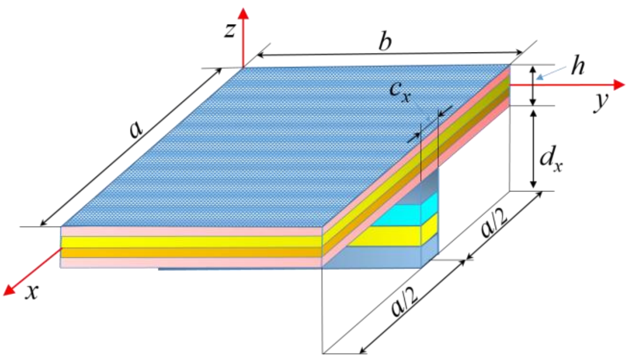

Figure 2. Model of a stiffened composite plate

Both plate and beam have four symmetric layers. The fiber orientation for layers of the plate is a set [θ_1 θ_2 θ_2 θ_1], and for the layers of the beam is [θ_3 θ_4 θ_4 θ_3]. The plate and beam are made by the same materials with $E_1 = 144.8\text{GPa}$, $E_2 = E_3 = 9.65\text{GPa}$, $G_{12} = G_{13} = 4.14\text{GPa}$, $G_{23} = 3.45\text{GPa}$, $\upsilon_{12} = \upsilon_{13} = \upsilon_{23} = 0.3$. The plate is subject to a uniform load f = 0.6895 (N/mm^2).

The optimization problem is now expressed as:

$$\begin{cases} \min\limits_{\boldsymbol{\theta}} & \mathbf{U} = \dfrac{1}{2}\mathbf{d}^T\mathbf{K}\mathbf{d} \\ \text{subject to} & 0 \le \theta_i \le 180, \quad i = 1,\ldots,4 \end{cases}$$

where U is strain energy and θ_i is fiber orientation of *i*th layer.

Firstly, static analysis for the case of square plate is carried out to verify the reliability of the finite element solution using CS-DSG3 [15]. The results compared with those by Li Li [17] and M. Kolli [16] are presented in Table 1 and show good agreement.

Table 1. Comparison of central deflection (mm) of the simply-supported square stiffened composite plates subjected to a uniform load f = 0.6895 N/mm^2

Orientation angle for both beam and plate	$[0^0/90^0/90^0/0^0]$			$[45^0/-45^0/-45^0/45^0]$	
Method	CS-DSG3	[16]	[17]	CS-DSG3	[11]
Central deflection	1.0917	1.0396	1.0892	2.5049	2.4912

In Table 2, a comparison of different types of DE algorithm for the case of rectangular plate is presented. The first two versions are the original different evolution (DE) and the adjusted one (ReDE). Both of them are used with continuous variables. We can see that, the difference of computational cost between the two versions is rather big. The cost from DE is nearly double in comparison with that of ReDE algorithm. The third version is ReDE algorithm with integer variables. And the result obtained from this type just equals 43% of that of ReDE with continuous variables. However, the values of the solution are still nearly the same. Therefore, in this paper, the ReDE with integer variables is utilized for the optimization process for saving the cost.

Table 2. Comparison of different types of DE

Type of stiffened plate	Method	Optimal angle [Degree]				Strain energy (N.m)	Computational cost (seconds)
		θ_1	θ_2	θ_3	θ_4		
Rectangular	DE	159.2	37	0	179.9	30300	11223
(a = 254 mm,	ReDE	159.2	37	0	179.9	30300	6787
b = 508 mm)	Int_ReDE	160	37	0	180	30366	2851

Next, the optimization analysis for two cases of square plate and rectangular plate is carried out. The results of fiber orientations obtained from ReDE are presented in Table 3. In this analysis, the value of the design variables is chosen to be integer for saving the time of computing. The results from the Table 3 show that the solutions by the DE agree very well with those by the GA. However, the computational cost for the case of square plate with the mesh size of 20x20 is less than 188 seconds. And in the case of rectangular plate with the mesh size of 20x40, the cost from GA is nearly double in comparison with the one from DE. This shows a big difference and proves the effectiveness of the proposed method.

It is also seen that the optimal fiber orientations of the square plate problem are quite different from those of the rectangular plate case under the same conditions. This implies that the geometric parameters of the structures also have influence to the results of the optimization problems.

Table 3. The optimal results of two problems

Type of stiffened plate	Method	Optimal angle [Degree]				Strain energy (N.m)	Computational cost (seconds)
		θ_1	θ_2	θ_3	θ_4		
Square (a = b = 254 mm)	ReDE	135	48	0	180	6183.2	2065
	GA	135	48	0	180	6183.1	2253
Rectangular (a = 254 mm, b = 508 mm)	ReDE	160	37	0	180	30366	2851
	GA	159	37	0	180	30300	4995

3.2 Constrained problem with thickness optimization

Consider the same composite plate stiffened by a composite beam according to x-direction as in Figure 2 under simply-supported condition. But in this case, the fiber orientations for layers of the plate and the beam are given. The problem here is to find the optimal thickness of the plate (t_p) and the beam (t_b) to minimize the weight of the stiffened composite plate under the constraints of displacement and stress. The analysis is also carried out with two cases of square and rectangular plate. For both cases, the optimal fiber angles found in the above unconstrained problems are used, respectively. In particular, the fiber angles of [*135 48 0 180*] is used for the square plate case and the fiber angles of [*160 37 0 180*] is used for the rectangular plate.

For composite materials, many failure criteria proposed to predict lamina failure. In this paper, the Tsai-Wu index defined below is used to predict the most likely failure point in a layer.

$$S_{tw} = \frac{\sigma_{11}^2}{X_t X_c} + \frac{\sigma_{22}^2}{Y_t Y_c} + \frac{\tau_{12}^2}{S^2} - \frac{\sigma_{11}\sigma_{22}}{\sqrt{X_t X_c Y_t Y_c}} + \left(\frac{1}{X_t} - \frac{1}{X_c}\right)\sigma_{11} + \left(\frac{1}{Y_t} - \frac{1}{Y_c}\right)\sigma_{22} \leq 1 \tag{8}$$

The point with the highest Tsai-Wu index is the point that will most likely fail. And this is considered as the stress constraint in this problem.
The optimization problem is then expressed as

$$\begin{cases} \min\limits_{t_p, t_b} & \text{Weight}(t_p, t_b) \\ \text{subject to} & \text{Displacement is less than 1 mm} \\ & S_{tw} \leq 1 \end{cases}$$

Table 4. The optimal results of two problems

Type of stiffened plate	Method	Optimal thickness		Weight (kg)	Computational cost (seconds)
		t_p	t_b		
Square (a = b = 254 mm)	ReDE	13	83	1.5269	1065
	GA	13	83		3659
Rectangular (a = 254 mm, b = 508 mm)	ReDE	18	20	4.6593	2606
	GA	18	20		7482

The results from the Table 4 show that the solutions by the ReDE agree very well with those by the GA. The objective function is almost the same but the computational costs from GA are about 3 times bigger. This shows that the effectiveness of ReDE in comparison with GA is much better.

It is also seen that the optimal thicknesses of the square plate are quite different from those of the rectangular plate under the same conditions. In the case of square plate, when the thickness of the plate decreases about 27% (from 18 to 13), the thickness of the stiffened beam increases 4 times (from 20 to 83). This implies that the thickness of the stiffened beam has not too much influence to the response of the whole structure as of the thickness of the plate. Therefore, in the problem of weight optimization, we can adjust the thickness of the plate and focus only on optimizing the thickness of the plate for saving the cost.

4. Conclusion

In this paper, the unconstrained and constrained optimization analysis with integer variables for the stiffened composite plate using new modified version of DE is presented. In both problems, the results obtained are agreed well with those of GA. However, the computational cost of ReDE algorithm is much cheaper than the one from GA. The results illustrated the efficiency and the accuracy of the adjusted Differential Evolution in solving the optimization problem of the stiffened composite plate.

References

[1] Shun-Fa, H., Ya-Chu H., & Yuder, C,. (2014) A genetic algorithm for the optimization of fiber angles in composite laminates. *Journal of Mechanical Science and Technology*, Vol. 28 (8), p. 3163-3169

[2] Nguyen-Thoi, T., Ho-Huu, V., Dang-Trung, H., Bui-Xuan, T., Lam-Phat, T. (2013) Optimization analysis of stiffened composite plate by sequential quadratic programming. *Journal of Science and Technology*, Vol. 51(4B), p. 156-165.

[3] Marin, L., Trias, D., Badallo, P., Rus, G., & Mayugo, J. A. (2012) Optimization of composite stiffened panels under mechanical and hygrothermal loads using neural networks and genetic algorithms. *Composite Structures,* Vol. 94, p. 3321-3326.

[4] Falzon, B. G., & Faggiani, A. (2012) The use of a genetic algorithm to improve the postbuckling strength of stiffened composite panels susceptible to secondary instabilities, *Composite Structures,* Vol. 94, p. 883-895.

[5] Storn, R., Price, K. (1997) Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces, *J. Glob. Optim,* p. 341–359. doi:10.1023/A:1008202821328.

[6] Z.W.Z. Wang, H.T.H. Tang, P.L.P. Li. (2009) Optimum Design of Truss Structures Based on Differential Evolution Strategy, 2009 Int. Conf. Inf. Eng. Comput. Sci. 0–4. doi:10.1109/ICIECS.2009.5365996.

[7] C.-Y. Wu, K.-Y. Tseng (2010) Truss structure optimization using adaptive multi-population differential evolution, Struct. Multidiscip. Optim. 42 (2010) 575–590. doi:10.1007/s00158-010-0507-9.

[8] Le-Anh, L., Nguyen-Thoi, T., Ho-Huu, V., Dang-Trung, H., & Bui-Xuan, T. (2015) Static and frequency optimization of folded laminated composite plates using an adjusted Differential Evolution algorithm and a smoothed triangular plate element. *Compos. Struct,* Vol. 127, p. 382–394. doi:10.1016/j.compstruct.2015.02.069.

[9] Ho-Huu, V., Nguyen-Thoi, T., Nguyen-Thoi, M. H., Le-Anh, L. (2015) An improved constrained differential evolution using discrete variables (D-ICDE) for layout optimization of truss structures, *Expert Syst. Appl,* doi:10.1016/j.eswa.2015.04.072.

[10] Ho-Huu, V., Nguyen-Thoi, T., Khac-Truong, T., Le-Anh, L., Nguyen-Thoi, M. H. (2015) A fast efficient differential evolution based on roulette wheel selection for shape and sizing optimization of truss with frequency constraints.

[11] Liu GR, Nguyen Thoi Trung. (2010) *Smoothed Finite Element Methods*. NewYork: CRC Press, Taylor and Francis Group.

[12] Bui-Xuan, T., Nguyen-Thoi, T., Pham-Duc, T., Phung-Van, P., & Ngo-Thanh, P. (2012) An analysis of eccentrically stiffened plates by CS-FEM-DSG3 using triangular elements. *The international conference on advances in computational mechanics*, 629-643.

[13] Lipowski, A., & Lipowska, A. (2012) Roulette-wheel selection via stochastic acceptance, *Physica A*, Vol. 391, p. 2193–2196. doi:10.1016/j.physa.2011.12.004.

[14] Padhye, N., Bhardawaj, P., & Deb, K. (2013) Improving differential evolution through a unified approach, *J. Glob. Optim,* Vol. 55, p.771–799. doi:10.1007/s10898-012-9897-0.

[15] Nguyen-Thoi, T., Phung-Van, P., Nguyen-Xuan, H., Thai-Hoang, C. (2012) A cell-based smoothed discrete shear gap method (CS-DSG3) using triangular elements for static and free vibration analyses of Reissner-Mindlin plates. *International Journal for Numerical Methods in Engineering*, p. 705-741.

[16] Kolli, M., & Chandrashekhara. K. (1996) Finite element analysis of stiffened laminated plates under transverse loading. *Composites Science and Technology 56*, p. 1355-1361.
[17] Li Li, Ren Xiaohui. (2010) Stiffened plate bending analysis in terms of refined triangular laminated plate element. *Composite Structures*, p. 2936-2945.
[18] Javidrad, F., & Nouri, R. (2011) A simulated annealing method for design of laminates with required stiffness properties. *Composite Structures,* Vol. 93, p. 1127-1135.