Active Vibration Control of a Vehicle Suspension System Based on Signal Differentiation

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Abstract

Real-time estimation and differentiation of signals are common tasks in diverse applications of active vibration control. In this paper, an asymptotic approach for signal differentiation is applied in an active vehicle suspension system. The synthesis of the differentiation approach evades the use of a mathematical model of the suspension system. Estimation of unknown exogenous disturbances due to irregular road surfaces are also estimated. Estimates of time derivatives of the output variable and disturbances are then used for the implementation of an active vibration control scheme. Some numerical results are provided to show the effectiveness of the real-time estimation of the unavailable signals as well as a reasonable vibration attenuation level on a linear quarter-vehicle active suspension system.

Keywords: Active Vibration Control, Vehicle Suspension System, Differential Flatness, Signal Differentiation, Disturbance Rejection.

Introduction

Real-time estimation of parameters and signals is an active research subject in vibration control. Several approaches about parameter and signal estimation for mass-spring-damper systems, vibration absorbers and rotor-bering systems have been proposed in [1, 2, 3, 4, 5, 6]. Time derivatives of some system variables (e.g., velocity and acceleration) could be also required for implementation of active vibration control schemes. In fact, error signal differentiation is demanded in classical Proportional-Integral-Derivative (PID) control which is applied in many industrial engineering systems. Moreover, availability of signal derivatives can be used to reconstruct disturbance forces affecting a vibration mechanical system. State vector estimation is commonly based on asymptotic observers designed for specific dynamical systems. In practice, differentiation of signals is also performed by real-time numerical computations from samplings of the available output signals. Nevertheless, numerical differentiation could deteriorate the efficiency and robustness of system identification or control when measurements are corrupted by noise.

Recently, an asymptotic differentiation approach of signals for angular acceleration estimation for DC motors has been proposed in [7]. This paper describes the application of this signal differentiation approach to approximately estimate time derivatives and disturbances in vibrating mechanical systems. Signal differentiation is applied to control an active vehicle suspension system as well. The synthesis of the differentiation approach evades the use of a mathematical model of the suspension system. Hence, the differentiation approach can be employed in vibration mechanical systems where time derivative of some signal is required. It is shown that unknown exogenous disturbances due to irregular road surfaces can be algebraically reconstructed from estimates of time derivatives. Estimates of time derivatives of the output variable and disturbances are then used for the implementation of an active vibration control scheme. Some numerical results are provided to show the effectiveness of the real-time estimation of the unavailable signals. A reasonable level of forced vibration attenuation on an active linear quarter-vehicle suspension system is also verified.

1 Mathematical Model of a Vehicle Suspension System

Firstly, consider the mathematical model (1) of the active quarter-vehicle suspension system schematically shown in Fig.1:

$$m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = u$$

$$m_u \ddot{z}_u + k_t (z_u - z_r) - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) = -u$$
(1)

where the sprung mass m_s represents the mass of the car-body part, the unsprung mass m_u denotes the mass of the assembly of the axle and wheel, c_s is the damper coefficient of suspension, k_s and k_t are the spring coefficients of the suspension and tire, respectively. The generalized coordinates are the displacements of both masses z_s and z_u , z_r is the terrain disturbance and u is the control force input provided by some (electromagnetic or hydraulic) actuator.



Figure 1: Quarter-vehicle suspension system: (a) passive suspension system, (b) active suspension system with an electromagnetic actuator, (c) active suspension system with a hydraulic actuator.

Defining the state variables as $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_u$ and $x_4 = \dot{z}_u$, mathematical model (1) adopts the state-space description

$$\dot{x}_{1} = x_{2}$$

$$\dot{x}_{2} = -\frac{k_{s}}{m_{s}}x_{1} - \frac{c_{s}}{m_{s}}x_{2} + \frac{k_{s}}{m_{s}}x_{3} + \frac{c_{s}}{m_{s}}x_{4} + \frac{1}{m_{s}}u$$

$$\dot{x}_{3} = x_{4}$$

$$\dot{x}_{4} = \frac{k_{s}}{m_{u}}x_{1} + \frac{c_{s}}{m_{u}}x_{2} - \frac{k_{s} + k_{t}}{m_{u}}x_{3} - \frac{c_{s}}{m_{u}}x_{4} - \frac{1}{m_{u}}u + \frac{k_{t}}{m_{u}}z_{r}$$
(2)

The active suspension system (2) is a differentially flat system, where a flat output y is given by [8, 9]:

$$y = m_s x_1 + m_u x_3 \tag{3}$$

Therefore, state and control variables can be expressed in terms of the flat output y and a finite number of its time derivatives. Indeed, from y and its time derivatives up to fourth order:

$$\dot{y} = m_s x_2 + m_u x_4
\ddot{y} = k_t (z_r - x_3)
y^{(3)} = k_t (\dot{z}_r - x_4)
y^{(4)} = \frac{1}{m_u} u + \frac{k_t}{m_u} x_3 - \frac{1}{m_u} (\mathcal{F}_{sc} + \mathcal{F}_{sk}) - \frac{k_t}{m_u} z_r + k_t \ddot{z}_r$$
(4)

with

$$\mathcal{F}_{sk} = k_s \left(x_1 - x_3 \right)$$

$$\mathcal{F}_{sc} = c_s \left(x_2 - x_4 \right)$$
(5)

the differential parameterization results as follows

$$x_{1} = \frac{1}{m_{s}}y + \frac{m_{u}}{k_{t}m_{s}}\ddot{y} - \frac{m_{u}}{m_{s}}z_{r}$$

$$x_{2} = \frac{1}{m_{s}}\dot{y} + \frac{m_{u}}{k_{t}m_{s}}y^{(3)} - \frac{m_{u}}{m_{s}}\dot{z}_{r}$$

$$x_{3} = -\frac{1}{k_{t}}\ddot{y} + z_{r}$$

$$x_{4} = -\frac{1}{k_{t}}y^{(3)} + \dot{z}_{r}$$

$$u = \frac{1}{b}\left(a_{0}y + a_{1}\dot{y} + a_{2}\ddot{y} + a_{3}y^{(3)} + y^{(4)} - \xi\right)$$
(6)

with

$$a_{0} = \frac{k_{s}k_{t}}{m_{s}m_{u}}, \quad a_{1} = \frac{c_{s}k_{t}}{m_{s}m_{u}}$$

$$a_{2} = \frac{k_{s}}{m_{s}} + \frac{k_{s} + k_{t}}{m_{u}}, \quad a_{3} = \frac{c_{s}}{m_{s}} + \frac{c_{s}}{m_{u}}$$

$$b = \frac{k_{t}}{m_{u}}$$
(7)

and

$$\xi(t) = \left(\frac{k_t}{m_s} + \frac{k_t}{m_u}\right)k_s z_r + \left(\frac{k_t}{m_s} + \frac{k_t}{m_u}\right)c_s \dot{z}_r + k_t \ddot{z}_r \tag{8}$$

Thus from (6) the flat output is governed by the perturbed input-output differential equation

$$y^{(4)} + a_3 y^{(3)} + a_2 \ddot{y} + a_1 \dot{y} + a_0 y = bu + \xi$$
(9)

Hence, the following active vibration control scheme based on differential flatness can be di-

rectly synthesised:

$$u = \frac{1}{b}(v + a_3 y^{(3)} + a_2 \ddot{y} + a_1 \dot{y} + a_0 y - \xi)$$
(10)

with

$$\upsilon = -\alpha_3 y^{(3)} - \alpha_2 \ddot{y} - \alpha_1 \dot{y} - \alpha_0 y$$

Nevertheless, implementation of the control law (10) needs measurements or estimates of some time derivatives of the flat output variable y. In addition, information of the profile of irregular road surfaces z_r could be also demanded.

On the other hand, note that from (4) the flat output y and its derivatives up to third order can be computed from state variables and disturbance z_r . Otherwise, time derivatives of the flat output can be also estimated directly. Moreover, the disturbance z_r can be calculated by

$$z_r = \frac{1}{k_t}\ddot{y} + x_3 = \frac{1}{k_t}\left(m_s\ddot{x}_1 + m_u\ddot{x}_3\right) + x_3 \tag{11}$$

Thus, in the next section it is described a signal differentiation approach to get approximate derivatives for some stable dynamical system [7].

2 A Signal Differentiation Approach

The synthesis of the signal differentiation scheme with respect to time is based on the local approximation of some bounded signal \mathscr{Y} by a family of Taylor polynomials of forth degree as

$$\mathscr{Y}(t) \approx \sum_{i=0}^{4} q_i t^i \tag{12}$$

where coefficients q_i are assumed to be unknown.

Therefore, the signal \mathscr{Y} can be locally reconstructed by the dynamical system

$$\begin{array}{rcl}
\dot{\mathscr{Y}}_{f} &=& \mathscr{Y}_{1} \\
\dot{\mathscr{Y}}_{1} &=& \mathscr{Y}_{2} \\
\dot{\mathscr{Y}}_{2} &=& \mathscr{Y}_{3} \\
\dot{\mathscr{Y}}_{3} &=& \mathscr{Y}_{4} \\
\dot{\mathscr{Y}}_{4} &=& \mathscr{Y}_{5} \\
\dot{\mathscr{Y}}_{5} &=& \mathscr{F}
\end{array}$$
(13)

where $\mathscr{Y}_1 = \mathscr{Y}, \mathscr{Y}_2 = \mathscr{Y}, \dots, \mathscr{Y}_5 = \mathscr{Y}^{(4)}, \mathscr{Y}_f = \int_0^t \mathscr{Y} dt$, and \mathscr{F} is considered as an unknown bounded perturbation signal including the influence of high frequency noise and small residual terms of the truncated Taylor polynomial expansion (12) (see [7]). Moreover, we have assumed that the time derivatives up to fifth order of \mathscr{Y} are uniformly absolutely bounded.

Hence, from (13) we propose the following state observer for asymptotic estimation of some

time derivatives of the signal \mathscr{Y} :

$$\widehat{\mathscr{Y}}_{f} = \widehat{\mathscr{Y}}_{1} + \beta_{5} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)
\widehat{\mathscr{Y}}_{1} = \widehat{\mathscr{Y}}_{2} + \beta_{4} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)
\widehat{\mathscr{Y}}_{2} = \widehat{\mathscr{Y}}_{3} + \beta_{3} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)
\widehat{\mathscr{Y}}_{3} = \widehat{\mathscr{Y}}_{4} + \beta_{2} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)
\widehat{\mathscr{Y}}_{4} = \widehat{\mathscr{Y}}_{5} + \beta_{1} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)
\widehat{\mathscr{Y}}_{5} = \beta_{0} \left(\mathscr{Y}_{f} - \widehat{\mathscr{Y}}_{f} \right)$$
(14)

which only uses information of the filtered output signal \mathscr{Y}_f . Here, we use the notation $(\widehat{\cdot})$ for the estimated signals.

Then, the estimation error dynamics is governed by

$$\dot{e}_{f} = e_{1} - \beta_{5}e_{f}$$

$$\dot{e}_{1} = e_{2} - \beta_{4}e_{f}$$

$$\dot{e}_{2} = e_{3} - \beta_{3}e_{f}$$

$$\dot{e}_{3} = e_{4} - \beta_{2}e_{f}$$

$$\dot{e}_{4} = e_{5} - \beta_{1}e_{f}$$

$$\dot{e}_{5} = -\beta_{0}e_{f}$$
(15)

which is completely independent of any coefficients q_i of the Taylor polynomial expansion of the output signal \mathscr{Y} . Here, $e_i = \mathscr{Y}_i - \widehat{\mathscr{Y}_i}$, i = 1, 2, ..., 5, $e_f = \mathscr{Y} - \widehat{\mathscr{Y}_f}$. Notice that, estimator gains should be properly selected in order to have a stable characteristic polynomial for the observer-based closed-loop system dynamics. Additionally, the estimation dynamics should be sufficiently fast to get estimates opportunely to be used by the active vibration control scheme. Note that, it is widely known that delays in measurements or estimations could become unstable a dynamical systems. Better estimates can be obtained by employing a Taylor polynomial model of higher order.

3 Simulation results

Effectiveness of the differentiation approach for approximate estimation of time derivatives and road disturbance signals required for implementation of the active vibration control scheme (10) for an active linear quarter-vehicle suspension system was verified by some preliminary computer simulations. The vehicle suspension system is characterized by the set of parameters described in Table 1 [10].

In Fig. 2 is shown the unknown exogenous disturbance excitation due to irregular road surfaces which is described by [11]:

$$z_{r}(t) = \begin{cases} f_{1}(t) + f(t) & \text{for } t \in [3.5, 5) \\ f_{2}(t) + f(t) & \text{for } t \in [5, 6.5) \\ f_{3}(t) + f(t) & \text{for } t \in [8.5, 10) \\ f_{3}(t) + f(t) & \text{for } t \in [10, 11.5) \\ f(t) & \text{else} \end{cases}$$
(16)

Parameter	Value
Sprung mass m_s	216.75 kg
Unsprung mass m_u	28.85 kg
Spring stiffness k_s	21700 N/m
Damping constant c_s	1200 Ns/m
Tire stiffness k_t	184000 N/m

 Table 1: Parameters of the vehicle suspension system.

with

j j

$$f_{1}(t) = -0.0592 (t - 3.5)^{3} + 0.1332 (t - 3.5)^{2}$$

$$f_{2}(t) = 0.0592 (t - 6.5)^{3} + 0.1332 (t - 6.5)^{2}$$

$$f_{3}(t) = 0.0592 (t - 8.5)^{3} - 0.1332 (t - 8.5)^{2}$$

$$f_{3}(t) = -0.0592 (t - 11.5)^{3} - 0.1332 (t - 11.5)^{2}$$

$$f(t) = 0.002 \sin (2\pi t) + 0.002 \sin (7.5\pi t)$$



Figure 2: Irregular profile of the road surface.

Fig. 4 describes a reasonable attenuation level of vibrations induced by irregular road surfaces (16) using the active vibration control scheme based on high-gain signal differentiation. To get a fast signal estimation the characteristic polynomial of the estimation error dynamics was set as

$$P_O(s) = (s^2 + 2\zeta_o \omega_o s + \omega_o^2)^3$$
(17)

with $\omega_o = 2000$ rad/s and $\zeta_o = 5$.

Acceptable approximate estimation of the disturbance signal z_r is depicted in Fig. 4. On the other hand, the active control force applied to the vehicle suspension system is illustrated in Fig. 5. The control gains were chosen to have the closed loop characteristic polynomial

$$P_c(s) = (s^2 + 2\zeta_c \omega_c s + \omega_c^2)^2 \tag{18}$$



Figure 3: Position responses of sprung and unsprung masses.





Figure 4: High-gain fast estimation of the irregular profile of the road surface.



Figure 5: Active control force applied to the vehicle suspension system.

4 Conclusions

The application of a signal differentiation approach with respect to time has been described for an active linear quarter-vehicle suspension system. Certain signal derivatives and unknown exogenous disturbances due to irregular road surfaces were estimated. Approximate estimates were then used for the implementation of an active vibration control scheme based on differential flatness. Numerical results illustrate an acceptable estimation of the disturbance signal due to irregular road surfaces. It was also shown that the active vibration control scheme archives a reasonable vibration attenuation level on a linear quarter-vehicle active suspension system when the estimation error dynamics is sufficiently fast with respect to the closed loop vehicle suspension system and disturbances. Thus, the effectiveness of the on-line signal estimation algorithm without employing some specific mathematical model of the controlled dynamical system requires fast velocities for signal processing and high estimation gains. Actually, high speed and precise sensors, DSP boards, and software with high computational performance operating at high sampling rates are now available. Hence, the described differentiation approach represents a good choice to approximately estimate disturbances and time derivatives for scenarios where evasion of the use of some mathematical model for the system or a minimal number of sensors are desired.

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