Numerical methods for structural dynamic responses based on radial basis functions approximation

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Abstract

A numerical method for solving structural dynamic response was proposed by combining the theory of radial basis functions (RBFs) approximation and the collocation point methods. To solve the problem that using basic RBFs point interpolation method will bring great numerical oscillations, a multivariate interpolation function with the linear combination of each order differential terms was developed and the arithmetic steps were given. Unlike other numerical methods, there were no theoretical supposes about changing rules of acceleration and load within time interval, so this method had an applicability to solve jerk and jerk (third-order) equations. Actual examples showed that RBFs approximation method had simple computational process and improved the convergency and stability effectively.

Keywords: Radial Basis Functions; Meshless Methods; Dynamic Response; Jerk; Jerk Equations; Initial Value Problems

Introduction

Problems of dynamic response of systems can often be come down to initial problems of second-order ordinary differential equations. At present the methods for structural dynamic response mainly include: mode superposition methods, direct integration methods and methods turning second-order into first [1], et al. Mode superposition methods are only used for integration methods of linear systems adopted only first several-order modes, so it is inapplicable to systems which the effects of high modes can't be neglected. Direct integration methods are appropriate for both linear systems and nonlinear systems, the most commonly used includes difference methods, linear acceleration methods and improved linear acceleration methods. Above Methods are all based on two following assumptions: (1) Continues time scale is divided into finite number of nodes where the motion differential equation is just satisfied and solutions of displacement, velocity and acceleration just are obtained; (2) There are some simple assumptions about change rules of acceleration or loads in time interval.

Theoretical defects of these methods made a lower precision with only first or second-order, and the calculating precision will poorer if actual acceleration belied these assumption in transient response phase. Furthermore, the uncontrollability of inherent algorithmic damping of direct integration method also causes great calculation errors. Because applicability of integrate methods depends on type of nonlinearity and load characteristics, etc, it can be hard to choose an appropriate differential scheme while solving a nonlinear problem. Precise integration methods [2] open up a new direction for solving dynamic responses, but when solving homogeneous equations under random loads, such as seismic waves and wind loads, it is necessary to make some assumption about change rules of loads in time interval.

Moreover, Jerk [3], the time rate-of-change of acceleration, has been increasingly applied in areas of chaos theory [4][5], nonlinear dynamics [6][7], mechanical design [8], and structural damage detection [9], etc. And jerk equation, third-order differential equation, of the form involving the third temporal derivative of displacement can describe some physical problems such as third-order mechanical oscillations [10][11]. Above methods are no longer able to be used to solve jerk and jerk equation because of their inherent basic assumption. At present a few effective numerical methods can be used, e.g., fourth-order accurate Runge-Kutta method with sufficient-small step-sizes [12]. With deeper research about the role of jerk in mechanics and applications, jerk calculating will have more important implications.

Radial basis functions (RBFs) have advantages of simple form, isotropic and independent of space dimensions, etc. scholars, at home and aboard, have proposed a sea of methods based on radial basis functions which have been widely applied in scientific and engineering calculating areas of hydrodynamics, computational mechanics, picture processing, etc. Methless methods [13][14] based on RBFs have been used to solve boundary value problems, and it has acquired a great of achievements, but RBFs have not yet been used to solve initial problems up to now and we will try to do it.

1 Radial Basis Function Approximation Methods

Radial Basis Function (RBF) is a kind of basis function with a distance variable. It uses the simple function φ defined in $[0, +\infty)$ and Euclidean norm $\|\cdot\|_2$ in R^d to represent d-dimensional function $\varphi=\varphi(R_i)$, in which $R_\mathrm{i} =\|x-x_i\|_2$, the distance from arbitrary point x to the point x_i, is independent variable. In essence, φ is a one-variable function —— function of distance, thus its simple form makes data convenient to store and calculate. Another important advantage of RBF is the powerful capacity of approximation that it can almost approximate all functions.

Depending on its scoped, RBFs can be divided into two categories: Globally supported RBFs (GS-RBFs) and compactly supported RBFs(CS-RBFs) [15][16][17]. It limits the former's use for calculating large structure that the calculation process will produce ill-conditioned matrixes. CS-RBFs can make the coefficient matrix has the characteristic of banded sparse.

The method combined RBFs approximation with collocation point methods has many advantages, such as meshless, simple form, no numerical integration and high calculation efficiency. But at present both domestic and overseas researches about solving fractional differential equations using RBFs are all related to boundary problems without time parameter. The main reason lies in the independent variable of radial basis function is spatial distance. Xu [18] have presented the concept of transformation from spatial distance to time interval, then we will try to solve initial problems by using RBFs.

First, we take a single-degree-freedom dynamic system (1) as an example to explain the numerical method.

$$\begin{cases} m\ddot{u} + c\dot{u} + ku = p(t) \\ u(0) = u_0, \dot{u}(0) = \dot{u}_0 \end{cases} \tag{1}$$

The time domain Ω can be discretisized with n nodes t_i , i = 1, 2, $\cdots$, n, then the approximate function $u^h(t)$ of displacement function $u(t)$ can use a linear combination of radial basis function $\varphi_i(t)$ which is taking t_i as the center to expresseitself as:

$$u^h(t) = \sum_{j=1}^{n} a_j \varphi_j(t) = \boldsymbol{\Phi}^\mathrm{T}(t)\boldsymbol{a} \tag{2}$$

Eq. (2) is a basic expression of RBFs interpolation, a_j denotes a series of unsolved coefficients, $\boldsymbol{a}=[a_1, a_2, \cdots, a_n]^\mathrm{T}$, $\boldsymbol{\Phi}(t)=[\Phi_1(t), \Phi_2(t), \cdots, \Phi_n(t)]^\mathrm{T}$. Put CS-RBF as interpolation cardinal function, in this paper, we use Wu function [15]:

$$\varphi(r) = (1-r)_+^5 \left(1 + 5r + 9r^2 + 5r^3 + r^4\right) \tag{3}$$

In Eq. (3), For this method, we have $r=\|t-t_i\|_2 / R_{\max i}$. And $R_{\max i}$ is support radius of t_i , the maximum distance of t_i to any other points, which means that the effective region of t_i is the whole domain. $(1-r)_+$ can be defined as $(1\text{-}r)_+ = \begin{cases} 1-r, 0 \le r \le 1 \\ 0 \quad , other \end{cases}$.

Because traditional collocation methods have large numerical oscillation, for the characteristic that the objective solution of system (1) is the second derivative of u, thus we present the interpolation function combined displacement with velocity, as show in Eq. (4):

$$u^h(t) = \sum_{j=1}^{n} a_j \varphi_j(t) + b_1 \frac{\mathrm{d}\varphi_1(t)}{\mathrm{d}t} \tag{4}$$

In Eq. (4), there add a linear combination of first derivative of the initial time. According to the authors' solving experience [19], if we add a second derivative term on the basis of Eq. (4), then get the Eq. (5), the numerical oscillation can be diminished significantly. And adding the initial condition of the second derivative term as a new constrain, this initial constrain has explicit physical and mathematical interpretations which are the acceleration of initial time and that the second derivative satisfies the differential equation in the initial time.

$$u^h(t) = \sum_{j=1}^{n} a_j \varphi_j(t) + b_1 \frac{\mathrm{d}\varphi_1(t)}{\mathrm{d}t} + b_2 \frac{\mathrm{d}^2\varphi_1(t)}{\mathrm{d}t^2} \tag{5}$$

We can find Eq. (5) requires the RBF has high-order continuity, and taking high-order derivative of CS-RBFs will cause an ill-conditioned coefficient matrix. So we finally presented substituting helper function for high-order derivative term, as follows:

$$u^h(t) = \sum_{j=1}^{n} a_j \varphi_j(t) + b_1 \frac{\mathrm{d}\varphi_1(t)}{\mathrm{d}t} + b_2 \xi_1(t) = \boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{a} \tag{6}$$

In Eq. (6), b_1, b_2 denotes additional coefficient, $\xi_1(t)$ is helper function, and we can use other CS-RBF as an available helper function, for example,

$$\xi(t) = (1-r)_+^6 \left(6 + 36r + 82r^2 + 72r^3 + 30r^4 + 5r^5\right).$$

Then plugging t_i (i = 1, 2, ⋯, n, 1, 1) into the interpolation Eq. (6), we can obtain n+2 linear equations Eq. (7):

$$\boldsymbol{A}\boldsymbol{a} = \boldsymbol{u} \tag{7}$$

In Eq. (7),

$$\boldsymbol{A} = \begin{bmatrix} \varphi_1(t_1) & \varphi_2(t_1) & \cdots & \varphi_n(t_1) & \frac{\mathrm{d}\varphi_1(t_1)}{\mathrm{d}t} & \xi_1(t_1) \\ \varphi_1(t_2) & \varphi_2(t_2) & \cdots & \varphi_n(t_2) & \frac{\mathrm{d}\varphi_1(t_2)}{\mathrm{d}t} & \xi_1(t_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \varphi_1(t_n) & \varphi_2(t_n) & \cdots & \varphi_n(t_n) & \frac{\mathrm{d}\varphi_1(t_n)}{\mathrm{d}t} & \xi_1(t_n) \\ \frac{\mathrm{d}\varphi_1(t_1)}{\mathrm{d}t} & \frac{\mathrm{d}\varphi_2(t_1)}{\mathrm{d}t} & \cdots & \frac{\mathrm{d}\varphi_n(t_1)}{\mathrm{d}t} & \frac{\mathrm{d}^2\varphi_1(t_1)}{\mathrm{d}t^2} & \frac{\mathrm{d}\xi_1(t_1)}{\mathrm{d}t} \\ \frac{\mathrm{d}^2\varphi_1(t_1)}{\mathrm{d}t^2} & \frac{\mathrm{d}^2\varphi_1(t_1)}{\mathrm{d}t^2} & \cdots & \frac{\mathrm{d}^2\varphi_n(t_1)}{\mathrm{d}t^2} & \frac{\mathrm{d}^3\varphi_1(t_1)}{\mathrm{d}t^3} & \frac{\mathrm{d}^2\xi_1(t_1)}{\mathrm{d}t^2} \end{bmatrix},$$

$\boldsymbol{u}=[u_1, u_2, \cdots, u_n , v_1 , v_2]^{\mathrm{T}}$ where v_1 、 v_2 denote additional unknowns which respectively represent the initial velocity and acceleration.

It is easy, from Eq. (7), to show that $\boldsymbol{a}=\boldsymbol{A}^{-1}\boldsymbol{u}$ and using $\boldsymbol{a}$ in Eq. (6) gives

$$u^h(t) = \boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{A}^{-1}\boldsymbol{u} = \boldsymbol{N}(t)\boldsymbol{u}\,. \tag{8}$$

Let $\boldsymbol{N}(t)=\boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{A}^{-1}$. We could definite $\boldsymbol{N}(t)$ is time characteristic function, similar to finite element shape function, and $\boldsymbol{A}$ is dynamical characteristic matrix. Eq. (8) is an analytic equation, then differentiating Eq. (8) with respect to t gives the expression of velocity Eq. (9),

$$\dot{\boldsymbol{u}}(t)=\boldsymbol{N}'(t)\boldsymbol{u} \tag{9}$$

In Eq. (9),

$$\boldsymbol{N}'(t)=\begin{bmatrix} \varphi_1'(t_1) & \varphi_2'(t_1) & \cdots & \varphi_n'(t_1) & \varphi_1''(t_1) & \xi_1'(t_1) \\ \varphi_1'(t_2) & \varphi_2'(t_2) & \cdots & \varphi_n'(t_2) & \varphi_1''(t_2) & \xi_1'(t_2) \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \varphi_1'(t_n) & \varphi_2'(t_n) & \cdots & \varphi_n'(t_n) & \varphi_1''(t_n) & \xi_1'(t_n) \end{bmatrix}\boldsymbol{A}^{-1}, \quad \varphi'(t)=\frac{\mathrm{d}\varphi}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}, \quad \frac{\mathrm{d}r}{\mathrm{d}t}=\frac{t_i-t}{R_iR_{\max i}}.$$

Similarly, the expression of acceleration Eq. (10) as follows:

$$\ddot{\boldsymbol{u}}(t)=\boldsymbol{N}''(t)\boldsymbol{u}. \tag{10}$$

By substituting Eq. (8), (9) and (10) into dynamic system (1) and combining the external loads vector $\boldsymbol{p}$, we obtain $m\boldsymbol{N}''\boldsymbol{u}+c\boldsymbol{N}'\boldsymbol{u}+k\boldsymbol{u}=\boldsymbol{p}(t)$, it follows that

$$\left(m\boldsymbol{N}''+c\boldsymbol{N}'+k\boldsymbol{E}\right)\boldsymbol{u}=\boldsymbol{p}(t). \tag{11}$$

And according to the initial conditions of velocity and adding initial acceleration, we have

$$\left\{\varphi_1'(t_1) \;\; \varphi_2'(t_1) \;\; \cdots \;\; \varphi_n'(t_1) \;\; \varphi_1''(t_1) \;\; \xi_1'(t_1)\right\}\boldsymbol{A}^{-1}\boldsymbol{u}=\dot{u}(0) \tag{12}$$

$$\left\{\varphi_1''(t_1) \;\; \varphi_2''(t_1) \;\; \cdots \;\; \varphi_n''(t_1) \;\; \varphi_1'''(t_1) \;\; \xi_1''(t_1)\right\}\boldsymbol{A}^{-1}\boldsymbol{u}=\ddot{u}(0) \tag{13}$$

Combining Eq. (11), (12) and (13), we get n+2 linear equations, thus the second-order differential equation is discretized into linear algebraic equations. Substituting the initial displacement conditions and solving the equations we obtain the solution of $\boldsymbol{u}$. Then plugging $\boldsymbol{u}$ back into Eq. (9) and (10) respectively gives the velocity and acceleration at every time note.

As demonstrated above, the radial basis function approximation method has no theoretical assumption except the interpolation, and $u^h(t)$ is an analytic expression, so taking the third derivative of Eq. (8) can solve the jerk effectively, we will elaborate on this in *Example 3.3*.

Next, for jerk equations of third-order dynamic system with the form of $\dddot{x}=J(x,\dot{x},\ddot{x})$, we can present the interpolation function Eq. (14)

$$u^h(t)=\sum_{j=1}^{n}a_j\varphi_j(t)+b_1\frac{\mathrm{d}\varphi_1(t)}{\mathrm{d}t}+b_2\xi_1(t)+b_3\frac{\mathrm{d}\xi_1(t)}{\mathrm{d}t}=\boldsymbol{\Phi}^{\mathrm{T}}(t)\boldsymbol{a}. \tag{14}$$

Using the similar process, the jerk equation is discretized into n+3 nonlinear algebraic equations

$$N'''x-J\left(x,N'x,N''x\right)=0,$$

where $\boldsymbol{N}$ denotes time characteristic matrix. And using initial displacement, velocity, acceleration and jerk as the constrains, by replacing functions at initial time with corresponding constrains, then using iterative method to calculate the nonlinear algebraic equations, we can obtain the solutions of the third-order equation. We will give a numerical example in *Example 2.4*.

2 Numerical Examples Analyses

2.1 Forced vibrations of a single degree of freedom

$$\begin{cases} \ddot{u}+4\dot{u}+5u=\sin(2t) \\ u(0)=\dfrac{57}{65}, \dot{u}(0)=\dfrac{2}{65} \end{cases}. \tag{15}$$

Analytic solution of system (15) is $u = e^{-2t}(\cos t+2\sin t)-[8\cos(2t)-\sin(2t)]/65$. If time domain is t = 60.8 s， time interval is Δt = 0.1 s, using RBFs approximation methods, choosing Eq. (4) as interpolation function and using $u(0)=57/65$ and $\dot{u}(0)=2/65$ as initial conditions, then choosing Eq. (6) and adding a second-order differential initial condition $\ddot{u}(0)=-293/65$ to solve system (15), calculating the relative errors of solutions in first six notes as shown in Table 1, we are confident that the latter can diminish numerical oscillations obviously improve solving accuracy.

Using other traditional methods to solve system (15) with also Δt = 0.1 s, and relative errors at some moments are displayed in Table 2. It is seen that, RBFs approximation methods, as compared to Newmark method (NM) and Wilson-θ method (W-θ), improve the solving accuracy greatly and it has high stability of accuracy for long periods.

Table 1. The relative errors for RBFs approximation methods with different interpolation function and initial conditions

t(s)	Interpolation function (6) with $\ddot{u}(0)$			Interpolation function (4) without $\ddot{u}(0)$		
	displacement	velocity	acceleration	displacement	velocity	acceleration
0	0	2.59×10^{-15}	1.82×10^{-15}	0	3.94×10^{-4}	2.39×10^{-1}
0.1	1.46×10^{-4}	6.20×10^{-3}	3.16×10^{-3}	3.30×10^{-3}	1.08×10^{-1}	5.62×10^{-2}
0.2	3.09×10^{-4}	1.24×10^{-3}	2.58×10^{-3}	6.59×10^{-3}	3.19×10^{-2}	6.30×10^{-2}
0.3	4.14×10^{-4}	7.92×10^{-4}	5.75×10^{-3}	8.98×10^{-3}	1.62×10^{-2}	0.12×10^{-1}
0.4	5.00×10^{-4}	2.23×10^{-4}	1.12×10^{-1}	1.08×10^{-2}	5.36×10^{-3}	2.50
0.5	5.63×10^{-4}	4.27×10^{-4}	4.71×10^{-3}	1.22×10^{-2}	8.42×10^{-3}	9.85×10^{-2}

Table 2. The relative error of the solution of displacement, velocity and acceleration with several numerical methods

t(s)	displacement			velocity			acceleration		
	NM	W-θ	RBF	NM	W-θ	RBF	NM	W-θ	RBF
2	0.0051	0.0150	3.70×10^{-4}	0.0048	0.0192	2.94×10^{-4}	0.0444	0.2328	2.34×10^{-3}
4	0.0166	0.0189	4.40×10^{-6}	0.0016	0.0132	4.18×10^{-9}	0.0086	0.0187	5.30×10^{-6}
16	0.0003	0.0193	1.40×10^{-6}	0.0039	0.0004	2.20×10^{-6}	0.0070	0.0227	1.96×10^{-6}
34	0.0085	0.0029	8.33×10^{-6}	0.0029	0.0174	3.30×10^{-6}	0.0019	0.0004	4.00×10^{-7}
40	0.272	0.6670	5.21×10^{-5}	0.0005	0.0114	9.70×10^{-8}	0.2666	0.6656	5.10×10^{-7}
60	0.0005	0.0199	1.58×10^{-5}	0.0036	0.0012	1.27×10^{-5}	0.0072	0.0233	3.50×10^{-6}

2.2 Bending Vibration of Simply Supported Beam

A simply supported beam with constant section is illustrated in Fig. 1, and with length L=6 m, high of cross section h=0.02m, width b=0.02m, cross sectional area $A=bh$, section inertia $I=bh^3/12$, density $\rho=4\times10^4$ kg/m^3, elastic modulus E=210 GPa and poisson ratio μ=0.3. Suppose the beam is damping-free, and there is a lateral load $q(x,t)=F_0\sin(\omega_0 t)\delta(x-L/2)$. The theoretical solution of vibration displacement of this beam is

$$w(x,t)=\frac{2F_0}{\rho AL}\sum_{r=1,3,5,\cdots}^{\infty}\frac{(-1)^{(r-1)/2}}{\omega_r^2[1-(\omega_0/\omega_r)^2]}\sin\frac{r\pi x}{L}\times\left(\sin(\omega_0 t)\ \frac{\omega_0}{\omega_r}\sin(\omega_r t)\right)$$

where $\omega_r=(r/\pi)^2\sqrt{EI/(\rho A)}$ is inherent frequency.

If F_0=1 kN, ω_0=4 Hz, t = 64 s, Δt = 0.2 s, dispersing the beam into ten cubic Hermite finite elements, using RBFs approximation method to solve this problem, some results are shown in Fig. 2. It is seen that, compared to the solutions [20] of newmark method and Wilson-Wilson-θ method, the solving accuracy of this method has improved and the errors do not accrue with time.

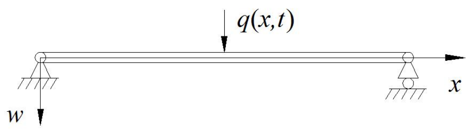
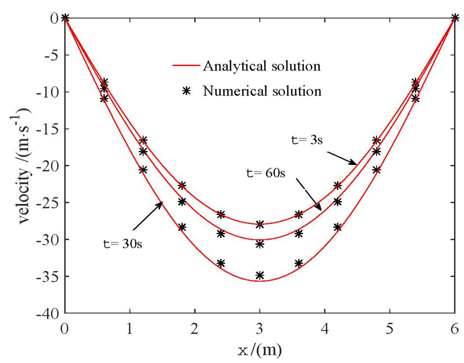
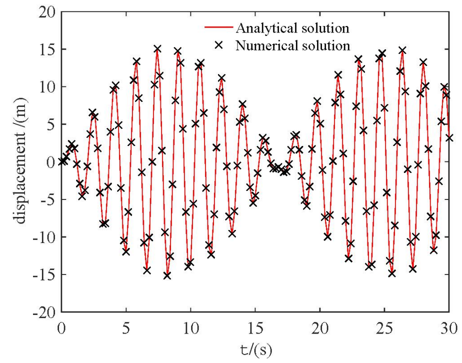


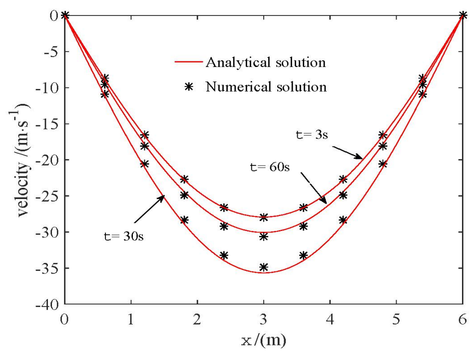
Figure 1. The sketch of simple supported beam bending vibration



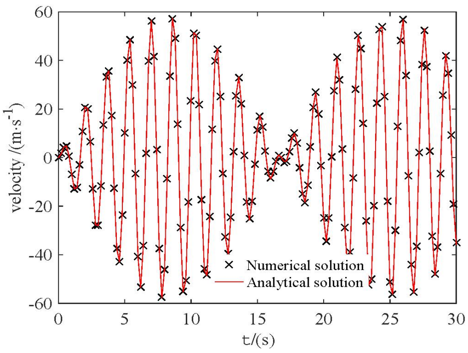
(a) Displacement



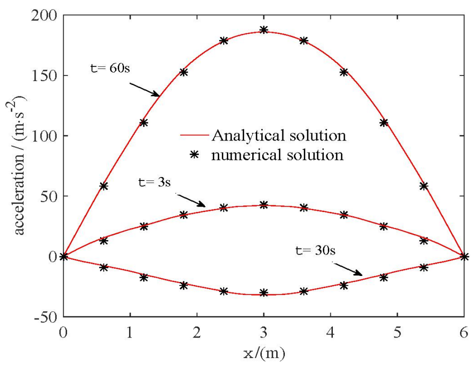
(d) Displacement



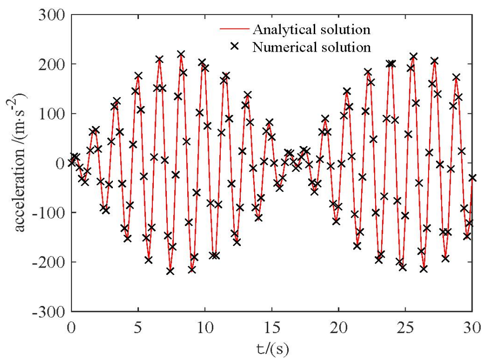
(b) Velocity



(e) Velocity



(c) Acceleration



(f) Acceleration

Figure 2. (a), (b) and (c) show the numerical solution and the exact solution at every node at t=3s,30s, 60s; (d), (e) and (f) show the numerical solution and the exact solution at the middle point in different time.

2.3 Three Stories Frame Structure

Fig. 3 shows a sketch of three-storey shear frame structure, the masses of each storey, including columns, are m_1=1.8×10^5 kg, m_2=2.7×10^5 kg, m_3=2.7×10^5 kg, respectively. Lateral stiffness are k_1=9.8×10^7 N/m, k_2=1.96×10^8 N/m, k_3=2.45×10^8 N/m. We will solve the dynamical responses and the jerk of this structure with Rayleigh damping under horizontal seismic excitation (as shown in Fig. 4), and the 1st and 2nd damping ratio are $\xi_1=\xi_2=0.05$. The initial conditions are $u(t)=0$, $\dot{u}(t)=0$, and adding $\ddot{u}(0)=0.014$, initial acceleration of ground vibration.

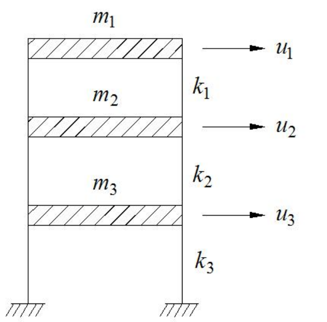


Figure 3. Sketch of three stories shear frame

The mathematical model can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{U}}+\boldsymbol{C}\dot{\boldsymbol{U}}+\boldsymbol{K}\boldsymbol{U}=-\boldsymbol{M}\ddot{\boldsymbol{U}}_g, \tag{16}$$

Where $\ddot{\boldsymbol{U}}_g$ and $\ddot{\boldsymbol{U}}$ are respectively horizontal acceleration caused by seismic and relative acceleration, and the former also can be called carrier acceleration. For jerk, let us assume $\ddot{\boldsymbol{U}}_g$ is derivable, we have

$$\boldsymbol{M}\dddot{\boldsymbol{U}}+\boldsymbol{C}\ddot{\boldsymbol{U}}+\boldsymbol{K}\dot{\boldsymbol{U}}=-\boldsymbol{M}\dddot{\boldsymbol{U}}_g, \tag{17}$$

Then

$$\boldsymbol{M}\dddot{\boldsymbol{U}}_{abs}=-\boldsymbol{C}\ddot{\boldsymbol{U}}-\boldsymbol{K}\dot{\boldsymbol{U}}, \tag{18}$$

$\dddot{\boldsymbol{U}}_{abs}=\dddot{\boldsymbol{U}}+\dddot{\boldsymbol{U}}_g$ is absolute jerk of the structure. Using RBFs approximation method, we solved the solutions and some of them are shown in Fig. 5.

It is important to note that the Eq.(18) can’t be used in traditional method, such as Newmark method, Wilson-θ method, and so on, because of the theoretical hypothesis. And in this example, although the jerk is also an object solving, we can't use the Eq.(14) because of a lack of initial jerk regarded as a constrain.

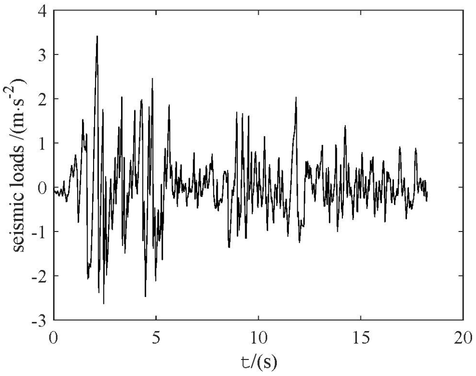
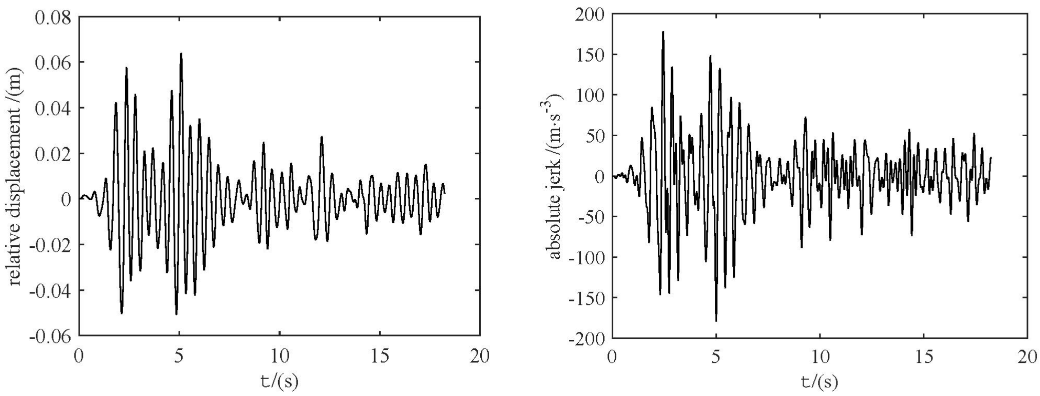


Figure 4. Transport acceleration causing by earthquake



(a) Displacement **(b) Absolute jerk**

Figure 5. the numerical results at the top storey

2.4 Third-order Mechanical Oscillations

Following Gottlieb [21], the most general jerk function which is invariant under time- and displacement- reversals is

$$\dddot{x} = -\gamma\dot{x} - \alpha\dot{x}^3 - \beta x^2\dot{x} + \delta x\dot{x}\ddot{x} - \varepsilon\dot{x}\ddot{x}^2 , \tag{19}$$

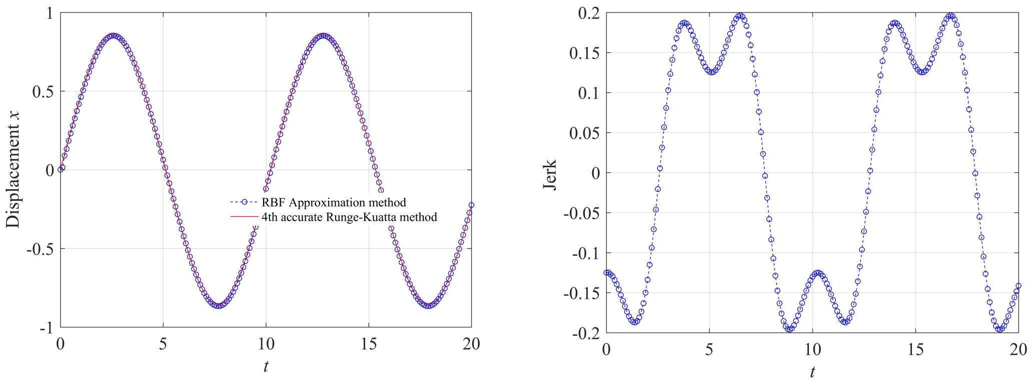
where α, β, γ, δ, and ε are all constants, and at least one of β, δ and ε should be different from zero. In addition, if ε=0, it is required that $\delta \neq$-2α such that the Eq. (19) is simply not the secon d-order ordinary differential equation. The corresponding initial conditions are

$$x(0) = 0\,, \dot{x}(0) = B\,, \ddot{x}(0) = 0\,.$$

In this paper, we consider the case for $\alpha = \beta$=1, γ= δ= ε=0 and B=0.5. For this situation, Eq. (19) becomes

$$\begin{cases} \dddot{x} = -\dot{x}^3 - x^2\dot{x} \\ x(0) = 0, \dot{x}(0) = 0.5, \ddot{x}(0) = 0 \end{cases} . \tag{20}$$

Using RBFs approximation method with Δt=0.1s and choosing Eq. (14) as interpolation function, we get the period of the solution is t=10.210655 s. And the period using fourth-order accurate Runge-Kutta method with Δt=0.001s is 10.210761s [12][21]. The solution of displacement and jerk are shown in Fig. 6.



(a) Displacement **(b) Jerk**

Figure 7. Numerical results of Eq. (20) using RBFs approximation method with Eq. (14)

3 Conclusions

This paper developed a new approach for solving structural dynamic responses and jerk based on the powerful approximation capability of RBFs. The practical calculation examples show that the RBFs approximation method has great astringency and high solving accuracy. Furthermore, this method also has the following advantages:

1. RBFs approximation method is different from stepwise direct integration methods. it needn't numerical integration, has high calculation efficiency, and has no recursive formulae and error accumulation.

2. We proposed the combined interpolation expression of all-order derivatives and it is necessary to add initial condition that the order is same as the differential equation, which can decrease the numerical oscillation significantly.

3. This method has no assumption of load changes and acceleration in time interval, can solve jerk and jerk equation effectively, and it breaks the limitation that stepwise direct integration methods are difficult to solve jerk because of inherent assumptions.

The method of RBFs approximation has clear advantages and it may well become a common method.

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