Cross-line method (CLM) for solving partial differential equations

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Abstract

In this paper, a new numerical method, Cross-Line Element Method (CLEM or abbreviated as CLM), is proposed for solving general two-dimensional (2D) and three-dimensional (3D) boundary value problems of partial differential equations (PDEs). Firstly, a family of isoparametric elements consisting of a set of cross lines are constructed using the Lagrange interpolation formulation. Secondly, the first and second order partial derivatives of functions with respect to global coordinates are derived based on the Element Differential Method (EDM). Thirdly, the derived spatial partial derivatives are substituted into the operator of PDEs and boundary conditions to set up the final system of equations.

The constructed cross-line elements are isoparametric elements, which can be used to represent both the geometry and physical variables. And the proposed CLM is a strong-form numerical method, which solves the boundary value problems of PDEs based on the direct differentiation of the shape functions of the cross-line elements. When solving a specific problem using the proposed method, only the governing differential equations and boundary conditions are needed, no additional mathematical or mechanical principles being required. A detailed description of CLM for solving general thermal and mechanics problems is presented in the paper, and a number of 2D and 3D numerical examples are given to demonstrate the correctness and efficiency of the proposed method.

Keywords: Cross-line method, CLM, CLEM, Element differential method, Cross-line element, Shape functions