The Effects of Quality and Shortages on the Economic Production Quantity Model in a Two-Layer Supply Chain

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Abstract

The purpose of this paper is to develop an economic production quantity model (EPQ) in a coordinated supplier-produced supply chain. This collaborative supply chain accounts for the quality of both finished product and raw material used in the production process. It is assumed that the raw material acquired from the supplier contains a percentage of good quality items. These items are detected through a screening process at the beginning of the production period. The quality of the finished items produced is checked continuously throughout the production period. The imperfect quality items are either reworked or rejected. The nature of this production/inventory problem necessitates the consideration of shortages. The mathematical model is formulated and the supply chain is optimized by determining the order quantity that maximizes the collaborative profit function. Numerical examples are provided to illustrate the model and the collaborative and non-collaborative models are compared.

Keywords: Inventory Control, Economic Production Quantity, Supply Chain, Quality, Screening, Rework, Reject.

Introduction

The two classical inventory control techniques known as economic order quantity (EOQ) and economic production quantity (EPQ) models have been widely used among researchers and industries (Bedworth and Bailey [1]) and (Simpson, [2]). The EOQ model aims to optimize the order size by balancing or trading off the ordering and holding costs. The EPQ model seeks to determine the optimal lot size by minimizing the total setup and carrying costs. Despite of their widespread usage and implementation, these models are based on idealistic assumptions that are rarely met in real life situations. In the past few decades, considerable research has been published whereby the underlying assumptions are relaxed so that the EOQ/EPQ models are examined under situations that closely resemble the actual inventories encountered in real life. The modified models account for factors that influence the inventory costs. These factors include deterioration, shortages, probabilistic demand, order quantity and demand dependent costs, inflation, time discounting and credit facilities.

One of the unrealistic assumptions of the classical EOQ/EPQ models is that all items received from the supplier and all items produced by the manufacturer are of a perfect quality. These assumptions initiated a new line of research in the field of inventory management that ensures quality. Porteus [3] studied the relationship between the lot size, process quality, and setup cost. Rosenblatt and Lee [4] examined a production system with defective finished items. Salameh and Jaber [5] introduced an EOQ model where each lot delivered by the supplier contains imperfect items, not necessarily defectives, which can be salvaged at a discounted price. This modeling approach has triggered numerous research papers extending this model. Hayek and Salameh [6] studied an EPQ model where the imperfect quality items are reworked. Chiu [7] considered a production process with random defective rate where the defective items are reworked and unsatisfied demand is backlogged. Ozdemir [8] proposed an

EOQ model with defective items where shortages are backordered. Khan et al. [9] presented an extensive survey of such articles.

A typical supply chain consists of suppliers, producers, distributors and retailers. The inventory problems of partners in a supply chain have been treated separately. Recently, numerous studies have been published in the field of inventory management dealing with the interaction between partners in a supply chain and aiming for the improvement of their joint performance. Khan et al. [9] reviewed articles related to EOQ/EPQ models in supply chains with imperfect quality items.

In a different direction, El-Kassar et al. [10] introduced an EPQ model with imperfect quality items of raw material used in the production process. El-Kassar et al. [11] examined the effect of time value of money on this model. The purpose of this paper is to examine the effects of the interaction between the supplier of raw material and the producer of the finished product on the joint performance of both partners in this supply chain. This is done by developing an EPQ model in a coordinated supplier-produced supply chain. This model accounts for the quality of both finished product and raw material in a collaborative supply chain. The raw material acquired from the supplier contains a percentage of good quality items. At the beginning of the production period, the good quality items are detected through a 100% screening process. Throughout the production period, the quality of the finished product is checked continuously. The imperfect quality items are either reworked or rejected. This model allows for shortages. The unsatisfied demand is assumed to be fully backordered.

The rest of this paper is organized as follows. In section two, the needed assumptions and the used notations are presented and the mathematical model is formulated so that the supply chain is optimized by determining the order quantity that maximizes the collaborative profit function. In section three, a numerical example is provided to illustrate the model and the collaborative and non-collaborative models are compared. In section four, a conclusion and some managerial implications are provided. Also, future research suggestions are stated.

The Mathematical Model

Assumptions:

1. Finished product items produced are checked for quality through a 100% error free screening process conducted throughout the production process.
2. The production rate of perfect quality items is greater than the demand rate.
3. Planned shortages are permitted and fully backordered.
4. Rework process starts at the end of the production process with no setup time.
5. Reworked items are processed at the same production rate.
6. The percentage of good quality items of raw material, the percentage of perfect quality finished items, and the percentage of scrap items are known constants.

Notation:

P	Production rate
D	Demand rate
x	Raw material screening rate
Q	Order size of raw material
S	Maximum shortage per cycle
T	Cycle length
t_1	Time to fulfill the backorder of size S
t_2	Time to build up the maximum inventory of perfect quality finished items
t_3	Rework time
t_4	Time to deplete on-hand inventory after rework
t_5	Time to build up the maximum shortage level of size S
t_s	Raw material screening period

t_p	Production time ($t_p = t_1 + t_2$)
A_p	Producer's fixed ordering cost of raw material
A_s	Supplier's fixed ordering cost of raw material
K	Production fixed setup cost
c_{ms}	Supplier's cost of one unit of raw material
c_{mp}	Producer's cost of one unit of raw material
c_p	Cost of producing one unit of finished product
c_r	Cost of reworking one unit of finished product
c_d	Cost of disposing of one unit of scrap item of the finished product
c_s	Cost per unit shortage per unit time
d_m	Cost of screening one unit of raw material
d_f	Cost of screening one unit of finished product
r	Selling price per unit of finished product
r_d	Discounted selling price per unit of raw material.
h_{ms}	Supplier's holding cost of raw material per unit per unit time
h_{mp}	Producer's holding cost of raw material per unit per unit time
h_p	Holding cost due to production per unit per time
h_r	Holding cost due to rework per unit per time
γ	Percentage of good quality items of raw material
π	Percentage finished product that are of perfect quality
ρ	Percentage of imperfect quality items that are reworked
$1-\rho$	Percentage of imperfect quality items that are scrapped (defective)
λ	Proportion of reworked items used to meet the demand
G_s	Supplier's profit function per unit time
G_p	Producer's profit function per unit time
G_c	Chain's profit function per unit time
N	Number of production cycles per one supplier's inventory cycle

In this coordinated two layer supply chain, the producer orders from the supplier Q units of raw material at the beginning of each production cycle. The raw material acquired contains a percentage γ of imperfect quality items. The γQ units of good quality items are detected through a 100% error free screening process and used in the production of the finished product. At the end of the screening period, the remaining $(1-\gamma)Q$ units of raw material are returned to the supplier who sells the items at a discounted unit price r_d. Since raw material is screened at rate of x units per unit time, the screening period is $t_s = Q/x$. Also, the γQ units of good quality items of raw material are processed into finished product at a rate of P, where $x>P$, so that the production period is $t_p = \gamma Q/P$.

Throughout the production period, the finished product is screened to detect perfect quality items. Since the percentage of perfect quality finished items is π, the number of perfect quality items produced is $\gamma\pi Q$ and the remaining $(1-\pi)\gamma Q$ finished items are of imperfect quality. A percentage ρ of the imperfect quality finished items are can be reworked into perfect quality finished items and the remaining $1-\rho$ are scraped. The number of reworked and scraped items are $\rho(1-\pi)\gamma Q$ and $(1-\rho)(1-\pi)\gamma Q$, respectively.

The $\gamma\pi Q$ perfect quality items are produced at a rate of $P\pi$ units per unit time. Since shortages are allowed, the perfect quality finished items produced at the beginning of the production period will be used to meet the demand, at a rate D, and to fulfill backorders at a rate $P\pi-D > 0$. Assuming that the inventory cycle begins with S units short, the time required to fulfill the backorders is $t_1 = S/(P\pi-D)$. Once all backorders are fulfilled, inventory of perfect quality finished items is accumulated at a rate of $P\pi-D$ until a level of $z_1 = t_2\,(P\pi-D)$, where $t_2 = t_p-t_1$, is reached at the end of the production period.

When regular production stops, the scraped items are disposed of at a unit cost of c_d. The remaining imperfect quality finished are reworked into perfect quality items at the same production rate P. During the rework period, from $t = t_p$ until $t = t_p + t_3 = t_p + \rho(1-\pi)\gamma Q/P$, the perfect quality finished items inventory increases at a rate of $P-D$ until a maximum level of z_2 is reached where $z_2 = z_1 + (P-D)t_3$. This accumulated inventory will be used to meet the demand at a rate D so that the time required to deplete this inventory is $t_4 = z_2/D$. During the remainder of the inventory period, the demanded items are backordered. The time required to build up the maximum shortage level of size S is $t_5 = S/D$. The inventory behavior of perfect quality items is depicted in Fig. 1.

In order to calculate the inventory holding cost, the behavior of both imperfect quality items and reworked items inventories must be determined. Since imperfect quality items are reworked at the end of the production period, such items are accumulated throughout this period at a rate of $(1-\pi)P$ until a maximum level of $z_3 = t_p(1-\pi)P = (\gamma Q/P)(1-\pi)P = (1-\pi)\gamma Q$ is reached. After the disposal of scraped items, the imperfect quality items inventory drops to a level of $z_4 = \rho(1-\pi)\gamma Q$. During the rework period, between time $t = t_p$ and time $t = t_p + t_3$, these items are reworked into perfect quality items at a rate P. The inventory behavior of the imperfect quality items is illustrated in Fig. 2.

In the following we construct the producer's profit function by determining the relevant cost and revenue. At the beginning of each production/inventory cycle the producer places an order of size Q of raw material at a unit purchasing cost of c_{mp} and an ordering cost of A_p. These items are screened to detect the good quality at a unit screening cost of d_m. The γQ good quality items are used to produce γQ units of the finished product at a unit production cost of c_p and a setup cost of K. The remaining $(1-\gamma)Q$ items are returned to the supplier at the end of the screening period. Therefore, the purchasing cost of raw material is $c_{mp}\gamma Q$, the screening cost is $d_m Q$, and the production cost is $c_p\gamma Q$. Throughout the production period, the finished items are screened to detect the perfect and imperfect quality items at a unit screening cost d_f so that the screening cost of items produced is $d_f\,\gamma Q$. The perfect quality items $\pi\gamma Q$ are sold at a unit selling price of r. The remaining $(1-\pi)\gamma Q$ imperfect quality items are classified as scrap items or as items that be reworked into perfect quality. The $(1-\rho)(1-\pi)\gamma Q$ the scrap items are disposed of at a unit cost of c_d. The remaining $\rho(1-\pi)\gamma Q$ items of imperfect quality are reworked at a unit cost of c_r and sold at the same unit selling price of r. The shortage cost per cycle is obtained by multiplying the average number of units short per cycle by the cycle length by the cost of having of unit short per unit time. Similarly, the holding costs of the various types of items on hand are calculated by multiplying the average number of units on hand per cycle by the cycle length by the holding cost per unit per unit time. In summary, the revenues and cost components per cycle are:

Sales of good quality items	$= r\pi\gamma Q + r\rho(1-\pi)\gamma Q$
Ordering/Setup Cost	$= A_p + K$
Purchasing cost of raw material	$= c_{mp}\,\gamma Q$
Screening cost of raw material	$= d_m\,Q$
Finished items production cost	$= c_p\,\gamma Q$
Screening cost of finished product	$= d_f\,\gamma Q$
Disposal cost of scrap items	$= c_d\,(1-\rho)(1-\pi)\gamma Q$
Imperfect quality items rework cost	$= c_r\,\rho(1-\pi)\gamma Q$

In addition, the shortage cost is given by

$$\text{Shortage cost per cycle} = \frac{1}{2}S(t_1 + t_5)c_s = \frac{1}{2}S\left(\frac{S}{P\pi - D} + \frac{S}{D}\right)c_s = \frac{S^2}{2D\left(1 - \frac{D}{P\pi}\right)}c_s, \qquad (1)$$

and the various holding costs are

$$\text{Holding cost of raw material} = Q^2\left(\frac{\gamma^2}{2P}+\frac{1-\gamma}{x}\right)\times h_{mp}$$

$$\text{Perfect quality items holding cost} = \frac{1}{2}\left(z_1t_2+(z_1+z_2)t_3+z_2t_4\right)\times(h_{mp}+h_p) \tag{2}$$

$$\text{Imperfect quality items holding cost} = \frac{1}{2}\left(z_3t_p+z_4t_3\right)\times(h_{mp}+h_p)$$

$$\text{Holding cost due to rework} = \frac{1}{2}z_4(t_3+t_4)\times h_r$$

From the above revenue and cost components as well as Eqs. (1) and (2), we have that the total profit per cycle function is

$$\begin{aligned} TP(Q,S) &= r\pi\gamma Q + r\rho(1-\pi)\gamma Q - A_p - K - c_{mp}\gamma Q - d_mQ - c_p\gamma Q - c_r\rho(1-\pi)\gamma Q \\ &\quad - d_f\gamma Q - c_d(1-\rho)(1-\pi)\gamma Q - \frac{S^2}{2D\left(1-\frac{D}{P\pi}\right)}c_s - Q^2\left(\frac{\gamma^2}{2P}+\frac{1-\gamma}{x}\right)\times h_{mp} \\ &\quad - \frac{1}{2}z_4(t_3+t_4)\times h_r - \frac{1}{2}\left(z_1t_2+(z_1+z_2)t_3+z_2t_4+z_3t_p+z_4t_3\right) \end{aligned} \tag{3}$$

Dividing by the inventory cycle length $T = Q(\pi+\rho-\pi\rho)/D$, we obtain the producer total profit per unit time function $G_p(Q,S)$.

Next, all revenue and cost components for the supplier are determined by assuming that the supplier inventory cycle is a multiple of the producer production cycle T. Let N be the number of production cycles in one supplier's inventory cycle. At the beginning of the cycle, the supplier orders NQ units of raw material at an ordering cost A_s and a unit cost of c_{ms}. These items will be delivered to the producer in batches each of size Q, where the first batch is delivered at the start of the supplier cycle so that the supplier maximum inventory level is $(N-1)Q$. The supplier inventory behavior is shown in Fig. 3. The producer keeps the γNQ good quality items and the producer sells the $(1-\gamma)NQ$ returned items at a discounted price r_d, where $r_d < c_{ms}$.

The supplier cost and revenue components per cycle are:

Sales of good quality items	$= c_{mp}\, N\gamma Q$
Sales of returned items	$= r_d\, N(1-\gamma)Q$
Ordering	$= A_s$
Purchasing cost of raw material	$= c_{ms}\, NQ$
Holding cost	$= QT\ hms\ N(N-1)/2$

The supplier total profit per cycle function is

$$TP(Q,S) = TP(Q,S) = c_{mp}N\gamma Q + r_dN(1-\gamma)Q - A_s - c_{ms}NQ - QTh_{ms}N(N-1)/2 \tag{4}$$

Dividing by the supplier inventory cycle length NT, we obtain the supplier total profit per unit time function $G_s(Q,S)$. The supply chain total profit per unit time function is obtained by adding Eqs. (3) and (4) so that

$$Gc(Q) = Gp(Q) + Gs(Q). \tag{5}$$

In a non-collaborative supply chain, the producer is the decision maker. In this case, the optimal solution Q^* is determined by maximizing the function $G_p(Q)$. The supplier then determines the integer N that maximizes $G_s(Q^*)$. In the case of a coordinated supply chain, the optimal solution is determined by maximizing the $G_c(Q)$ for each value of N and selecting the value corresponding to the largest maximum total profit for the supply chain.

Numerical Example

Consider a production process where the demand rate for an item is 100 units per day and the production rate is 400 units per day. The raw material used in production is ordered from a supplier where 80% of the items received are of good quality. Screening for good quality items of the raw material is conducted at a rate of 1000 items per day and at a cost of $0.25 per unit. The ordering cost for the raw material is $5,000 and the production setup cost is $5,000. The holding cost of raw material is $0.02 per unit per day while the holding cost due to production is $0.05 per unit per day. Hence, the holding cost of one unit of the finished product is $0.07 per day. 75% of the items produced are of perfect quality. 80% of the imperfect quality items produced can be reworked and the remaining 20% are scrap items. The screening cost for detecting imperfect quality finished items is $0.5 per unit. If an item is reworked, an additional holding cost of $0.01 per unit per day is incurred. The purchasing cost of one item of raw material is $10, the unit production cost is $20, and the rework cost per unit is $5. The selling price is $50 per unit. The scrap items are disposed of at the end of production period at a cost of $2 per unit. Planned shortages are permitted, where the cost of having one perfect quality finished short is $0.3 per day. The supplier cost of one item of raw material is $5. A returned item of raw material can be sold at a discount price of $2. The supplier holding cost of raw material is $0.01 per unit per day.

The parameters of the problem are $D = 100$, $P = 400$, $x = 1000$, $\gamma = 0.8$, $\pi = 0.75$, $\rho = 0.80$, $Ap = 5000$, $K = 5000$, $h_{mp} = 0.02$, $h_p = 0.05$, $h_r = 0.01$, $C_{mp} = 10$, $C_p = 20$, $C_r = 5$, $C_d = 2$, $d_m = 0.25$, $d_f = 0.50$, and $r = 50$, $As = 3000$, $h_{ms} = 0.01$, $C_{ms} = 10$, and $r_d = 50$.

In a non-collaborative supply chain, the optimal solution obtained by maximizing the function $G_p(Q,S)$ via a numerical search. The search resulted in the following:

Optimal Order Size = $Q^* = 8000$

Optimal Planned Shortage = $S^* = 800$

Producer Total Daily Profit = $1318.77

Using the optimal order size, the supplier determines the best value of N=3 with a supplier total profit of $64.43 per day so that the supply chain's total profit is equal to $1383.20.

On the other hand, if the supply chain is coordinated, the best value of N is found to be 1, and the optimal solution is:

Optimal Order Size = $Q^* = 10{,}000$

Optimal Planned Shortage = $S^* = 1000$

Supply Chain Total Daily Profit =$1693.86

Conclusion

The effects of the interaction between the supplier of raw material and the producer of the finished product on the joint performance of both partners in this supply chain were examined. An EPQ model in a coordinated supplier-produced supply chain was developed. The model accounted for both the quality of finished product and raw material in a collaborative supply chain. A mathematical model was formulated so that the supply chain is optimized by determining the order quantity that maximizes the collaborative profit function. A numerical example was provided to compare the collaborative and non-collaborative models. This study showed how collaboration between supply chain members can increase their overall profits.

It is recommended that future research consider probabilistic percentages of good quality items in both raw material and finished products. In another direction, this model can be extended to incorporate other factors such as time value of money, and credit facilities.

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Figure 1: Perfect Quality Inventory Level

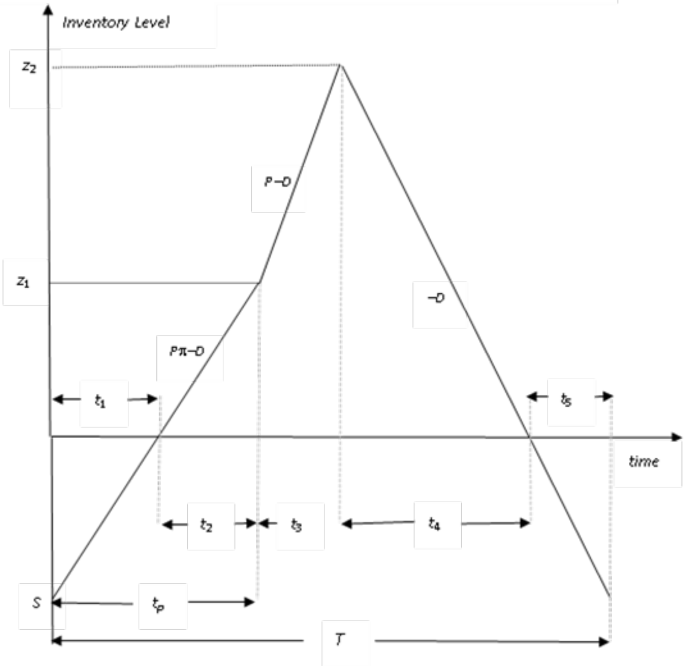

Figure 2: Finished Product Inventory Level

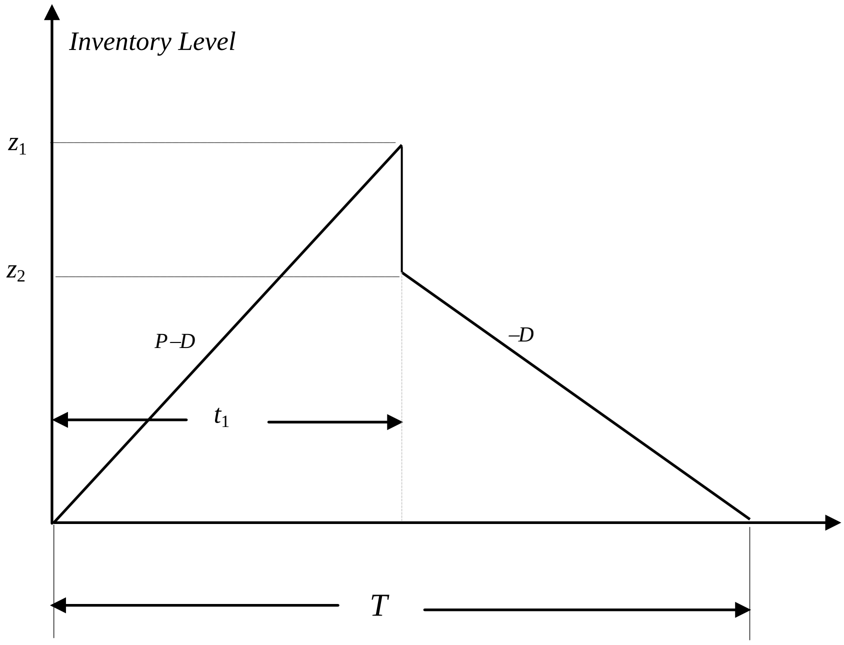

Figure 3: Supplier Inventory Level