A bounding method for contact stress using fractional programming

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Abstract

One branch of the promising verification research field of computational mechanics involves computing an upper and lower bound on a quantity of interest in engineering with finite element method. Several methods have been proposed to provide bounds on these quantities, especially those that are linear functionals of the displacement — for example, the local displacements, local stresses, stress intensity factors, etc. All of these methods need the aid of an adjoint problem with the quantity of interest as the external force for providing the upper and lower bounds. This becomes more complicated for a quantity of interest that is a nonlinear functional of the displacement, especially a quantity that cannot be expressed in an explicit formulation with respect to the displacement, e.g., the contact stress.

In this presentation, a method is shown for computing an upper bound on the contact stress of elastic bodies. A duality of an optimization model of contact forces is at first formulated for the elastic bodies discretized by finite elements. A formulation of the total contact force is derived with only the normalized nodal contact forces as the constrained variables in a standard simplex. Then a bound is obtained for the sum of the nodal contact forces, which is solved by first maximizing the fraction function subject to the standard simplex and then using Dinkelbach's algorithm for fractional programming to find the maximum — since the fraction function is pseudo concave in a neighborhood of the solution. The bound is solved with the problem dimensions being only the number of contact nodes or node pairs, which are much smaller than the dimension for the original problem, namely, the number of degrees of freedom. And a scheme for constructing an upper bound on the contact stress is proposed that uses the bound on the sum of the nodal contact forces obtained on a fine finite element mesh and the nodal contact forces obtained on a coarse finite element mesh, which are problems that can be solved at a lower computational cost.

The proposed method is verified through some examples concerning both frictionless and frictional contact. The bound requires solving a fractional programming problem with only a constraint of a simplex. Examples are provided show that the method can provide the upper bounds on the contact stress calculated by a finite element method on fine meshes.

Keywords: Contact stress; Upper bound; Finite elements; Fractional programming