REAL-TIME MULTISCALE MODELING VIA PROJECTION-BASED MODEL REDUCTION

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Multiscale problems are ubiquitous in science and engineering. Numerical methods that attempt to resolve all scales exhibited by such problems lead to massive discretizations. For example, the μ-finite element analysis of an orthopedic component may lead to finite element meshes with $O\ (10^8)$ elements [1, 2]. For this reason, a multitude of multiscale methods have been introduced to model highly heterogeneous materials without requiring a monolithic spatial discretization. Unfortunately, despite these advances, many numerical multiscale methods remain today computationally intensive. To address this issue, this lecture will present a parametric, nonlinear, projection-based model reduction framework for making multiscale methods not only affordable, but even operable in real-time. At each scale, this computational framework relies on the Proper Orthogonal Decomposition (POD) method based on solution snapshots to lower the dimensionality of the governing equations, and on the Energy Conserving Sampling and Weighting (ECSW) hyper reduction method [3, 4] to reduce the computational complexity associated with evaluation of the resulting nonlinear low-dimensional terms. It efficiently transmits mechanical information between the scales, without incurring high-dimensional operations. It treats periodic boundary conditions at the microscales as linear multipoint constraints, and reduces them via projection onto the span of a basis formed by compressing Lagrange multiplier snapshots using the Singular Value Decomposition. The proposed framework features a two-phase training strategy for the resulting hyper reduced multiscale model. In the first phase, it constructs a microscale hyper reduced model in-situ in order to yield significant speedups, even in a non-parametric setting. In the second phase, it adopts a traditional offline-online training approach to produce a fully reduced multiscale model capable of real-time multiscale simulations. Furthermore, the offline cost of the macroscale training is minimized by using its in-situ counterpart to accelerate the acquisition of solution snapshots. Speedups of several orders of magnitude will be demonstrated during the lecture for several challenging dynamic multiscale problems featuring large deformations, microscale damage, plasticity, and viscoelasticity.

References

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