3D Nonlinear dynamical analysis of cable-stayed offshore structures

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Abstract

The nonlinear vibrations of the moored floating structures under horizontal sinusoidal excitations are studied in three dimensions. Four mooring lines are connected to the floating structure and fixed to the sea bed. The nonlinear equations of motions of the mooring lines are formulated using the cable elements formulated based on the extended Hamilton principle. The floating structure is considered as a rigid body with six degrees of freedom. Then the equations of motion of the floating structure and mooring lines are formulated through their connection conditions. In the last, the equations of motion of the whole structure are analyzed numerically. The influences of different sag-to-span ratio and inclined angle of the mooring cables on the responses of the floating structure are studied.

Keywords: Floating structure, mooring lines, cable elements, connection conditions

Introduction

The moored floating structures can find their applications in ocean engineering to exploit marine resources such as oil, gas and minerals. It consists of the floating platform and mooring cables. If the floating body is subjected to external excitations, the movements of floating body can induce the geometry change of mooring lines. The geometric nonlinearity of the mooring lines plays an important role in the dynamical analysis due to their flexibility. Therefore accurate modeling of mooring cables is necessary for the vibration analysis of the whole structure. Some researches simplified the mooring lines as a linear spring [1, 2, 3] to support the floating body for convenient and efficient analysis. The constant stiffness of the spring is derived and added to the linear stiffness matrix of the floating body. The mooring lines were also modeled as nonlinear spring [4, 5, 6, 7]. The restoring forces from mooring lines are determined on the static analysis of catenary cables with the assumption that the floating body moves slowly. However, the above two methods cannot reflect the real behavior of the cable. Therefore, fully modeling of cable is required. The lumped mass model [8, 9, 10, 11] or the bar element [12, 13] was used for the vibration analysis of mooring lines. The cables were modeled using the finite element method based on the principle of minimum energy including strain energy due to bending and torsion [14, 15], in which the equations of motions of the mooring lines and those of floating body were solved separately and iteratively.

In this paper, the nonlinear vibrations of three-dimensional floating structure and mooring system under the horizontal sinusoidal excitations are studied. The nonlinear equations of motions of the mooring lines are formulated using the 3D cable elements formulated based on the extended Hamilton principle [16, 17]. The cable element is simplified as a flexible tension member without considering its bending and torsion stiffness because of the extremely large ratio of length over cross-section dimension. The floating body is considered as a rigid body with six degrees of freedom, i.e., three translational displacements and three rotational displacements. The equations of motions of both the floating body and mooring system are formulated through their connection conditions and they are solved numerically as a whole.

Problem Statement

Consider the floating structure and mooring system as shown in Fig. 1. It consists of the floating body and four catenary mooring lines C_1, C_2, C_3 and C_4. The floating body and mooring lines are connected through four nodes *A*, *B*, *C* and *D*. *O* is the centroid of the floating body. The other ends of the mooring lines are fixed on the sea bed. w_a, w_b and w_c are the length, height and width of the floating body, respectively; *h* is the depth of sea; h_s is the submerged height of the floating body in the sea in static state. The top view and side view of the three-dimensional floating system are shown in Fig. 2. The mooring lines C_1, C_2 and C_3, C_4 are symmetric about the *y*-axis in the plane x_1Oy and x_2Oy, respectively. θ, *l* and *d* are the inclined angle, inclined length and initial sag of the mooring line, respectively. w_l is the distance between the nodes *A* and *B*.

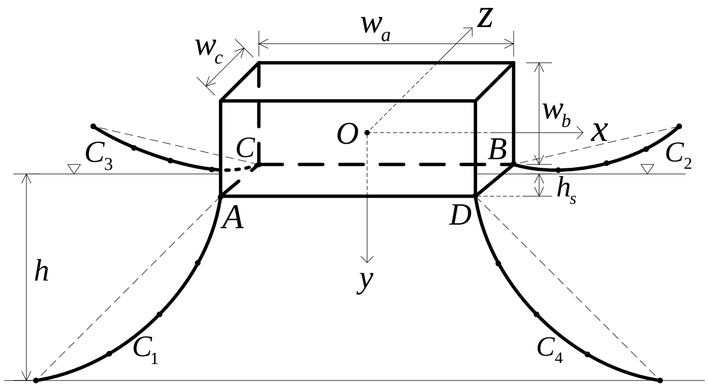
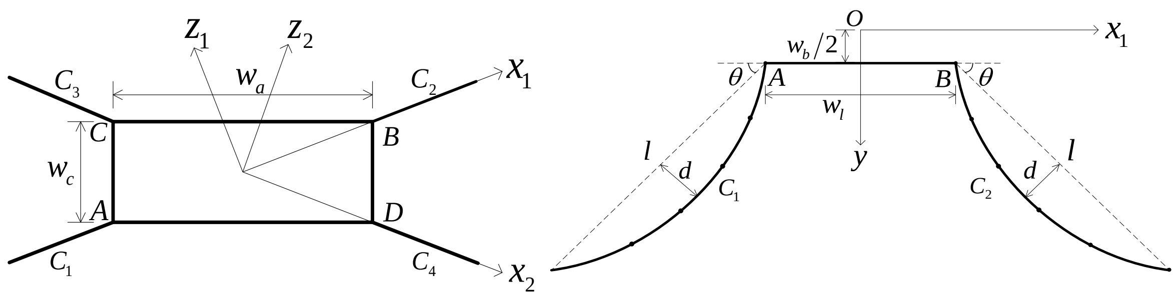


Figure 1. Configuration of the three-dimensional floating system



(a) (b)

Figure 2. (a) Top view (b) Side view of the three-dimensional floating system

The exact catenary profile of the mooring cables in static state is governed by the initial pretension and self-weight of cable. The catenary profile of cable is needed for given sag-to-span ratio *d*/*l*.

The submerged height h_s of the floating body in static state is obtained as follows referring to Fig. 3. F_{A1}, F_{A2}, F_{A3}, F_{B1}, F_{B2}, F_{B3}, F_{C1}, F_{C2}, F_{C3}, F_{D1}, F_{D2}, F_{D3} are the components of cable pretensions at nodes *A*, *B*, *C*, and *D* in *x*, *y*, *z* axes, respectively, and

$$F_f + F_{A2} + F_{B2} + F_{C2} + F_{D2} = Mg \tag{1}$$

where *M* is the mass of the floating body; F_f is the buoyancy of the floating body in the sea and expressed by $F_f = \rho_s g h_s w_a w_c$, in which ρ_s is the density of sea water.

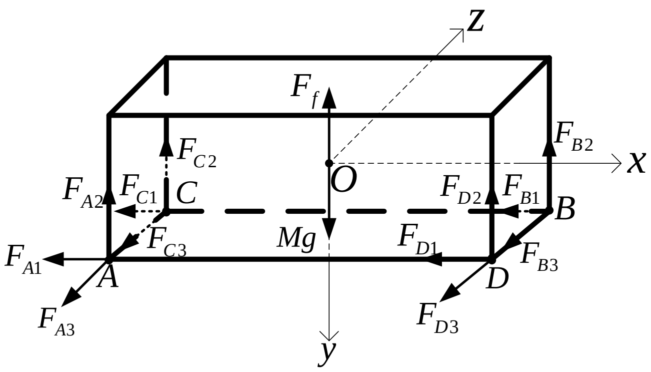


Figure 3. Static equilibrium of the floating body

Nonlinear Dynamical Analysis of the Moored Floating System

The equations of motions of the mooring lines and the floating body are derived individually first and then they are assembled together through their connection conditions.

Finite Element Formulation for the Dynamics of Cable

The equations of motion of the mooring cables are formulated with finite element. The cable element is formulated based on the extended Hamilton principle in the following.

Consider the differential cable element in dynamical state as shown in Fig.4. Let ds and ds' denote the length of cable element in static state and dynamical state, respectively. u, v and w are the dynamical displacements in x, y and z directions, respectively. Then

$$\begin{aligned}(ds)^2 &= (dx)^2 + (dy)^2 \\ (ds')^2 &= (dx+du)^2 + (dy+dv)^2 + (dw)^2\end{aligned} \tag{2}$$

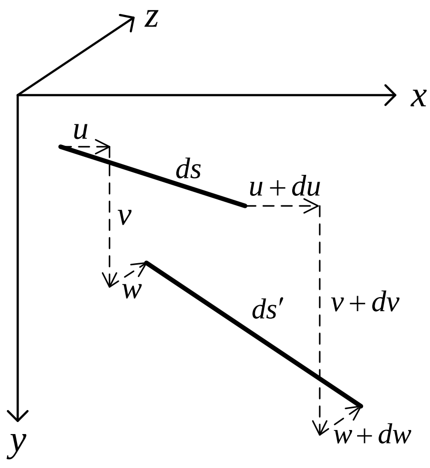


Figure 4. Differential cable element in dynamical state

Retaining the terms up to second order, the axial strain of cable is given by

$$\varepsilon = \frac{ds' - ds}{ds} = x'u' + y'v' + \frac{1}{2}(u')^2 + \frac{1}{2}(v')^2 + \frac{1}{2}(w')^2 \tag{3}$$

where $(\ )' \equiv \partial/\partial s$. Taking the derivatives of ε with respect to u', v' and w', respectively, it gives

$$\frac{\partial\varepsilon}{\partial u'} = x' + u', \quad \frac{\partial\varepsilon}{\partial v'} = y' + v', \quad \frac{\partial\varepsilon}{\partial w'} = w' \tag{4}$$

From which we have the following variation of ε,

$$\delta\varepsilon = \left(x' + u'\right)\delta u' + \left(y' + v'\right)\delta v' + w'\delta w' \tag{5}$$

The variation of potential energy relative to the unstressed state is given by

$$\delta\Pi = \int_0^L \left(T + EA\varepsilon\right)\delta\varepsilon ds \tag{6}$$

The variation of kinetic energy is given by

$$\delta K = -\int_0^L \rho A\left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right)ds = 0 \tag{7}$$

The variation of virtual work associated with gravity and damping force is given by

$$\delta W = \int_0^L \left(\rho Ag\delta v + f_1\delta u + f_2\delta v + f_3\delta w - c_1\dot{u}\delta u - c_2\dot{v}\delta v - c_3\dot{w}\delta w\right)ds \tag{8}$$

where f_1, f_2 and f_3 are the external distributed loads per unit length along x, y and z directions, respectively; c_1 , c_2 and c_3 are the damping coefficient per unit length along x, y and z directions, respectively.

Substituting Eq. (5) into Eq. (6), the variation of potential energy can be expressed as

$$\delta\Pi = \int_0^L \left\{EA\varepsilon\left[\delta u'\left(x' + u'\right) + \delta v'\left(y' + v'\right) + \delta w'w'\right] + T\left[\delta u'\left(x' + u'\right) + \delta v'\left(y' + v'\right) + \delta w'w'\right]\right\}ds \tag{9}$$

With Eqs. (7)-(9) and applying the static equilibrium equations of cable element into $\int_{t_1}^{t_2}\left(\delta K - \delta\Pi + \delta W\right)dt = 0$, we have

$$\int_{t_1}^{t_2}\left(\delta K - \delta\Pi' + \delta W'\right)dt = 0 \tag{10}$$

where Π' is the potential energy relative to static equilibrium state and W' is the virtual work done by the external forces from static equilibrium state and done by the damping forces.

The variation of potential energy relative to the static state is expressed as

$$\delta\Pi' = \int_0^L \left\{EA\varepsilon\left[\delta u'\left(x' + u'\right) + \delta v'\left(y' + v'\right) + \delta w'w'\right] + T\left(\delta u'u' + \delta v'v' + \delta w'w'\right)\right\}ds \tag{11}$$

and the variation of the virtual work done from the state of initial profile is expressed as

$$\delta W' = \int_0^L \left(f_1\delta u + f_2\delta v + f_3\delta w - c_1\dot{u}\delta u - c_2\dot{v}\delta v - c_3\dot{w}\delta w\right)ds \tag{12}$$

Let $\mathbf{u}^e = \left\{u^e, v^e, w^e\right\}^T = \mathbf{N}^e\mathbf{d}^e$ and $\mathbf{d}^e = \left\{u^i, v^i, w^i, u^j, v^j, w^j\right\}^T$ for element e, where i and j are two node numbers of element e and $\mathbf{d}^e$ is the displacement vector of element e in local coordinate system O-x_1yz_1 or O-x_2yz_2 of the cable.

The linear shape function of element e is given as follows,

$$\mathbf{N}^e = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 \end{bmatrix} \tag{13}$$

where $N_1 = 1 - s/l^e$, $N_2 = s/l^e$ and l^e is the length of element e. Denoting $\mathbf{D}^e = d\mathbf{N}^e/ds$ and $\mathbf{x}^e = \{x', y', 0\}^T$ for element e, the strain of element e or ε^e can be expressed as

$$\begin{aligned} \varepsilon^e &= \mathbf{x}^{eT}\mathbf{u}'^{eT} + \frac{1}{2}\mathbf{u}'^{eT}\mathbf{u}'^e = \mathbf{x}^{eT}\mathbf{D}^e\mathbf{d}^e + \frac{1}{2}\mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e\mathbf{d}^e \\ &= \left(\mathbf{x}^{eT}\mathbf{D}^e + \frac{1}{2}\mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e\right)\mathbf{d}^e = \mathbf{B}_1^e\mathbf{d}^e \end{aligned} \tag{14}$$

where $\mathbf{B}_1^e = \mathbf{x}^{eT}\mathbf{D}^e + \frac{1}{2}\mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e$. The variation of strain $\delta\varepsilon^e$ is obtained to be

$$\delta\varepsilon^e = \left(\mathbf{x}^{eT}\mathbf{D}^e + \frac{1}{2}\mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e\right)\delta\mathbf{d}^e = \mathbf{B}_2^e\delta\mathbf{d}^e \tag{15}$$

where $\mathbf{B}_2^e = \mathbf{x}^{eT}\mathbf{D}^e + \mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e$. Therefore, the variation of the potential energy is given by

$$\begin{aligned} \delta\Pi' &= \sum_{e=1}^{N^e}\int_0^{l^e}\left[EA\left(\delta\varepsilon^e\right)^T\varepsilon^e + T^e\delta\mathbf{u}'^{eT}\mathbf{u}'^e\right]ds \\ &= \sum_{e=1}^{N^e}\int_o^{l^e}\left[EA\delta\mathbf{d}^{eT}\mathbf{B}_2^T\mathbf{B}_1\mathbf{d}^e + T^e\delta\mathbf{d}^{eT}\mathbf{D}^{eT}\mathbf{D}^e\mathbf{d}^e\right]ds \\ &= \sum_{e=1}^{N^e}\delta\boldsymbol{d}^{eT}\int_0^{l^e}\left[EA\mathbf{B}_2^T\mathbf{B}_1 + T^e\mathbf{D}^{eT}\mathbf{D}^e\right]\mathbf{d}^e ds \end{aligned} \tag{16}$$

where N^e is the total number of elements. The stiffness matrix of element e is then obtained to be

$$\mathbf{k}^e = \int_0^{l_e}\left[EA\mathbf{B}_2^T\mathbf{B}_1 + T^e\mathbf{D}^{eT}\mathbf{D}^e\right]ds \tag{17}$$

The variation of kinetic energy of the whole system is given by

$$\delta K = -\sum_{e=1}^{N_e}\int_0^{l^e}\rho A\delta\mathbf{u}^{eT}\ddot{\mathbf{u}}^e ds = -\sum_{e=1}^{N_e}\delta d^{eT}\int_0^{l^e}\rho A\mathbf{N}^{eT}\mathbf{N}^e\ddot{\mathbf{d}}^e ds \tag{18}$$

Thus the mass matrix of element e is obtained to be

$$\mathbf{m}^e = \int_0^{l^e}\rho A\mathbf{N}^{eT}\mathbf{N}^e ds = \frac{\rho A l^e}{6}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix} \tag{19}$$

The lumped mass matrix $\mathbf{m}_l^e$ of element e is

$$\mathbf{m}_l^e = \frac{\rho A l^e}{2}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

In the following numerical analysis, the lumped mass matrix of element e is used for simplicity.

The equations of motion for the element e in local coordinate systems $O\text{-}x_1yz_1$ and $O\text{-}x_2yz_2$ are given as follows.

$$m_l^e \ddot{\mathbf{d}}^e + \mathbf{c}^e \dot{\mathbf{d}}^e + \mathbf{k}^e(\mathbf{d}^e)\mathbf{d}^e = \mathbf{f}^e \tag{21}$$

where $\mathbf{c}^e$ and $\mathbf{f}^e$ are the damping matrix and force vector of element e.

Denote $\mathbf{d}_g^e = \left\{u_g^i, v_g^i, w_g^i, u_g^j, v_g^j, w_g^j\right\}^T$ for element e, which is the displacement vector of element e in global coordinate system $O\text{-}xyz$ of the cable. Using the transformation matrix $\mathbf{T}$ between the local and global coordinate systems and the relationship $\mathbf{d}^e = \mathbf{T}\mathbf{d}_g^e$, Eq. (21) becomes

$$\mathbf{M}_g^e \ddot{\mathbf{d}}_g^e + \mathbf{C}_g^e \dot{\mathbf{d}}_g^e + \mathbf{K}_g^e(\mathbf{d}_g^e)\mathbf{d}_g^e = \mathbf{F}_g^e \tag{22}$$

where $\mathbf{M}_g^e = \mathbf{T}^T\mathbf{M}_l^e\mathbf{T}$, $\mathbf{C}_g^e = \mathbf{T}^T\mathbf{C}_l^e\mathbf{T}$, $\mathbf{K}_g^e = \mathbf{T}^T\mathbf{K}_l^e\mathbf{T}$, and $\mathbf{F}_g^e = \mathbf{T}^T\mathbf{F}_l^e$, which are the stiffness matrix, damping matrix, stiffness matrix and force vector of element e in global coordinate system $O\text{-}xyz$ of the cable.

The equations of motion of the mooring cables are then formulated as follows using the above cable element when ρ is replaced by $\rho - \rho_s$.

$$\mathbf{M}_m\ddot{\mathbf{U}}_m + \mathbf{C}_m\dot{\mathbf{U}}_m + \mathbf{K}_m(\mathbf{U}_m)\mathbf{U}_m = 0 \tag{23}$$

where the subscript m denotes the number of mooring lines and $m = 1$, 2, 3 or 4 for this structure. $\mathbf{U}_m$ denotes the global displacement vector of the mth mooring cable; $\mathbf{K}_m(\mathbf{U}_m)$ denotes the global stiffness matrix of the mth mooring cable; $\mathbf{M}_m$ denotes the global mass matrix of the mth mooring cable; $\mathbf{C}_m$ denotes the global damping matrix of the mth mooring cable; and $\mathbf{F}_m$ denotes the force vector of the mth mooring cable.

Dynamics of the Floating Body

The floating body is considered as a rigid body with six degrees of freedom, which are displacements u_f, v_f, w_f along x, y, z axes and rotations α, β, γ in xOy, xOz, yOz planes, respectively. The equations of motion of the floating body are given as follows referring to Fig. 5.

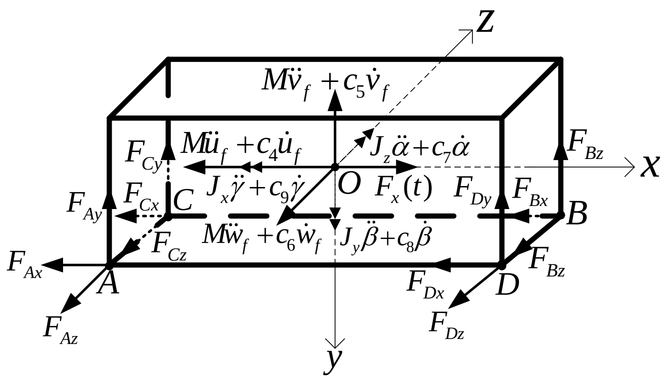


Figure 5. Forces applied on floating body

$$\sum F_x = 0: \ M\ddot{u}_f + c_4\dot{u}_f + F_{Ax} + F_{Bx} + F_{Cx} + F_{Dx} = F_x(t) \tag{24}$$

$$\sum F_y = 0: \ M\ddot{v}_f + c_5\dot{v}_f + F_{Ay} + F_{By} + F_{Cy} + F_{Dy} + F_b = 0 \tag{25}$$

$$\sum F_z = 0: \ M\ddot{w}_f + c_6\dot{w}_f + F_{Az} + F_{Bz} + F_{Cz} + F_{Dz} = 0 \tag{26}$$

$$\begin{aligned} \sum M_z = 0: J_z\ddot{\alpha} + c_7\dot{\alpha} + F_{Ax}\frac{w_b}{2} + F_{Bx}\frac{w_b}{2} + F_{Ay}\frac{w_a}{2} - F_{By}\frac{w_a}{2} \\ + F_{Cx}\frac{w_b}{2} + F_{Dx}\frac{w_b}{2} + F_{Cy}\frac{w_a}{2} - F_{Dy}\frac{w_a}{2} + F_{b1}\frac{2w_a}{3} = 0 \end{aligned} \tag{27}$$

$$\begin{aligned} \sum M_y = 0: J_y\ddot{\beta} + c_8\dot{\beta} + F_{Ax}\frac{w_c}{2} - F_{Bx}\frac{w_c}{2} - F_{Az}\frac{w_a}{2} + F_{Bz}\frac{w_a}{2} \\ - F_{Cx}\frac{w_c}{2} + F_{Dx}\frac{w_c}{2} - F_{Cz}\frac{w_a}{2} + F_{Dz}\frac{w_a}{2} = 0 \end{aligned} \tag{28}$$

$$\begin{aligned} \sum M_x = 0: J_x\ddot{\gamma} + c_9\dot{\gamma} + F_{Ay}\frac{w_c}{2} - F_{By}\frac{w_c}{2} + F_{Az}\frac{w_b}{2} + F_{Bz}\frac{w_b}{2} \\ - F_{Cy}\frac{w_c}{2} + F_{Dy}\frac{w_c}{2} + F_{Cz}\frac{w_b}{2} + F_{Dz}\frac{w_b}{2} + F_{b2}\frac{2w_c}{3} = 0 \end{aligned} \tag{29}$$

where $F_x(t)$ is the external force applied on floating body in x axis; J_z, J_y and J_x are the moment of inertia of the floating body in xOy, xOz and yOz planes, respectively; F_{Ax}, F_{Ay}, F_{Az}, F_{Bx}, F_{By}, F_{Bz}, F_{Cx}, F_{Cy}, F_{Cz}, F_{Dx}, F_{Dy}, F_{Dz} are the dynamical tensions of the cable at node A, B, C, and D in x, y, z axes, respectively, which are induced by the displacements of the elements of mooring lines connected to floating body; F_b, F_{b1} and F_{b2} are the dynamical buoyancy of the floating body due to the change of submerged volume of the floating body, which are expressed as

$$\begin{aligned} F_b &= \rho_s g w_a w_c v_f \\ F_{b1} &= \frac{1}{8}\rho_s g w_a^2 w_c \alpha \\ F_{b2} &= \frac{1}{8}\rho_s g w_a w_c^2 \gamma \end{aligned} \tag{30}$$

Connection Conditions

In order to formulate the equations of motion of the mooring cables and the floating body as a whole, the connection conditions between the mooring lines and floating body are needed. Their relationships are

$$u_A = u + \frac{w_b}{2}\alpha + \frac{w_c}{2}\beta,\ v_A = v + \frac{w_a}{2}\alpha + \frac{w_c}{2}\gamma,\ w_A = w - \frac{w_a}{2}\beta + \frac{w_b}{2}\gamma \tag{31}$$

$$u_B = u + \frac{w_b}{2}\alpha - \frac{w_c}{2}\beta,\ v_B = v - \frac{w_a}{2}\alpha - \frac{w_c}{2}\gamma,\ w_B = w + \frac{w_a}{2}\beta + \frac{w_b}{2}\gamma \tag{32}$$

$$u_C = u + \frac{w_b}{2}\alpha - \frac{w_c}{2}\beta,\ v_C = v + \frac{w_a}{2}\alpha - \frac{w_c}{2}\gamma,\ w_C = w - \frac{w_a}{2}\beta + \frac{w_b}{2}\gamma \tag{33}$$

$$u_D = u + \frac{w_b}{2}\alpha + \frac{w_c}{2}\beta,\ v_D = v - \frac{w_a}{2}\alpha + \frac{w_c}{2}\gamma,\ w_D = w + \frac{w_a}{2}\beta + \frac{w_b}{2}\gamma \tag{34}$$

where u_A, v_A, w_A, u_B, v_B, w_B, u_C, v_C, w_C, u_D, v_D, w_D are the displacements of the nodes *A*, *B*, *C*, and *D* in *x*, *y*, *z* axes, respectively. Then the equations of motion about the nodes A, B, C and D in Eq. (23) are removed and replaced by Eqs. (24)-(29) using the connections conditions given by Eqs. (31)-(34). The variables of displacements related to nodes A, B, C and D in other equations of motion in Eq. (23) are also expressed by Eqs. (31)-(34). Thus the final equations of motion of the whole system are obtained to be

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}(\mathbf{U})\mathbf{U} = \mathbf{F} \tag{35}$$

where **U** is the global displacement vector of the whole system; **K**(**U**) is the global stiffness matrix of the whole system; **C** is the Rayleigh damping matrix.

Numerical example

Consider the mooring cables and floating body with their parameter values shown in Table 1 and Table 2, respectively. The density of sea water is $\rho_s = 1.025\times10^3$ kg/m^3.

Table 1. Properties of mooring cables

Parameter	Value
Young's modulus E (N/m^2)	2×10^{11}
Diameter D (m)	0.1
Mass density ρ (kg/m^3)	8×10^3
Damping ratio ξ	0.03
Sea depth h (m)	100
Inclined angle θ (degree)	50
Sag-to-span ratio d/l	1/60

Table 2. Properties of floating body

Parameter	Value
Length w_a (m)	26
Height w_b (m)	5
Width w_c (m)	10
Mass M (kg)	1.2×10^5

The submerged height of the floating body in static state is calculated with Eq. (1) to be $h_s = 0.896$ m, Consider that the floating body is subjected to the sinusoidal force $F_x(t) = \mathrm{A}\sin(\omega t)$ with amplitude A being 10^5 N and $\omega = 1.6$ rad/s. Each mooring line is divided into 11 elements. The system starts to move from static state. The time history of the responses of the floating body and the maximum cable tensile force is shown in Fig. 6.

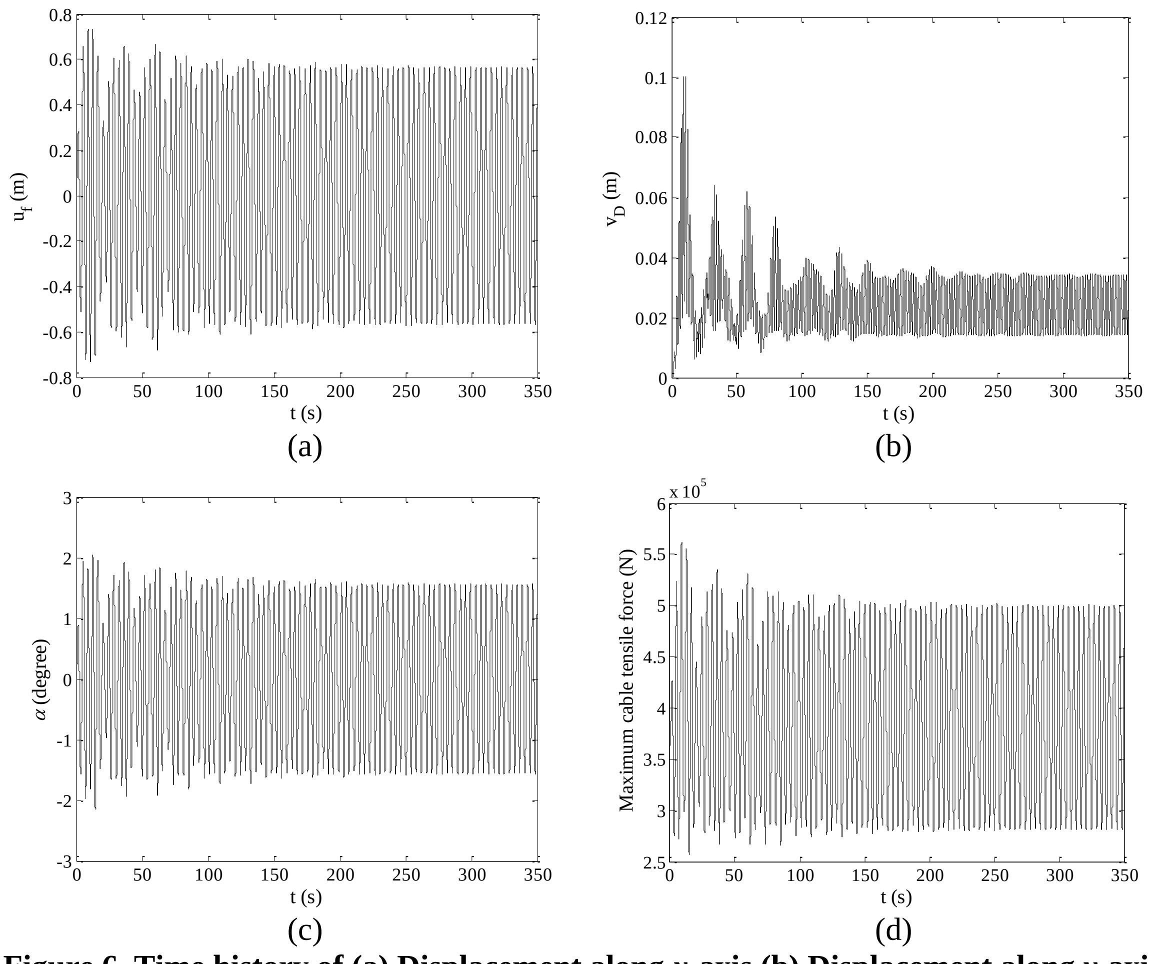


Figure 6. Time history of (a) Displacement along *x*-axis (b) Displacement along *y*-axis (c) Rotational angle in *xOy* plane (d) Maximum tensile force in cable with $d/l = 1/60$ and $\theta = 50^\circ$

When the initial inclined angle equals 50°, the response amplitudes of the floating body and the maximum tensile force in the cable at steady state are presented in Fig. 7 for different sag-to-span ratios of cable. It is seen that the displacements along *x*-axis, y-axis and the rotational angle in *xOy* plane of floating body decreases obviously as the sag-to-span ratio decreases from 1/45 to 1/80. This means that the displacements along *x*-axis, y-axis and the rotational angle in *xOy* plane of floating body are much influenced by the sag-to-span ratio of the cable. The maximum cable tensile force decreases as the sag-to-span ratio decreases from 1/45 to 1/60. After that, it increases as the sag-to-span ratio decreases from 1/60 to 1/80. It means that the maximum cable tensile force is much influenced by the sag-to-span ratio of the cable.

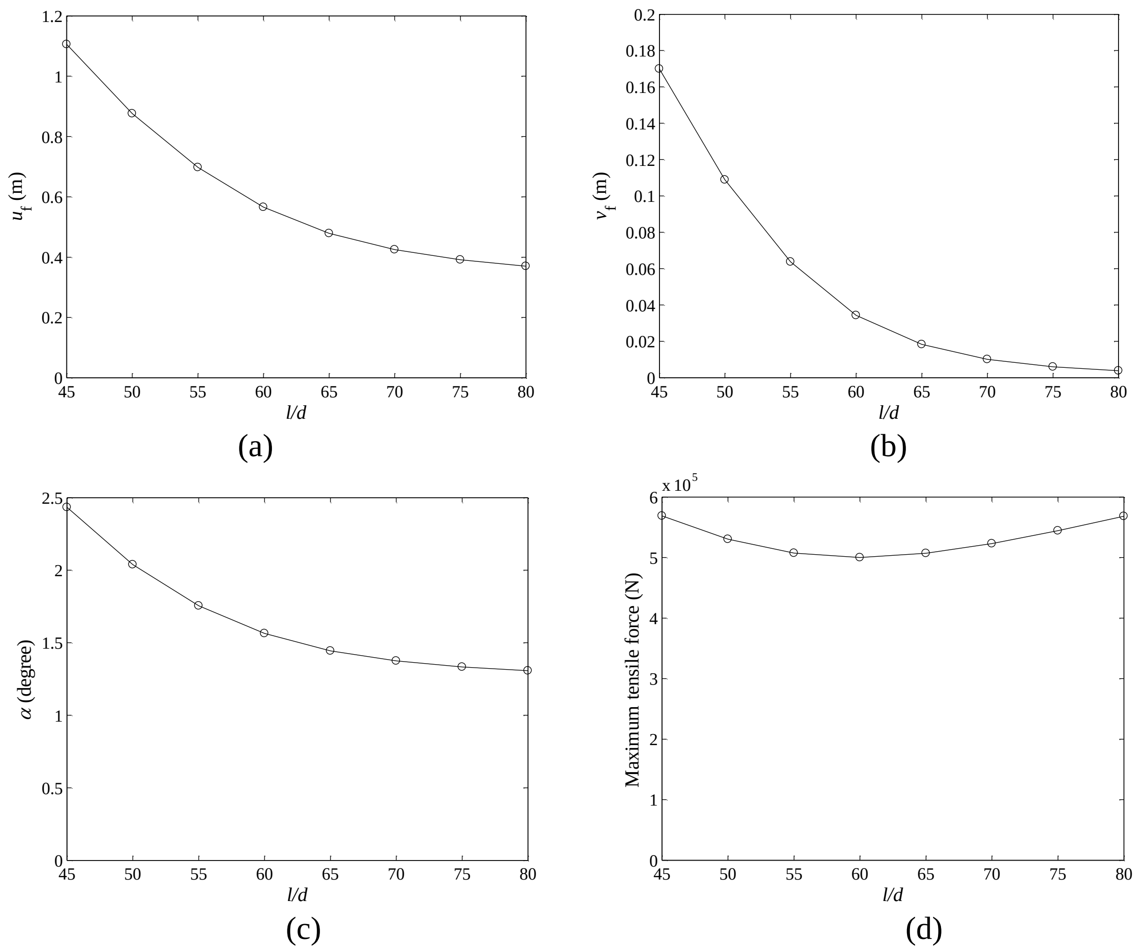


Figure 7. Amplitude of (a) Displacement along *x*-axis (b) Displacement along *y*-axis (c) Rotational angle in *xOy* plane (d) Maximum tensile force in cable at steady state for different sag-to-span ratios of cable with $\theta = 50^\circ$

When the sag-to-span ratio equals 1/60, the response amplitudes of the floating body and the maximum tensile force in the cable at steady state are presented in Fig. 8 for different initial inclined angles of cable. It is seen that the displacements along *x*-axis, y-axis and the rotational angle in *xOy* plane of floating body decreases obviously as the inclined angle increases from 40° to 54°. This means that the displacements in *x*-axis, y-axis and the rotational angle in *xOy* plane of floating body are much influenced by the initial inclined angle of the cable. The maximum cable tensile force decreases as the inclined angle increases from 40° to 50°. After that, it increases as the inclined angle increases from 50° to 54°. It means that the maximum cable tensile force is much influenced by the inclined angle of the cable.

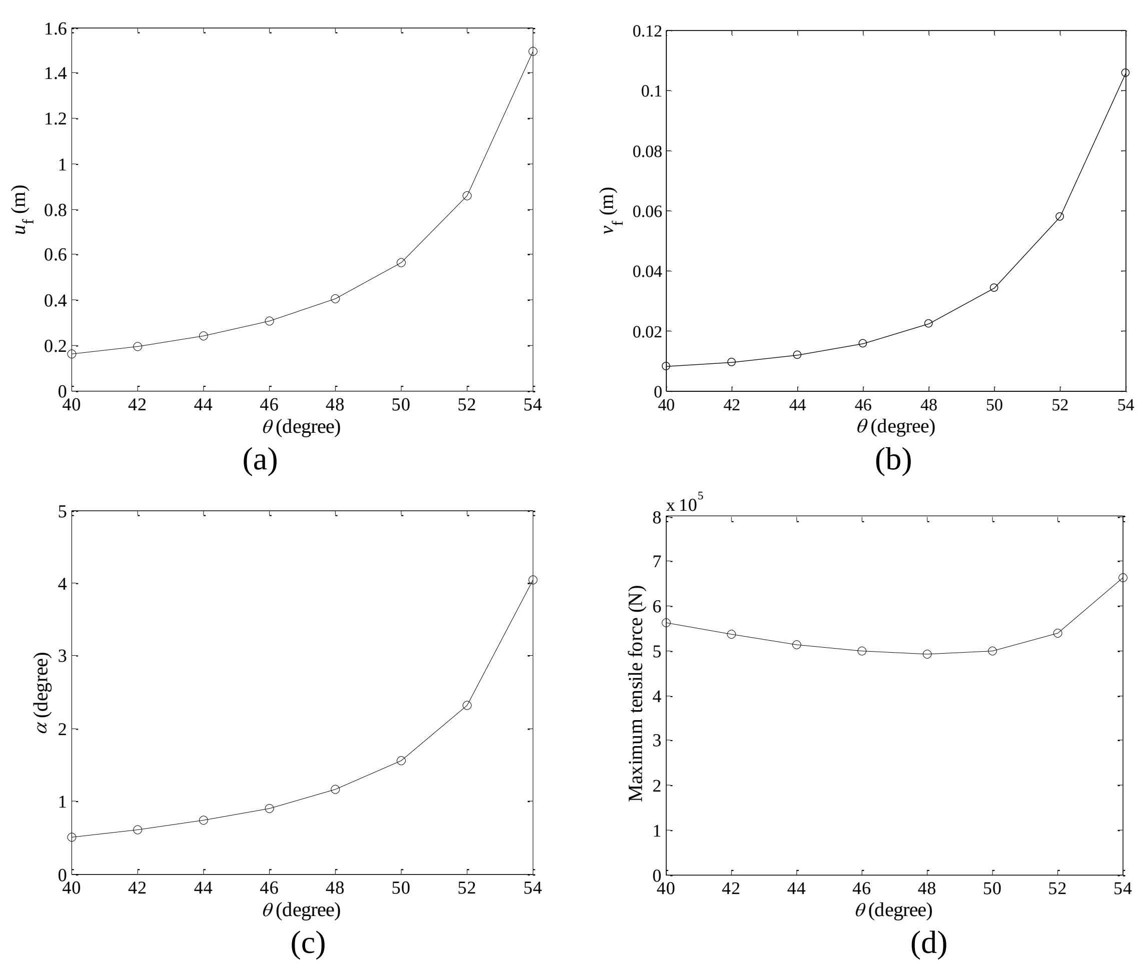


Figure 8. Amplitude of (a) Displacement along *x*-axis (b) Displacement along *y*-axis (c) Rotational angle in *xOy* plane (d) Maximum tensile force in cable at steady state for different initial inclined angle of cable with $d/l = 1/60$

Conclusions

The nonlinear vibrations of the three-dimensional floating structure moored by cables are analyzed. The floating body is modeled as a rigid body with six degrees of freedom. The mooring cables are modeled with the 3D nonlinear cable elements which are formulated with the extended Hamilton principle. The connection conditions between the mooring cables and the floating platform are introduced and hence the nonlinear equations of motions of both the mooring cables and floating platform are formulated as a whole through these connection conditions. Then the nonlinear equations of motion of the system under horizontal sinusoidal excitations are solved numerically as a whole. The influence of the mooring cables on the responses of the floating body and the maximum cable tensile force are discussed for different values of initial sag-to-span ratio or initial inclined angle of the mooring cables. It is seen from the numerical results that the initial sag-to-span ratio and the initial inclined angle of the mooring cables at static state have much influence on the dynamical displacements and rotations of the floating platform and the cable tensile forces.

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