# Fast Analysis and Reanalysis for Structures with Nonlinear Supports

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## Abstract

A fast reanalysis method for structures with nonlinear supports is developed based on Indirect Factorization Updating (IFU) in this study. The famous Newton-Ralfson's method is employed to solve the nonlinear equation, therefore, the tangent stiffness matrix should be calculated and factorized repeatedly in the iterative process. The nonlinearity of the supports as well as structural modifications will lead to change of tangent stiffness matrix. In order to improve the efficiency of solving process, the IFU method is applied to deal with the change of tangent stiffness matrix. The numerical example shows that the proposed method is effective for structures with nonlinear supports.

**Keywords:** Reanalysis, Nonlinear supports, Indirect factorization updating

## 1 Introduction

Nonlinear reanalysis is one of the most challenging problem in reanalysis research area. Some achievements about nonlinear reanalysis have been gained in recent decades. Kirsch [[1]] developed a general reanalysis approach – Combined Approximation (CA), which can be used for nonlinear problems. Leu and Tsou [[2]] developed Kirsch's method for nonlinear dynamic analysis of framed structures. Akgün et al. [[3]] extended SMW (Sherman-Morrison-Woodbury) formulas to some nonlinear reanalysis problems. Deng et al. [[4]] developed a pseudoforce method for nonlinear analysis and reanalysis of structural systems. Hurtado [[5]] proposed a method based on Shanks transformation for both linear and nonlinear reanalysis problems. Materna et al. [[6]] proposed a nonlinear reanalysis method based on residual increment approximations.

Generally, nonlinear reanalysis methods are developed based on linear approaches. Because of the high requirement of accuracy, exact reanalysis methods are more suitable for nonlinear problems. Recently, Huang et al. [[7]] proposed an Indirect Factorization Updating (IFU), which is exact and suitable for structures with low-rank modifications. In this study, the IFU method is extended for reanalysis of structures with nonlinear supports.

## 2 Fast initial analysis for structures with nonlinear supports

A brief example for structures with nonlinear supports is shown in Fig. 1a. The equilibrium equation of the structure can be stated as

$$\mathbf{K}_0\mathbf{u} = \mathbf{F}(\mathbf{u}), \tag{1}$$

where, $\mathbf{K}_0$ is the stiffness matrix, $\mathbf{u}$ is the displacement vector, and $\mathbf{F}$ is the load vector, which depends on $\mathbf{u}$.

By using Newton-Ralfson's method, Eq. (1) can be solved by solving

$$\mathbf{K}_T^{(i)}\Delta\mathbf{u}^{(i)} = \Delta\mathbf{F}^{(i)}, \left(i = 0,1, \ldots\right), \tag{2}$$

where, $\mathbf{K}_T^{(i)}$ and $\Delta\mathbf{F}^{(i)}$ are tangent stiffness matrix and residual load vector of i-th iteration, respectively, which are calculated as

$$\mathrm{K}_T^{(i)} = \mathrm{K}_0 - \left(\frac{\partial \mathrm{F}}{\partial \mathbf{u}^{(i-1)}}\right)^T, \tag{3}$$

and

$$\Delta \mathrm{F}^{(i)} = \mathrm{K}_0 \mathbf{u}^{(i-1)} - \mathrm{F}\left(\mathbf{u}^{(i-1)}\right). \tag{4}$$

Assume that the nonlinear supports are applied on several degree of freedoms (DOFs), which are numbered as di (i=1,2,…,s). In this case, only the di-th diagonal member of $\left(\frac{\partial \mathrm{F}}{\partial \mathbf{u}^{(i-1)}}\right)^T$ are non-zero. Therefore, $\left(\frac{\partial \mathrm{F}}{\partial \mathbf{u}^{(i-1)}}\right)^T$ can be expressed as

$$\left(\frac{\partial \mathrm{F}}{\partial \mathbf{u}^{(i-1)}}\right)^T = \begin{bmatrix} -\mathrm{p}_{d_1} & -\mathrm{p}_{d_2} & \cdots & -\mathrm{p}_{d_s} \end{bmatrix} \begin{bmatrix} \mathrm{e}_{d_1} & \mathrm{e}_{d_2} & \cdots & \mathrm{e}_{d_s} \end{bmatrix}^T = \mathrm{P}\mathrm{E}^T, \tag{5}$$

where, pj indicates the j-th column of $\left(\frac{\partial \mathrm{F}}{\partial \mathbf{u}^{(i-1)}}\right)^T$, and

$$\mathrm{e}_j = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}^T, \text{ (1 is the j-th member).} \tag{6}$$

Using SMW formula, Eq. (2) can be solved as

$$\Delta \mathbf{u}^{(i)} = \left(\mathrm{K}_0^{-1} - \mathrm{K}_0^{-1}\mathrm{P}\left(\mathrm{I} + \mathrm{E}^T\mathrm{K}^{-1}\mathrm{P}\right)^{-1}\mathrm{E}^T\mathrm{K}_0^{-1}\right)\Delta \mathrm{F}^{(i)}. \tag{7}$$

Therefore, only $\mathrm{K}_0^{-1}$ (or factorization of $\mathbf{K}_0$) need to be calculated before solving process, and Eq. (7) can be calculated very efficiently.

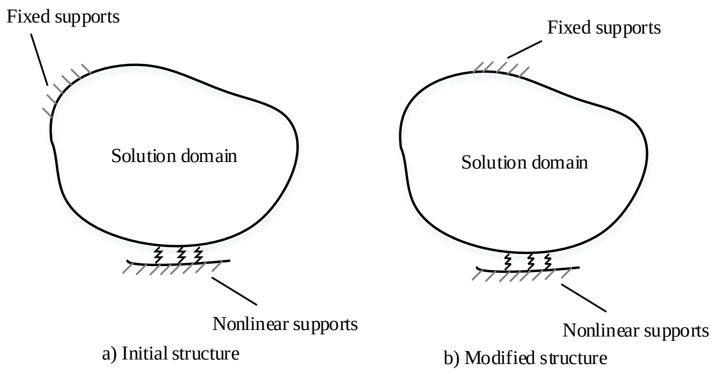



**Fig. 1 A structure with nonlinear supports**

## 3 Fast reanalysis for structures with nonlinear supports

Assume that a local modification (such as change of fixed supports as shown in Fig. 1b) is than applied on the structure, and the equilibrium equation becomes

$$\mathrm{K}\mathrm{u} = \mathrm{F}(\mathrm{u}), \tag{8}$$

and the tangent stiffness matrix in Eq. (2) becomes

$$K_T^{(i)} = K + \left(\frac{\partial F}{\partial u^{(i-1)}}\right)^T. \tag{9}$$

Define

$$K = K_0 + \Delta K, \tag{10}$$

$$\Delta K_T = \Delta K + \left(\frac{\partial F}{\partial u^{(i-1)}}\right)^T. \tag{11}$$

Equation (2) becomes

$$(K_0 + \Delta K_T)\Delta u^{(i)} = \Delta F^{(i)}. \tag{12}$$

In order to obtain an exact solution of Eq. (12) efficiently, the IFU method [[7]] is employed.

## 4 Numerical example

As shown in Fig. 2 is a tow-dimensional beam. Two different work condition is considered: cantilever beam as the initial structure and simply supported beam as the modified structure. A nonlinear support is applied on the middle-bottom of the beam as shown in Fig. 2. The law of the support is

$$f = -kx^3, \tag{13}$$

where, x is the deformation of the support, and

$$k = 1 \times 10^9 N / mm^3. \tag{14}$$

The material parameters are modulus of elasticity $E = 70 \times 10^3 MPa$, and Poisson's ratio $\upsilon = 0.3$. The forces in both Fig. 2a and 2b are linearly increased from 0 to 10N. The analysis results are shown in Fig. 3, and the comparisons of computational efficiency are shown in Table 1. Fig. 3 shows that the results of the SMW formula based initial analysis and the IFU based reanalysis are almost the same as the ones of the full analysis. From Table 1, it appears that the computational efficiency of the SMW formula based initial analysis and the IFU based reanalysis are higher than that of full analysis.

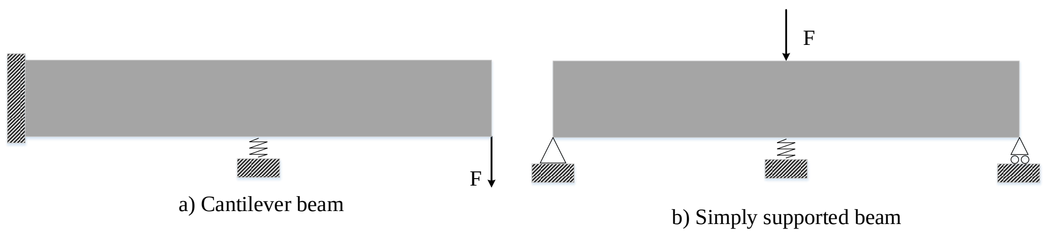
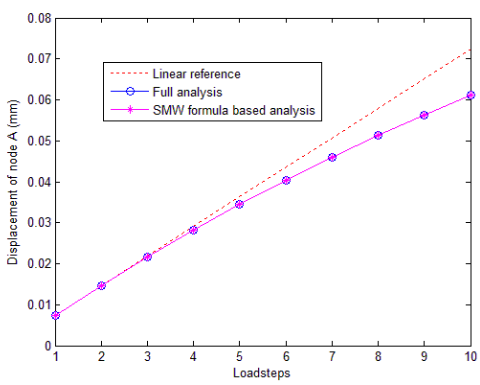


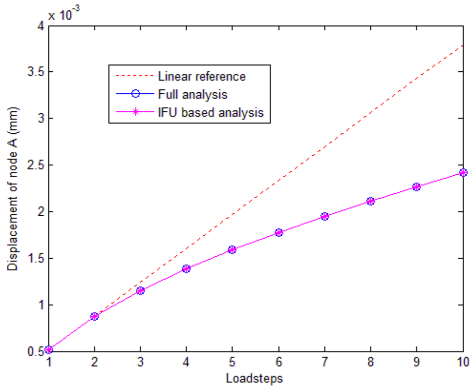

**Fig. 2 Models of the two-dimensional beam**

**Table 1 Comparison of computation efficiency**

| Models | Analysis methods | Computational cost (s) |
|---|---|---|
| Initial analysis | Full analysis | 11.3176 |
| | SMW formula based analysis | 2.2826 |
| Reanalysis | Full analysis | 11.4454 |
| | IFU based analysis | 6.2782 |



a) Results of the cantilever beam



b) Results of the simply supported beam

**Fig. 3 Analysis results of the two-dimensional beam**

## 5 Summary

This study developed a fast analysis and reanalysis method for structures with nonlinear supports. The SMW formula is applied in initial analysis, and the IFU method is adopted in reanalysis. The numerical example shows that the computational efficiency of the proposed fast analysis and reanalysis is high than that of full analysis, and exact solutions can still be guaranteed.

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