# The nonlinear analysis by using the sub-domain meshless method

***†Y.-M. Guo[1], T. Hamada[1], G. Yagawa[2] and S. Kamitani[1]**

[1]Department of Mechanical Engineering, Kagoshima University, Japan
[2]Emeritus Professor, University of Tokyo and Toyo University, Japan

*Presenting author: k0246682@kadai.jp
†Corresponding author: k0246682@kadai.jp

## Abstract

As is well known, a sub-domain method is often used in computational mechanics. The conforming sub-domains, where the sub-domains are not separated nor overlapped each other, are often used, while the nonconforming sub-domains could be employed if needed. In the latter cases, the integrations of the sub-domains may be performed easily by choosing a simple configuration. Then, the meshless method with nonconforming sub-domains is considered one of the reasonable choices for the large-scale computational mechanics without the troublesome integration. We have proposed the sub-domain meshless method (SDMM). It is noted that, since the method can employ both the conforming and the nonconforming sub-domains, the integration for the weak form is necessarily accurate and easy by selecting the nonconforming sub-domains with simple configuration. However, in the boundary of the analysis domain with a complicated shape, it is difficult to select a sub-domain of simple configuration. To overcome this problem, we apply the collocation approach to the nodes on the boundary. In addition, in order to satisfy the positivity conditions for the boundary nodes, the over-range points are added. The mixed boundary value problems about the Poisson equation and the Helmholtz equation have been analyzed by using the SDMM. In this work, a nonlinear problem is analyzed by using the proposed SDMM. The numerical solutions are compared with the exact solutions and the solutions of the collocation method, showing that the relative errors by using the SDMM are smaller than those by using the collocation method and that the proposed method possesses a good convergence.

Numerical solutions of a 2D nonlinear equation

$$\nabla^2 u + \varepsilon u^2 = e^x + 4e^{2y} + \varepsilon(e^{2x} + 2e^x e^{2y} + e^{4y}) \tag{1}$$

are obtained over an 0.3×0.1 domain of (0, 0)×(0.3, 0.1) by using the SDMM and the collocation method (CM). A mixed problem, the essential boundary condition is imposed at nodes on top and bottom boundaries and the natural boundary condition is prescribed at nodes on left and right boundaries, is solved. Regular (taking the same nodal interval $h$) nodal models of $h$=1/60 (197 nodes), $h$=1/80 (305 nodes) and $h$=1/100 (437 nodes) are, respectively, used to study the convergence with the nodal model refinement. In order to solve the challenge issue of strong nonlinear problems, six kinds of $\varepsilon$ which are $\varepsilon = 0.1$, $\varepsilon = 1$, $\varepsilon = 10$, $\varepsilon = 100$, and $\varepsilon = 200$ are used. The sub-domain of integration is chosen as a square configuration in this paper, and the area of the sub-domain of integration is $c^2$. To find optimal values of $c$, the problems with $\varepsilon = 0.1$ and $\varepsilon = 1$ are first calculated by using the SDMM. The results of relative errors are shown in Fig. 1 with $c = kh$ and $0 < k < 2$. For the regular nodal models, $k = 1$ means that the sub-domains of integration are conforming, $k < 1$ separated and $k > 1$ overlapped sub-domains, respectively. Fig. 1 shows that the most accurate results are given at $k = 1.3$ for $\varepsilon = 0.1$ and $k = 0.8$ for $\varepsilon = 1$. The relative errors for $\varepsilon = 0.1$ and $\varepsilon = 1$ by using the SDMM and the CM are shown in Figs. 2 and 3, respectively, showing that the relative errors by using the SDMM are smaller than those by using the CM, and the relative errors of become smaller with the decrease of the nodal interval $h$. The relative errors for all the values of $\varepsilon$ by using the SDMM with $k = 1.3$ are shown in Fig. 4. It can be seen from this figure that although the error levels become higher with the increase of $\varepsilon$ in general, the error levels using $\varepsilon = 0.1$, $\varepsilon = 1$ and $\varepsilon = 10$ are rather low, those using $\varepsilon = 100$ and $\varepsilon = 200$ are also lower, and the relative errors using all the values of $\varepsilon$ become smaller with the decrease of the nodal interval $h$.

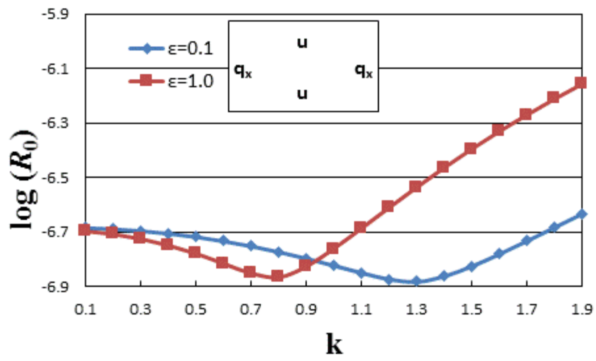


Figure 1. Changes of relative error $R_0$ with $k$ for $\varepsilon = 0.1$ and $\varepsilon = 1$

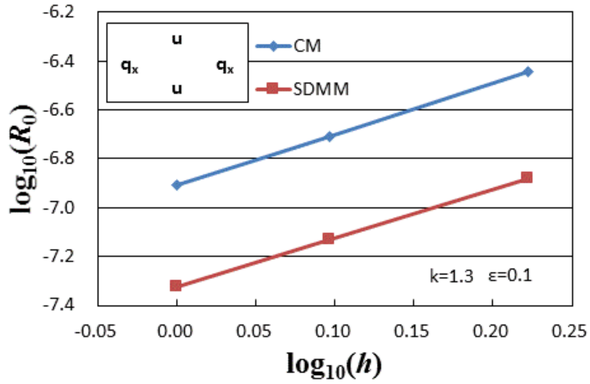


Figure 2. The relative error $R_0$ with $h$ for $\varepsilon = 0.1$

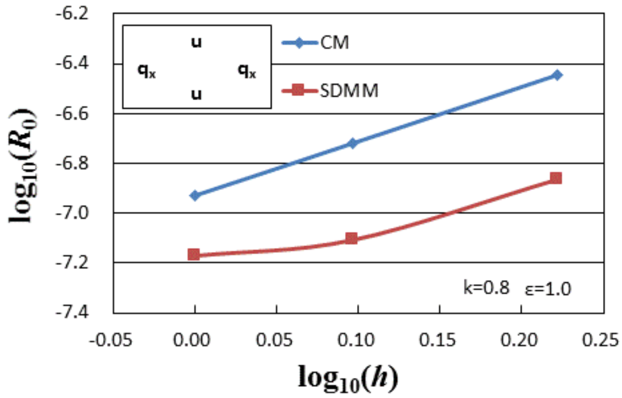


Figure 3. The relative error $R_0$ with $h$ for $\varepsilon = 1$

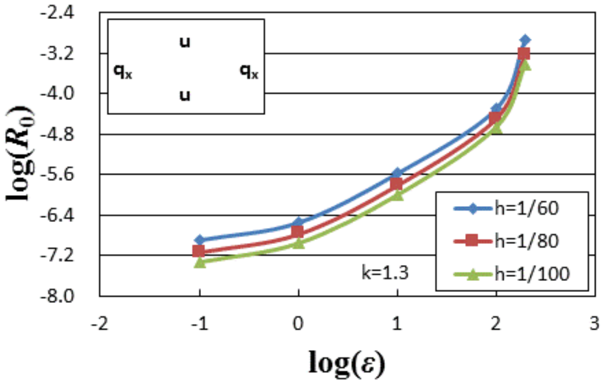


Figure 4. The relative error $R_0$ using all the values of $\varepsilon$

**Keywords:** Sub-domain method, Meshless method, The SDMM, Weak form, Easy integration