On more efficient and flexible peridynamics-based computational tools

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Abstract

This document describes two newly developed computational tools that greatly improve the efficiency and the applicability of peridynamics-based software to computational mechanics.
The first contribution is concerned with a new, flexible and accurate way to couple peridynamic grids to FEM meshes. The coupling produces a new computational method that applies FEM or PD discretisations where required and couples them in a way that keeps the advantages of both approaches and avoids their pitfalls.
The second contribution is about a simple and accurate implicit solution of static problems in which the usual prototype microelastic brittle material (elasto-brittle) model is adopted.
Finally a computational example is presented to show the potentialities of the new computational techniques.

Keywords: Peridynamics, FEM, coupling, statics, implicit solution.

1. Introduction

Peridynamics (PD) is a family of non-local continuum theories proposed in a series of papers since the year 2000 [1]-[4]. Similar approaches were also proposed previously by other authors [5]-[6]. PD is an integral formulation that does not make use spatial derivatives. For that reason computational methods based on it do not encounter any difficulty if they face the problem of crack initiation, propagation and bifurcation. PD-based computational tools have been successfully applied to the solution of dynamic crack propagation in brittle materials [7]-[10]. These simulations can easily describe phenomena involving many coexisting and interacting cracks, complex crack patterns and fragmentation in 3D conditions in a way which is not currently possible for most computational methods based on the more common local theory of continuum mechanics.
However some aspects of PD-based computational tools require improvements:
- given the non-local framework in which they are set up PD-based computational methods are usually more computationally expensive than their counterpart based on local approaches, such as FEM. In a PD code a point interacts with all material points within its horizon, which can be many, whereas a FEM node interacts only with the nodes of the adjacent elements. It would be very convenient to couple peridynamic grids to FEM meshes so that a numerical method, that possesses the advantages of both computational techniques and avoids their pitfalls, can be obtained [11]-[16]. A small part of the model, where cracks are more likely, would be usually discretized with a PD grid, whereas the rest would be modelled with the more efficient FEM.
- most PD works on fragile fracture adopt the prototype microelastic brittle material (PMB) [11] which assumes that the relation between bond stretch and bond force is linear up to a maximum stretch value, beyond which the bond force drops to zero (elasto-brittle material). The PMB model can be easily introduced in explicit dynamic codes, but its discontinuous definition makes its use in implicit static codes more problematic. This is probably one of the reasons why static crack propagation phenomena, such as fatigue crack propagation, are not commonly addressed by the PD community [17], [18].

This document describes two newly developed computational tools that greatly improve the efficiency and the applicability of peridynamics-based software to computational mechanics. Section 2 presents the first contribution, concerned with a new, flexible and accurate way to couple peridynamic grids to FEM meshes. Section 3 introduces the second contribution, about a simple and accurate way to perform an implicit solution of static problems in which the usual PMB model is adopted. Section 4 presents a computational example and finally section 5 provides the conclusions.

2. A new PD-FEM coupling

The new coupling method makes possible the coupling between FEM and PD models with different grid spacings. Let us consider a 1-d coupled problem, as shown in fig. 2. In this case

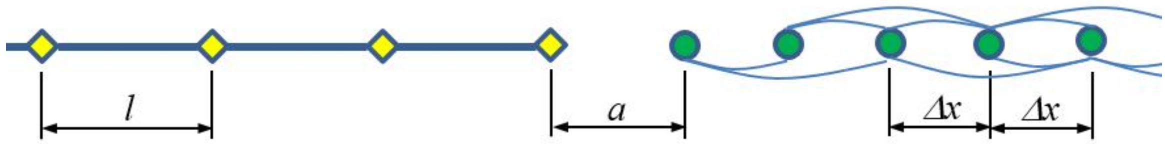

Figure 1: portions of a 1-d model, diamonds are FEM nodes, circles PD nodes.

$l \neq \Delta x$. The two portions of the model have to be connected so that they represent a unique physical system. The distance a between the two portions is taken to be smaller than both, the element length l, and the horizon δ, but bigger than the PD grid spacing Δx. In order to perform the coupling we introduce a fictitious FEM node, node 1, at a distance l from the last real FEM node, and two fictitious PD nodes, nodes 2 and 3, at a distance respectively of Δx and $2\Delta x$ from the first real PD node, as shown in figure 2. In the case of figure 2 $\delta=2\Delta x$.

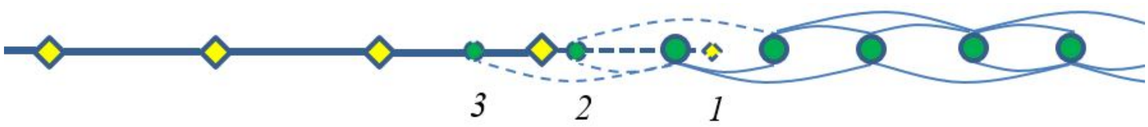

Figure 2: fictitious nodes: 1, 2, 3; dashed lines are coupling element and bonds.

Then the coupling elements and bonds are introduced as shown by dashed lines in figure 2**.** The displacements of the fictitious nodes are defined by interpolation of the real node displacements and are then used to define the forces in the coupling element/bonds, which are applied only to real nodes.

3. Implicit solution for static problems

The PMB constitutive law assumes that the relation between bond stretch and bond force is linear up to a maximum stretch value, beyond which the bond force drops to zero. Therefore it is a discontinuous constitutive law which prevents a straightforward application of standard Newton-Raphson algorithms. A PD model, in which geometric non-linearity is negligible,

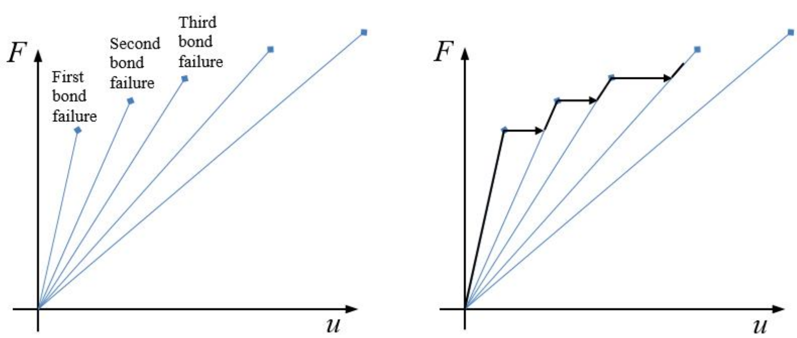

Figure 3: piecewise linear behavior of a PMB-PD structure.

under static monotonic loads can be considered as a piecewise linear system the stiffness of which changes whenever one (or more) bond(s) breaks. The general behavior of such a system can be represented as in figure 3. Initially the structure is in an undamaged condition, and therefore reacts to the applied load with its initial stiffness. As the load grows so does the stretch in the bonds up to the point where (generically) one bond fails. After the failure of the first bond the stiffness of the structure is reduced and the process is repeated up to a complete failure of the structure. The simple approach described in figure 3 can be easily implemented in a PD code.

4. Example

Figure 4 shows the crack path computed in a static way in a coupled model.

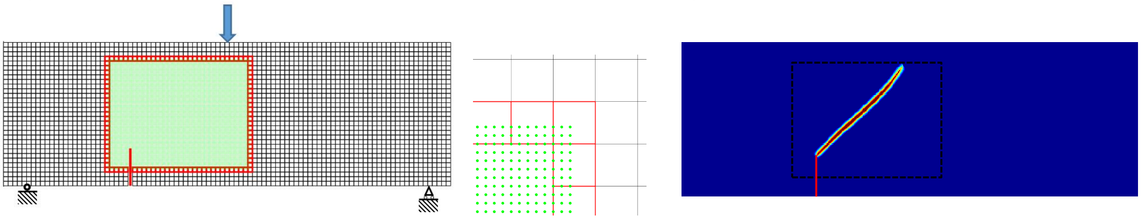

Figure 4: crack-path computed with an implicit static algorithm in a coupled model.

5. Conclusions

The paper presents two ways to increase efficiency and flexibility of PD-based codes for computational mechanics applications: a new way to couple PD to FEM and a new implicit solution procedure.

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