Closed Loop Algebraic Parametric Identification of a DC Shunt Motor

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Abstract

Real-time system identification for electric machinery is an active research topic. Diagnostic or adaptive control tasks could demand the knowledge of the energy conversion system parameters and, possibly, load mechanical torque as well. In this paper, an on-line identification scheme is proposed for estimation of all parameters and load torque for an efficiently controlled DC shunt motor. The parameters of the nonlinear electromechanical system and load torque are estimated algebraically and quickly. A PI control law is also described for regulation and tracking tasks on this nonlinear energy conversion system. Some numerical simulation results are provided to show the effective closed-loop estimation of all system parameters and mechanical torque.

Keywords: DC shunt motor, System identification, Algebraic identification, PI control.

Introduction

In recent decades, several applications of electric motors can be found at industry and homes. In fact, motor-driven equipment is approximately 60% of manufacturing final electricity worldwide [1]. Among them are direct current (DC) motors and, particularly, shunt connection allows advantages over those well-known permanent magnet motors. This configuration is commonly applied for operation conditions of variable load torque with a reduced effect on the rotor speed. Further, it does not handle high currents as series DC motors, therefore, it is a useful configuration that allows starting and nominal torques with relatively low currents in transient and steady state operation. Several control schemes for DC electrical machines have been reported in the literature (see, e.g., [2, 3, 4]). However, most of them are focused on permanent magnet or separately excited, limiting the load torque operation mainly in starting and tracking variable speed condition. In addition, a priori knowledge of the motor parameters and, possibly, load torque are required to get an efficient control performance under variable velocity operation scenarios. Thus, parameter identification techniques have been commonly employed [5, 6, 7]. Nevertheless, this requirement is a difficult aspect to guarantee because a DC shunt motor is a nonlinear dynamical system with parameters changing in time. The parameter identification area for electrical machines is very extensive, where important aspects of implementation are searched. In general, closed loop parametric identification should be performed on-line and fast to be used simultaneously with some control technique applied to the motor [8].

There are numerous research works that propose different parameter identification techniques [9, 10, 11]. Recent contributions based on neural networks, fuzzy logic, Kalman filter, complementary, and using optimization procedures such as genetic algorithms, ant colony, particle swarm have been proposed for motor parameters estimation [10, 11]. They are suitable for on-line or off-line application depending mostly on high computational requirements. One of the drawbacks of some of the proposed strategies is that correct parameters estimation is not guaranteed because a nonlinear and coupled nature of the interest variables are not included. Additionally, identification schemes have a weak performance in some operation conditions due to in many cases the complex behavior presented in electrical motors are not considered in the design stage. In general, there are some considerations that must be taken into account in parameters estimation as continuous variations of the load torque, the impact of the electronic controllers in transient response and noise included in measured variables, and tracking speed. Such schemes must meet high precision in face to continuous motor changes with low computational cost for implementation in real time platform. On the other hand, recent algebraic parametric identification methods have shown an excellent and fast performance [12]. In fact, algebraic identification have been successfully applied to estimate parameters and signals in flexible mechanical systems [13, 14, 15]. Numerical and experimental results have confirmed that algebraic identification represents an very good choice for the synthesis of on-line parameter estimators.

In this paper, an on-line identification scheme is proposed for estimation of all parameters and load torque for an efficiently controlled DC shunt motor. The parameters of the nonlinear electromechanical system and load torque are estimated algebraically and quickly. A PI control law is also described for regulation and tracking tasks on this nonlinear energy conversion system. Some numerical simulation results are provided to show the effective closed-loop estimation of all system parameters and mechanical torque.

1 Mathematical Model of a Controlled DC Shunt Motor

Consider the nonlinear mathematical model of a DC motor with field and armature circuits connected in parallel

$$
\begin{aligned}
L_f \frac{d}{dt} i_f =& - R_f i_f + u \\
L_a \frac{d}{dt} i_a =& - R_a i_a - L_{af} i_f \omega + u \\
J \frac{d}{dt} \omega =& - b\omega + L_{af} i_f i_a - \tau_L \\
y =& \omega
\end{aligned} \tag{1}
$$

where the positive parameters of the field circuit are the inductance L_f and resistance R_f. L_a and R_a are the inductance and resistance of the armature circuit, respectively, and L_{af} is the mutual inductance. J and b are the inertia moment and viscous damping of the mechanical subsystem. Here, u is the voltage control input, $y = \omega$ is the controlled output angular velocity and τ_L is the constant load torque. The field and armature current signals are respectively i_f and i_a.

From basic control fundamentals, one can very that the dynamics of the output angular velocity $y = \omega$ around some desired equilibrium operation state $(\omega^e, i_a^e, i_f^e, u^e)$ is governed by

$$
\ddot{y} + a_1 \dot{y} + a_0 y = \gamma u + \phi \tag{2}
$$

with

$$
\begin{aligned}
a_1 &= \frac{b}{J} + \frac{R_a}{L_a} \\
a_0 &= \frac{L_{af}^2}{JL_a}\left(i_f^e\right)^2 + \frac{R_a b}{JL_a} \\
\gamma &= \frac{L_{af}}{JL_a} i_f^e + \frac{L_{af}}{JL_f} i_a^e
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\phi &= \left[\frac{L_{af}^2}{JL_a}\left(i_f^e\right)^2 + \frac{R_a b}{JL_a}\right]\omega^e - \left(\frac{L_{af}}{JL_a} i_f^e + \frac{L_{af}}{JL_f} i_a^e\right) u_a^e \\
&\quad + \left[\frac{L_{af}}{J}\left(\frac{R_a}{L_a} - \frac{R_f}{L_f}\right) i_a^e - \frac{L_{af}^2}{JL_a} i_f^e \omega^e\right] i_{f\delta}
\end{aligned}
\tag{4}
$$

Notice that, constants a_0, a_1 and γ depend on the system parameters and desired operation state. Thus, high operation efficiency levels could require information about some approximated values of the parameters of the motor subjected to completely unknown load torque disturbances.

In the design of some classical control law, ϕ could be considered as an completely unknown disturbance signal depending on the equilibrium operation state specified for the electromechanical system. Moreover, for a constant operation velocity $y = \overline{\omega}$, $i_f \longrightarrow i_f^e$ and, as a consequence, $\phi \longrightarrow \phi^e =$ constant. Therefore, we propose the following PI angular velocity tracking controller:

$$
u = -k_p e - k_i \int_0^t e \, dt \tag{5}
$$

where the proportional and integral control gains, k_p and k_i, should be chosen such as the characteristic polynomial associated to the closed loop tracking error dynamics, $e = \omega - \omega^\star$,

$$
P(s) = s^3 + a_1 s^2 + (a_0 + \gamma k_p) s + \gamma k_i \tag{6}
$$

is a Hurwitz polynomial. Hence, closed loop system stability can be verified. Notice that, the control gains should be also selected properly in accordance with the equilibrium operation point for the motor. Certainly, the on-line knowledge of the system parameters and load torque allows to tune easily the control gains during the operation of the machine.

The main objective of this paper is to propose an alternative choice for on-line estimation of all parameters and load torque for a DC Shunt Motor. Thus, in the next section estimators are synthesized to get estimates of the system parameters algebraically and on-line.

2 On-line Algebraic Parameter Identification

Firstly, consider the dynamics of the electrical subsystem. Multiplication of the first two equations of model (1) by $\Delta = t - t_i$, and integrating the resulting expressions twice with respect to time yields to

$$\begin{aligned} L_f \left[\Delta i_f - \int_{t_i}^{t} i_f \, dt \right] + R_f \int_{t_i}^{t} \Delta i_f \, dt & \\ = \int_{t_i}^{t} \Delta u_f \, dt & \\ L_a \left[\Delta i_a - \int_{t_i}^{t} i_a \, dt \right] + R_a \int_{t_i}^{t} \Delta i_a \, dt + L_{af} \int_{t_i}^{t} \Delta i_f \omega \, dt & \\ = \int_{t_i}^{t} \Delta u_a \, dt & \end{aligned} \tag{7}$$

where $t_i > 0$ is the start time to perform the parameter identification process.

By integrating up to twice Eqs. (7), we get the following equation systems

$$A_i \theta_i = B_i, \quad i = f, a \tag{8}$$

where $\theta_f = \begin{bmatrix} L_f & R_f \end{bmatrix}^T$ and $\theta_a = \begin{bmatrix} L_a & R_a & L_{af} \end{bmatrix}^T$ are the parameter vectors associated with the electrical subsystem to be identified on-line. Matrices A_i and B_i are given by

$$\begin{aligned} A_f &= \begin{bmatrix} a_{11,f} & a_{12,f} \\ a_{21,f} & a_{22,f} \end{bmatrix} \\ A_a &= \begin{bmatrix} a_{11,a} & a_{12,a} & a_{13,a} \\ a_{21,a} & a_{22,a} & a_{23,a} \\ a_{31,a} & a_{32,a} & a_{33,a} \end{bmatrix} \\ B_f &= \begin{bmatrix} b_{1,f} \\ b_{2,f} \end{bmatrix} \\ B_a &= \begin{bmatrix} b_{1,a} \\ b_{2,a} \\ b_{3,a} \end{bmatrix} \end{aligned} \tag{9}$$

with

$$
\begin{aligned}
a_{11,f} &= \Delta i_f - \int_{t_i}^{t} i_f \, dt \\
a_{12,f} &= \int_{t_i}^{t} \Delta i_f \, dt \\
a_{21,f} &= \int_{t_i}^{t} a_{11,f} \, dt \\
a_{22,f} &= \int_{t_i}^{t} a_{12,f} \, dt \\
b_{1,f} &= \int_{t_i}^{t} \Delta u \, dt \\
b_{2,f} &= \int_{t_i}^{t} b_{1,f} \, dt \\
a_{11,a} &= \Delta i_a - \int_{t_i}^{t} i_a \, dt \\
a_{12,a} &= \int_{t_i}^{t} \Delta i_a \, dt \\
a_{13,a} &= \int_{t_i}^{t} \Delta i_f \omega \, dt \\
b_{1,a} &= \int_{t_i}^{t} \Delta u_a \, dt \\
a_{kh,a} &= \int_{t_i}^{t} a_{k-1h,a}(\tau_1) d\tau_1 \\
b_{k,a} &= \int_{t_i}^{t} b_{k-1,a}(\tau_1) d\tau_1
\end{aligned} \tag{10}
$$

with $k = 2, 3$ and $h = 1, 2, 3$.

Therefore, from (8) the electrical subsystem parameters can be computed algebraically as

$$
\theta_i = A_i^{-1} B_i \tag{11}
$$

Nevertheless, parameter identifiers (11) could present problems of singularities when $det A_i = 0$. Hence, we propose the following algebraic identifiers to get estimates of the parameters of the electrical subsystem without singularities $\forall t_i > 0$:

$$
\begin{aligned}
\widehat{L}_f &= \frac{\int_{t_i}^{t} |\Delta_{1,f}| \ dt}{\int_{t_i}^{t} |\Delta_f| \ dt} \\
\widehat{R}_f &= \frac{\int_{t_i}^{t} |\Delta_{2,f}| \ dt}{\int_{t_i}^{t} |\Delta_f| \ dt} \\
\widehat{L}_a &= \frac{\int_{t_i}^{t} |\Delta_{1,a}| \ dt}{\int_{t_i}^{t} |\Delta_a| \ dt} \\
\widehat{R}_a &= \frac{\int_{t_i}^{t} |\Delta_{2,a}| \ dt}{\int_{t_i}^{t} |\Delta_a| \ dt} \\
\widehat{L}_{af} &= \frac{\int_{t_i}^{t} |\Delta_{3,a}| \ dt}{\int_{t_i}^{t} |\Delta_a| \ dt}
\end{aligned} \tag{12}
$$

Now, consider the dynamics of the mechanical subsystem described by third equation of model (1). By applying the same procedure explained before, one can get the following estimators for the mechanical parameters and load torque:

$$\begin{aligned}
\widehat{J} &= \frac{\widehat{L}_{af}}{\sigma_1} \\
\widehat{b} &= \frac{\sigma_2}{\sigma_1}\widehat{L}_{af} \\
\widehat{\tau}_L &= \frac{\sigma_3}{\sigma_1}\widehat{L}_{af}
\end{aligned} \tag{13}$$

where σ_j, $j = 1, 2, 3$, are given by

$$\begin{aligned}
\sigma_1 &= \frac{\int_{t_i}^{t} |\Delta_{1,m}| \ dt}{\int_{t_i}^{t} |\Delta_m| \ dt} \\
\sigma_2 &= \frac{\int_{t_i}^{t} |\Delta_{2,m}| \ dt}{\int_{t_i}^{t} |\Delta_m| \ dt} \\
\sigma_3 &= \frac{\int_{t_i}^{t} |\Delta_{3,m}| \ dt}{\int_{t_i}^{t} |\Delta_m| \ dt}
\end{aligned} \tag{14}$$

with

$$\theta_m = \begin{bmatrix} \sigma_1 & \sigma_2 & \sigma_3 \end{bmatrix}^T = A_m^{-1} B_m \tag{15}$$

$$\begin{aligned}
A_m &= \begin{bmatrix} a_{11,m} & a_{12,m} & a_{13,m} \\ a_{21,m} & a_{22,m} & a_{23,m} \\ a_{31,m} & a_{32,m} & a_{33,m} \end{bmatrix} \\
B_m &= \begin{bmatrix} b_{1,m} \\ b_{2,m} \\ b_{3,m} \end{bmatrix}
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
a_{11,m} &= \int_{t_i}^{t} \Delta i_f i_a \ dt \\
a_{12,m} &= -\int_{t_i}^{t} \Delta\omega \ dt \\
a_{13,m} &= -\int_{t_i}^{t} \Delta \ dt \\
b_{1,m} &= \Delta\omega - \int_{t_i}^{t} \omega \ dt \\
a_{kh,m} &= \int_{t_i}^{t} a_{k-1h,m}(\tau_1) d\tau_1 \\
b_{k,m} &= \int_{t_i}^{t} b_{k-1,m}(\tau_1) d\tau_1
\end{aligned} \tag{17}$$

with $k = 2, 3$ and $h = 1, 2, 3$.

3 Simulation results

Effectiveness of the proposed parameter estimation scheme was verified by computer simulations. The parameter values of the DC motor are described in Table 1.

Table 1: Parameters of the DC motor.

$R_a = 7.5\ \Omega$	$L_{af} = 2.2881$ H
$L_a = 0.0553$ H	$J = 0.0013$ Kg m^2
$R_f = 469.75\ \Omega$	$b = 0.001$ Nms
$L_f = 2.4123$ H	$\tau_L = 0.5$ Nm

The reference velocity trajectory $\omega^\star$ planned for the electromechanical system is shown in Fig. 1 and described by

$$\omega^\star(t) = \begin{cases} 0 & \text{for } 0 \leq t < T_i \\ \varpi\,(t, T_i, T_f)\,\bar{\omega} & \text{for } T_i \leq t \leq T_f \\ \bar{\omega} & \text{for } t > T_f \end{cases} \tag{18}$$

where $\bar{\omega} = 10$ rad/s, $T_i = 0$ s, $T_f = 5$ s, $\varpi\,(t, T_i, T_f)$ is a Bézier interpolation polynomial, with $\varpi\,(T_i, T_i, T_f) = 0$ and $\varpi\,(T_f, T_i, T_f) = 1$, given by

$$\begin{aligned} \varpi(t) \;\; = \;\; & \left(\frac{t - T_i}{T_f - T_i}\right)^5 \left[d_1 - d_2\left(\frac{t - T_i}{T_f - T_i}\right)\right. \\ & \left. + d_3\left(\frac{t - T_i}{T_f - T_i}\right)^2 - \ldots - d_6\left(\frac{t - T_i}{T_f - T_i}\right)^5\right] \end{aligned}$$

with $d_1 = 252$, $d_2 = 1050$, $d_3 = 1800$, $d_4 = 1575$, $d_5 = 700$, $d_6 = 126$. This profile was established to efficiently take the motor from a rest state to a low operation velocity of 10 rad/s in 5 seconds.

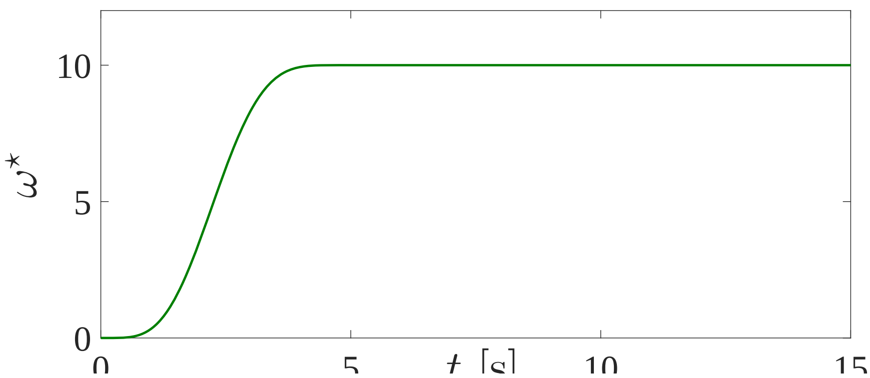

Figure 1: Reference angular velocity planned for the DC motor.

Fig. 2 depicts the satisfactory closed loop tracking of the desired velocity reference trajectory (18). A small velocity tracking error is clearly observed. The responses of the control voltage, current signals and electric powers are shown in Figs. 3 and 4. It can be observed that the properly controlled motion planning (18) avoids high peaks of the electric signals of voltage, currents and powers in presence of load torque from the start. Moreover, it is known widely that large fluctuations of voltage could cause control saturations and system instability. Thus, planning motion tracking control represents an excellent choice to reduce these undesirable issues. PI controller gains were conveniently set as: $k_p = 100$ and $k_i = 10$. Nevertheless, the controller gains can be easily adjusted on-line for diverse operation states for the electric motor, including uncertain changes in the system parameters and load mechanical torque.

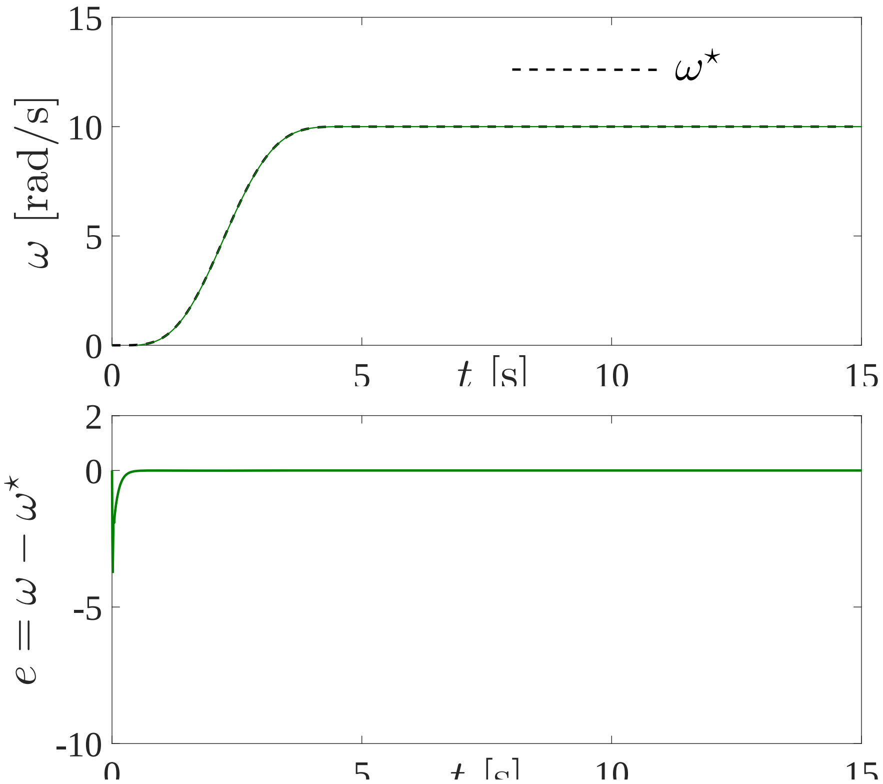

Figure 2: Closed loop tracking of the reference velocity trajectory.

On the other hand, the efficient performance of the parameter and torque identification scheme is presented in Figs. 5-7. An effective and fast estimation of the system parameters and load torque is confirmed. Estimates of all parameters and mechanical torque are quickly obtained before 1 second. Thus, those estimates can be used to tune the controller gains to improve the dynamic performance of the closed loop system and guarantee asymptotic stability around possibly varying-time desired operation states for the electric machine. Moreover, estimators could be updated continually for possible changes of the parameters and load torque during the motor operation.

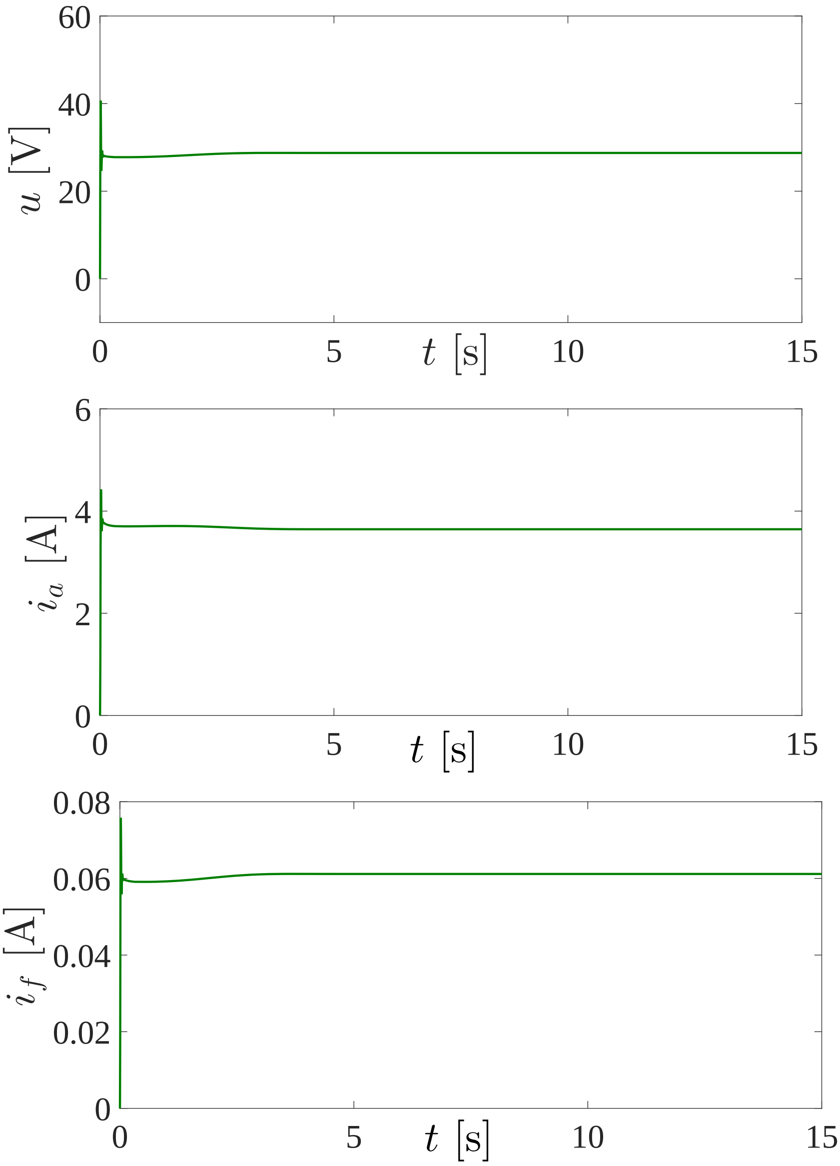

Figure 3: Closed loop responses of the control voltage and electric current signals.

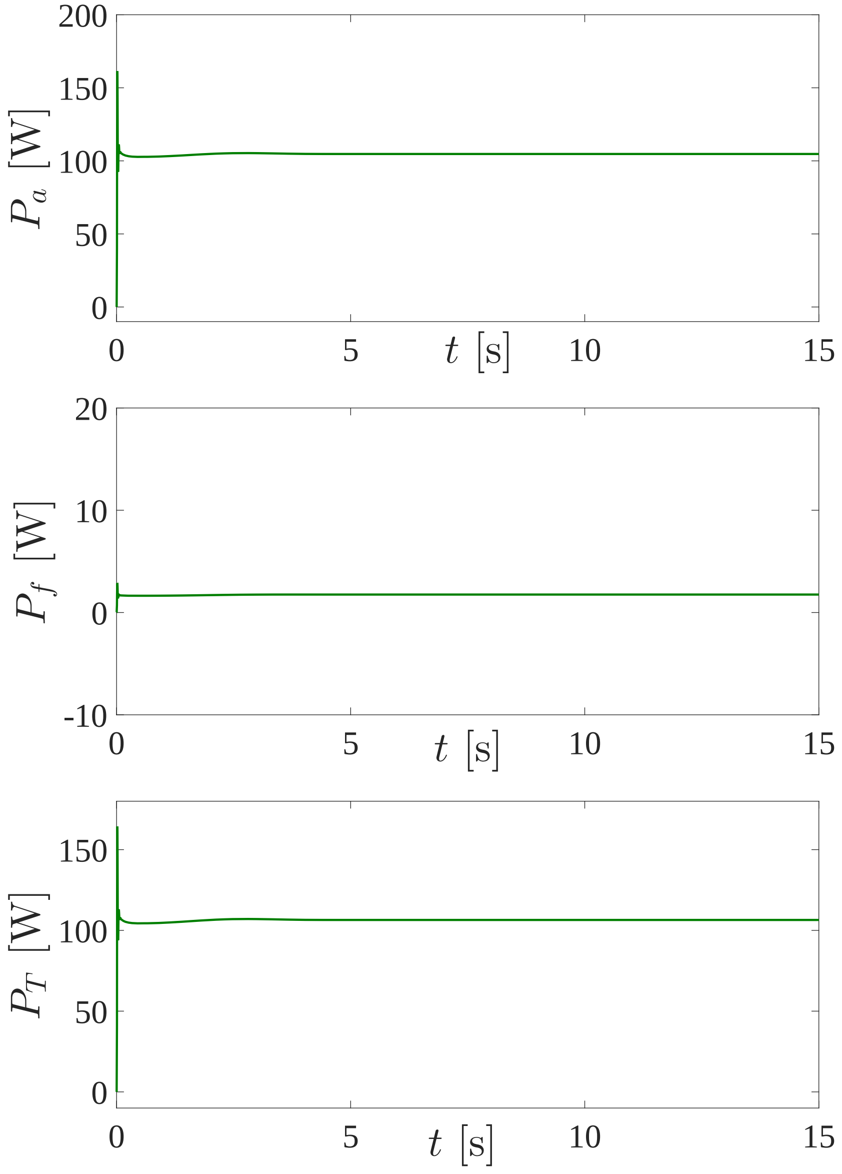

Figure 4: Closed loop responses of the electric powers of the armature circuit P_a, field P_f and total $P_T = P_a + P_f$.

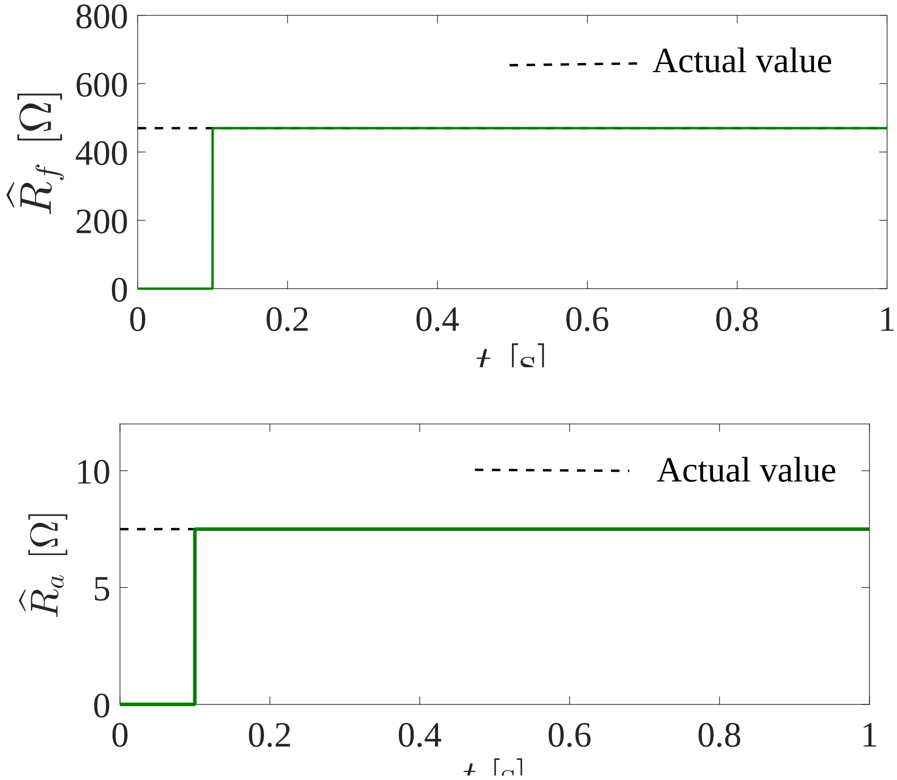

Figure 5: Algebraic estimation of the resistance parameters.

4 Conclusions

In this paper we have proposed an on-line estimation scheme for parameters and load torque for DC shunt motors. Connection in parallel of the field and armature windings of the motor results in a nonlinear system dynamics. The proposed estimation approach is performed algebraically and on-line. In addition, a PI tracking control law was also described to take the motor from a rest state toward a desired operation velocity. Controlled motion planning was established by a Bézier interpolation polynomial. It was shown that the suitable trajectory tracking avoid large fluctuations of voltage and, as a consequence, in the electric current signals as well. Analytical and numerical results show the effectiveness of the parameter and torque estimation for tracking tasks of reference angular velocity trajectories. Therefore, tracking control can be combined with on-line and algebraic estimation of system parameters and load mechanical torque to get satisfactory efficiency levels for DC shunt motors.

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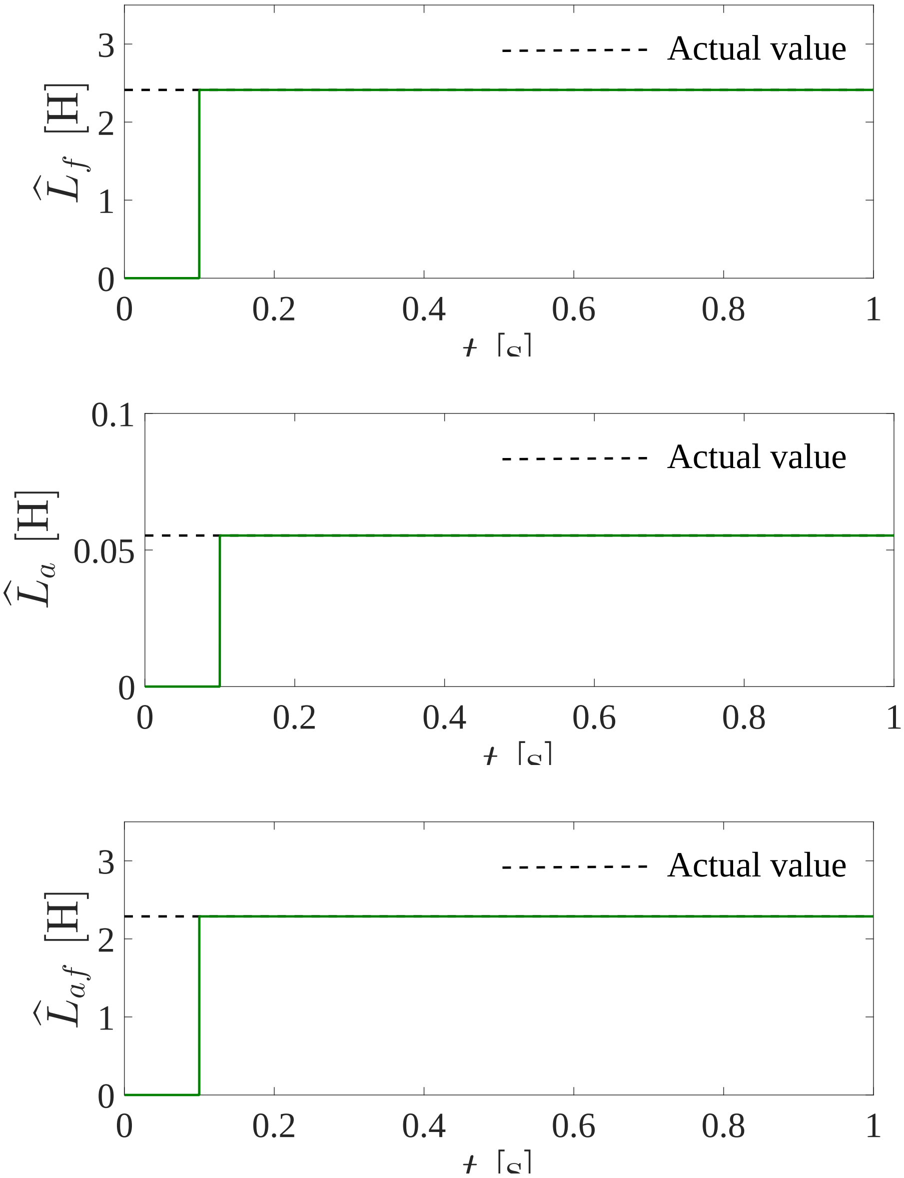

Figure 6: Algebraic estimation of the inductance parameters.

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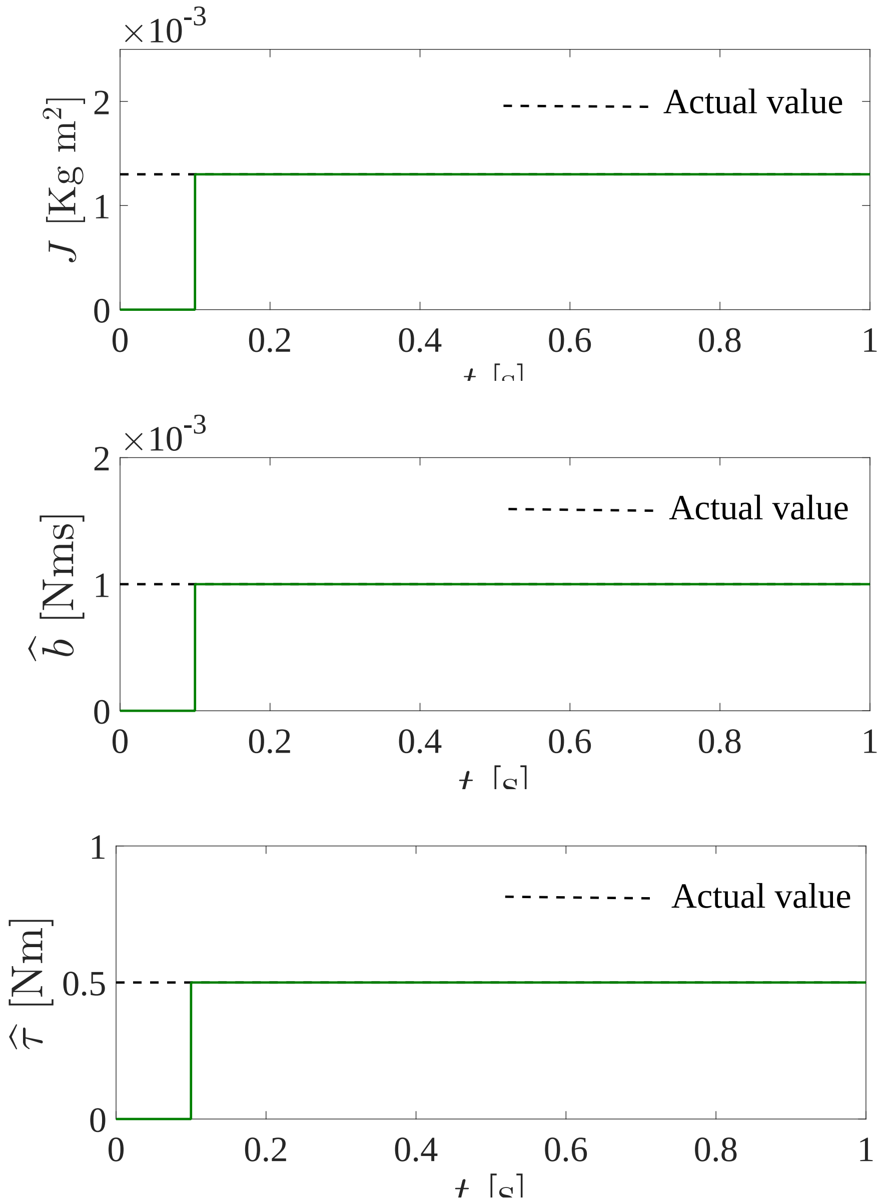

Figure 7: Algebraic estimation of the mechanical subsystem parameters and load torque.

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