State of the art-Reanalysis: recent development review

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Abstract

Reanalysis is a fast computational method to predict the response of a modified structure based on the initial analysis results. Due to its advantages in efficiency, the reanalysis method has made significant progress in recent several decades. With the increase of requirements for computational performance in numerical methods, the reanalysis method should be a more important role in complex product design. Therefore, this paper investigates and discusses some characteristics of recent developed direct and approximation methods from the view of static, dynamical and nonlinear problems. Moreover, some suggestions for advancing the reanalysis method have been also given according to the state of the art reanalysis.

Keywords: Reanalysis; Direct method; Approximation method; Static; Dynamic

Introduction

Reanalysis is a fast computational method to predict the response of a modified structure based on the computational information of initial analysis. The reanalysis method should reduce the computational cost significantly by avoiding a complete analysis of modified structure. The earliest reanalysis algorithm was proposed at 1950s and it was just used to calculate the inversion of modified stiffness matrix. However, it has been rapidly developed in recent years and can be used to solve many complex engineering problems. There are many different reanalysis methods have been developed. Generally, they can be mainly divided into two categories: direct methods (DMs) and approximate methods.

DMs can update the inverse of modified stiffness matrix quickly by the Sherman-Morrison (SM) formula [1] and the Sherman-Morrison-Woodbury lemma [2] and obtain the exact response of the modified structure, but usually it can only solve the problems of local or low-rank modifications. It is difficult to apply the DMs to the real engineering computation since the efficiency will greatly reduce with the increase of stiffness matrix modification.

Compared with DMs, the approximate methods can estimate the response of the modified structure by reducing the dimension of the stiffness matrix. The approximate methods mainly include local approximations (LA), global approximations (GA), iterative approximations (IA) and combined approximations (CA). Local approximations are based on information calculated at a single point, so these methods are also called single-point approximations, such as the first-order Taylor series expansion or the binomial series expansion about a given design point. These methods are very efficient but they are effective only for small changes in the design variables. For large changes in the design the accuracy of the approximations often deteriorates and the results may become meaningless. Global approximations (called also multi-point approximations), such as polynomial fitting (PFM) [3], response surface (RSM) [4] or reduced basis (RBM) [5] methods. These approximations are obtained by analyzing the structure at a number of design points, and they are valid for the whole design space, but global approximations may require much computational effort, particularly in problems with large numbers of design variables. Iterative approximations are derived from the iterative method of solving linear equations, such as the preconditioned conjugate gradient (PCG) [6], symmetric successive over-relaxation (SSOR) [7] preprocessing algorithm and so on. Combined approximations might be the most popular approximate reanalysis method [8]. In these approaches the global qualities are attempted to give to local approximations. This can be achieved by considering the terms of local (series) approximations as basis vectors in a global (reduced basis) expression.

In recent decades, with the rapid development of CAE technology in different fields, the reanalysis algorithm has been developed rapidly in the fields of static, dynamic and nonlinear domain. Therefore, this paper discusses the main achievements and research status of the reanalysis algorithm in the fields of linear static domain, dynamic problems and nonlinear domain.

Static reanalysis methods

Direct methods

DMs are usually suitable for local or low-rank modifications, and can obtain the exact solution of the modified equilibrium equation. In recent years, many achievements have been gained. Song et al. suggested a novel direct reanalysis algorithm based on the binary tree characteristic to update the triangular factorization in sparse matrix solution [9]. Compared with the SMW formula, the computational efficiency has been improved significantly and it can be used to solve large-scale engineering problems. Huang and Wang proposed an Independent Coefficients (IC) method [10] for large-scale problems which only requires the displacement result as input. By this way, the decomposition of initial stiffness matrix can be omitted and it is easy to combine with any other initial analysis method including the iterative method. Gao and Wang presented an alternative reanalysis algorithm [11] based on the block matrix to address problems with local modifications by divided the modified stiffness into three parts: influenced region, stationary region and an interface region between them (shown in Figure 1). This method can obtain an accurate response for large modification with lower computational cost because the main computation cost concentrates on the influenced region by this specific blocked strategy. It is worth mentioning that the geometric position of the structure needs to be found before calculation by this blocked strategy, so this method needs to enhance its adaptability much more. Huang and Wang also developed an exact reanalysis method named Indirect Factorization Updating (IFU) [12] for the structure with local modifications, including boundary modifications. This method divided the modified equations into two parts: balanced equations and unbalanced equations. Then the balanced equations can be obtained from the fundamental solution system by using the SMW formula and the unbalanced equations can be satisfied by a unique solution which is derived from the general solution of the balanced equations.

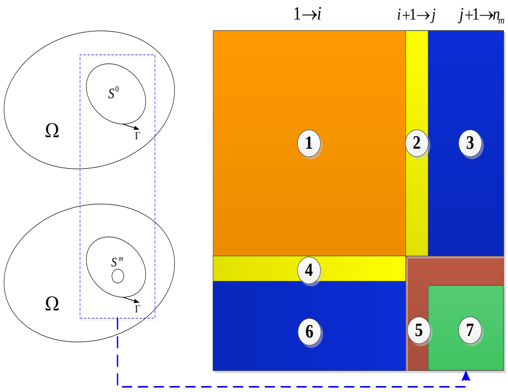

Figure 1. Basic idea of exact blocked reanalysis method

Approximation methods

In recent years, approximation methods have been well developed. Wu et al. developed an efficient implementation of a vector-valued rational approximation in structural reanalysis problems which based on combining the power series expansion with a rational approximation [13]. Compared with local approximations, the accuracy of result has been improved much more for large changes in design variables. Ha-Rok et al. proposed a static combined iterative (CI) method combined with the successive matrix inversion (SMI) and binomial series iterative (BSI) methods [14]. It makes it possible to perform a sequential reanalysis for both symmetric and non-symmetric coefficient matrices. Kirsch et al. introduced a classical iterative algorithm, Preconditioned Conjugate Gradient (PCG) for structural reanalysis [15]. Wu et al. developed the PCG method for both removal of Degrees of Freedom (DOFs) and added DOFs. The condition number of the modified stiffness matrix has been largely reduced and the convergence rate of iterative method has been significantly accelerated by using the preconditioned operator which constructed by fully using the Cholesky factorization of the initial stiffness matrix [16]. Xu and Cheng presented a structural static reanalysis of topological modifications by selected a suitable preconditioned operator based on Lanczos algorithm [17]. Then Wu et al. developed a preconditioned Richardson's iterative method and the relaxation parameter is determined by a very simple formula derived from the corresponding potential energy function [18]. Materna et al. proposed a residual increment approximations for both linear and nonlinear reanalysis [19]. In contrast to other existing reanalysis methods which are based on the evaluation of changed stiffness matrices, only residual vectors have to be computed and stored and it can be done very fast and efficient. To improve the efficiency of reanalysis method, a parallel CA method using single GPU to improve the computational efficiency was first developed by Wang et al. [7] and Wang et al. also developed a CAD/CAE integrated parallel reanalysis design system to handle some complicated problems [20]. Recently, Huang and Wang proposed a Multi-Grid (MG) assisted reanalysis [21] method which doesn't require consistency between the modified mesh and the initial mesh. This method use the MG method to establish a connection between modified mesh and initial mesh and can solve more complex engineering problems even the modified model has been re-meshed as shown in Figure 2.

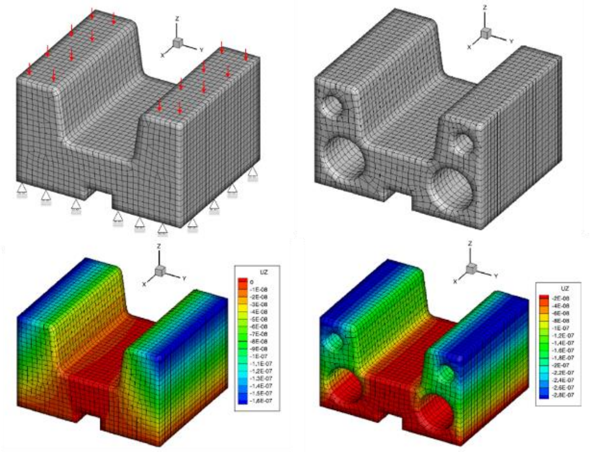

Figure 2 Multi-Grid assisted reanalysis for concave mold

Combined approximations developed by Kirsch might be the most popular approximation reanalysis method due to its generality [8]. By combining local and global approximation methods, the CA method inherited the efficiency of local approximation and the accuracy of global approximation. Initially, the classical CA method is commonly used to solve the problem with small modifications. Sequentially, the CA method has been applied to multiple disciplines, such as structural static reanalysis [22], eigenvalue reanalysis [23], topological optimization [24], vibration reanalysis [25], linear dynamic reanalysis [26], nonlinear dynamic reanalysis [27], sensitivity reanalysis [28] and others. Recently, some alternative versions have been suggested for some complicated engineering problems. Sun et al. proposed an adaptive reanalysis method for structural optimization which determines the minimum number of basis vectors using the K-condition number [29]. This method bridged a closer relationship between the CA method and structural optimization. Zuo et al. presented a sensitivity reanalysis of vibration problem using CA method [30]. The sensitivity reanalysis formulations of eigenvalues and eigenvectors are derived from the vibration equation reduced by CA method, where the eigenvector sensitivity is solved by Nelson's method. Then Feng and Cui utilized the CA method to reduce the computational effort in transient nonlinear heat transfer analysis [31], and transient nonlinear temperature fields are then calculated efficiently without solving complete set of nonlinear system equations.

Dynamic reanalysis methods

There are many methods which are used to solve the implicit dynamic equations, such as mode superposition method, Wilson-θ method, Newmark-β method and so on. But these methods are usually time-consuming because the implicit finite dynamic equations are required to be solved in every time steps. Therefore, the dynamic reanalysis methods have been well developed in recent decades. Generally, these methods can be divided into two parts: frequency-based and time-based reanalysis methods.

Frequency-based dynamic reanalysis methods

As for direct dynamic reanalysis methods, a procedure for the dynamic reanalysis of linear systems subjected to deterministic or stochastic loads is presented by Cacciola et al. and the dynamic response of modified structural system can be evaluated form the knowledge of the transition matrix and the eigenvectors of the original system [32]. Then Kashiwagi proposed a reanalysis method for eigensolution of local systems based on the inverse power method and it enables exact solutions for eigenvalues to be found quickly because only the DOFs of modified part were used by this method [33].

For approximate dynamic reanalysis methods, Stresta et al. presented a dynamic reanalysis of simple beam structures using a polynomial regression method which is used to obtain the natural frequencies of cantilever beam structure after changing the width and depth of the beam [34]. Chen and Guo proposed a method for computing a second-order sensitivity matrix and the Hessian matrix of eigenvalues and eigenvectors of multiple parameter structures. By combined with the second-order Taylor expansion and second-order perturbation method, the changes of eigenvalues and eigenvectors can be estimated when the design parameters are changed [35]. Massa et al. developed a structural modal reanalysis method using homotopy perturbation, projection techniques and Taylor series expansion and a reduced basis is generated using perturbed eigenvectors and the iterative Gram Schmidt normalisation algorithm [36]. Chen et al. proposed an efficient method for evaluating the natural frequencies of structures with uncertain but bounded parameters by using the epsilon-algorithm to accelerate the convergence of the Neumann series [37]. Recently, Sayin and Cigeroglu presented a structural modification method which the modal data of the original system and system matrices of the modifying structure are used to predict the modal response of the modified system [38]. Brizard and Besset developed a determinantal method based an improved sub-structuring method to deal with both undamped and damped systems and this method is applied on the last stage of a space launcher (shown in **Figure 3**) [39]. Zheng and Wu suggested a block CA method for vibration reanalysis based on block CA [40] with shifting which computes several eigenpairs of the modified structure at a time and it has been use to deal with the modal analysis of complex vehicle body as shown in **Figure 4**. In addition, reanalysis method can also handle the problem of near defective systems based on the theory of modal reanalysis method [41].

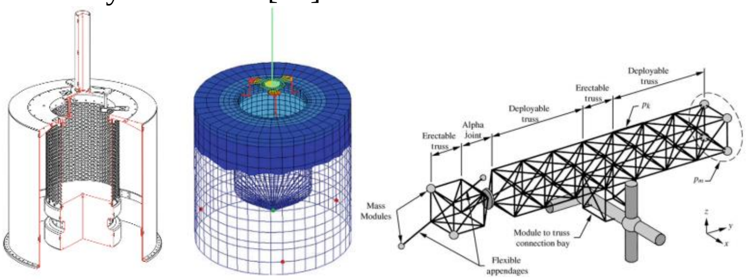

Figure 3 DM method for dynamical engineering problems

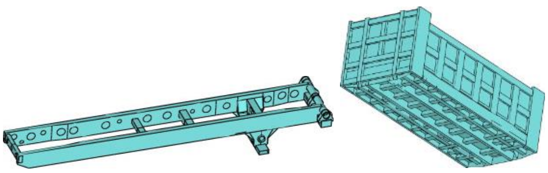

Figure 4 Blocked reanalysis method for frame of truck

Time-based dynamic reanalysis methods

As for the time-based dynamic reanalysis methods, there are also many methods developed in recent decades. Based on Neumman series and epsilon-algorithm, an efficient computation for dynamic responses of systems with arbitrary time-varying characteristics is investigated by Ma and Chen. Avoiding the calculation for the inverse of the equivalent stiffness matrices in each time step, the computational effort of the proposed method is reduced compared with the full analysis of Newmark method [42]. Chen and Ma also developed a dynamic response reanalysis method for modified structures under arbitrary excitation and the approximate response in each time step can be obtained by using the epsilon-algorithm which is based on Newmark method [43]. Then Gao and Wang suggested an adaptive time-based global algorithm for Newmark-β method. In order to obtain the accurate response of modified structure, the global information of original structure should be constructed by sample points which generated by Latin Hypercube Design (LHD). Moreover, the adaptive strategy is used for minimizing the accumulated error in time domain [44], and the analysis of error is shown in Figure 5.

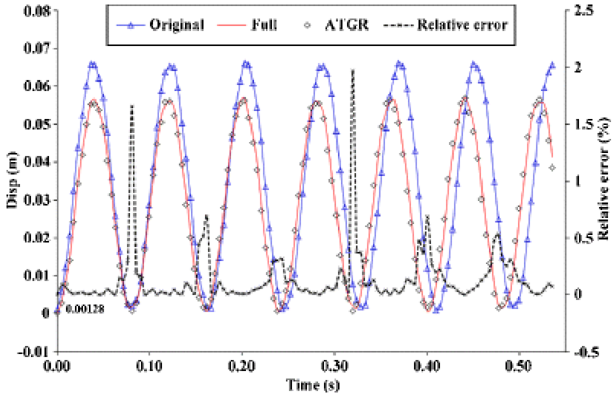

Figure 5 Global dynamical reanalysis method for a truss problem

Nonlinear reanalysis methods

With the rapid development of the reanalysis algorithm, the computational accuracy of the reanalysis algorithm is improved continuously. Since the solution of the nonlinear finite element is usually treated as an iterative process of linear solution, many scholars began to study the structural nonlinear reanalysis.

Kirsch has proved that the CA method can be applied to geometric and material nonlinear problems [8]. Then Kirsch and Bogomolni proposed an efficient procedure for calculations of response surface (CARS) to solve the nonlinear problems [45]. Meanwhile, in order to the difficulty in nonlinear dynamic reanalysis by mode superposition, Kirsch and Bogomolni adopted the CA method into the original method to obtain effective solutions of non-linear dynamic reanalysis problems [46]. Yang and Xu suggested a Multi-sample Compression (MSC) algorithm to obtain convergent results of each sample in the nonlinear FEM of multi-samples [47]. Sayin and Cigeroglu used a harmonic balance method (HBM) with modal superposition approach to convert the resulting nonlinear differential equations of motion into nonlinear algebraic equations, and the finite element model of the structure is divided into two parts representing the modified and unmodified sections. Then the response of modified section can be predicted by the initial modal response and the modified stiffness matrix. It can be observed in Figure 6, the more number of modes used, the more accurate results provided [48].

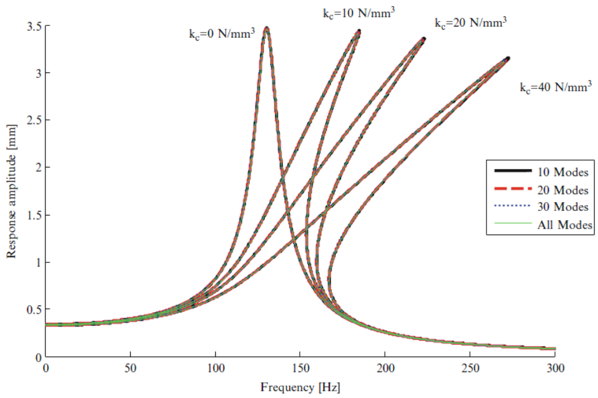

Figure 6 Reanalysis methods for nonlinear problems

Conclusions

This paper investigates and discusses some characteristics of recent developed direct and approximation methods from the view of static, dynamical and nonlinear problems. Reanalysis methods have been rapidly developed in recent decades, but there are still many works needed to do when meeting some more complex engineering problems. Thus, we considered that the following problems of current reanalysis methods are needed to be resolved.

For the methodological aspect, it is an important research focus in the field of reanalysis to solve the static linear problems with high accuracy and efficiency. In face of the dynamic problems, the main challenge is how to control the error and the computational efficiency in a relatively stable equilibrium state.

For the applied aspect, the current popular reanalysis methods are limited in macroscopic field of traditional computational solid mechanics, such as stiffness, strength, modal analysis and so on. Moreover, the potential of reanalysis method has not yet been tapped in the field of application. For example, in the case of dynamic iterations of small grid changes, how to maximize the advantages of reanalysis method when solving the problems of the fluid mechanics, crack propagation, transient temperature and multi-physics analysis will be the future work. For the very hot computational field of multi-scale, the report of reanalysis method has not yet been published.

In addition, as a tool of fast computational solver, the current reanalysis methods are mainly combined with FEM, so how to combine the reanalysis methods to other numerical methods (such as meshless method, X-FEM, and material point method) is also an important subject in the field of reanalysis.

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