A group of benchmark problems for local stress analysis of real cantilever plate using high accuracy and high performance BEM

*†**Z.H. Yao, and X.P. Zheng**

School of Aerospace Engineering, Tsinghua University, China

*Presenting author: demyzh@tsinghua.edu.cn
†Corresponding author: demyzh@tsinghua.edu.cn

Abstract

Since 1979 to enter the field of BEM, the first author regarded it as an important supplement of the FEM. In more than 37 years he has firsthand experienced its research progress in the depth and breadth, but in engineering applications BEM is increasingly marginalized. To explore the reason, it is found that in BEM literature, it is always relied on the comparison of the results with analytical solution or FEM results to verify its accuracy. To let BEM to be a necessary supplement, the verification of BEM results should be independent from FEM, and its advantages of high accuracy should be given full play. In recent years he has presented a new high-accuracy BEM (HABEM), and the authors are developing a new high-performance BEM (HPBEM).

During the analysis of practical engineering or scientific problems, a mathematical model should be formulated at first; for example, for the elastic stress analysis, a continuum model of elasticity can be formulated. During this stage some modeling errors have been introduced. Such error could be found out by the comparison of the analytical solution and the corresponding experimental results. For the complex practical problems, the numerical methods have to be applied, the continuum model should be converted into discretized model. During this stage the discretization error has been introduced. Based on the discretized model, for example a BE model, the integration of kernel function shape function product on each element should be calculated, to obtain the coefficients of the resulted system of linear algebraic equations, and then the resulted equation system should be solved by efficient solver. During this stage, some errors will also be introduced, including the integration error and the error of equation solver, which belong to the calculation error.

The new type of HABEM based on the Rizzo type boundary integral equation with singular kernel function, the main calculation error is integration error, provided the high-accuracy direct solver as Gaussian elimination was applied. The key point of the HABEM is to introduce the error indicator, which is based on the discontinuity of the boundary effective stress plot between adjacent elements, correspondingly all elements should be displacement continuous. In order to reduce the discretization error, the mesh refinement is guided by the error indicator, which should be correlated with the convergence degree of interested variables, and even more difficult to be converged. In the HPBEM, fast algorithm should be introduced into the HABEM, correspondingly, the additional calculation error of the fast algorithm and corresponding iterative solver should be considered.

As an application of the HABEM and HPBEM, the local stress analysis of real beam, plate and shell structure is investigated. Beam, plate and shell structures are widely applied in engineering, and the theory of beam, plate and shell, and the corresponding FE analysis have been successfully applied in the design and analysis of such structures. But if the maximum stress is appeared at the end of beam or on the boundary of plate and shell, the obtained stresses will be inaccurate, because the basic assumptions of the beam, plate and shell theory are not satisfied at such region. However the local stress concentration is very important for the strength of such structures, especially for the structures made of high-strength brittle material. To obtain the accurate local stress in such structures, the computational model should preserve practical geometric details and practical boundary conditions at the beam end and the boundary of plate and shell structures, therefore it should be 3D (or 2D for thin-plate beam and other simplified cases) analysis together with the peripheral structure or the foundation.

In this paper a group of benchmark problems for the local stress analysis of real cantilever plate using HABEM and HPBEM is presented. In HABEM, the main calculation error is integration error. To guarantee the integration accuracy, the improved equal accuracy Gaussian quadrature is applied for the regular, including nearly singular integral and weakly singular integral, and the Cauchy principal integral is determined indirectly using the special solution of rigid body displacements. To reduce the discretization error, the error indicator based on the discontinuity of the boundary effective stress plot between adjacent elements is applied. The converged results with satisfied accuracy for a kind of real cantilever square plate, the length to thickness ratio up to 30, are presented. In order to reduce the number of degrees of freedom, symmetry of the problem has been applied. For thinner cases of real cantilever plate, it is difficult to be computed on a laptop using HABEM, HPBEM with ACA algorithm has been applied. For the investigation of the additional calculation error, the results of HPBEM should be compared with the available results of HABEM. Based on such investigation, HPBEM can be applied to deal with thinner real cantilever plates, some results are presented up to the length to thickness ratio of 100.

Keywords: Boundary element method; high-accuracy BEM; high-performance BEM; real beam, plate and shell structure; local stress analysis; benchmark problem.