Simulation of sound transmission through thin elastic shell by the Coupled FEM/BEM

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Abstract

The coupled finite element/boundary element method is developed to simulate sound transmission through elastic shell. The vibration of the thin elastic shell is simulated using the finite element method and the acoustic fields both inside and outside the elastic shell are simulated using the boundary element method. To avoid the nonuniqueness problem occurring in the exterior boundary element method, the Burton and Miller formulation is applied. The algorithm is validated using the simulation of a fluid-filled submerged elastic spherical shell excited by a point sound source located at the center.

Keywords: FEM, BEM, Sound transmission

1. Introduction

Acoustic radiation and scattering from fluid-loaded elastic shells may be encountered in the aeronautical and naval industries as well as in underwater acoustics[1]. It is well known that the presence of fluid modifies considerably the resonance characteristics of the structure. In the mean time, the propagation of sound in fluids is altered by the presence of the elastic structure, which causes serious noise problems. The radiated noise from a vibrating structure is important for underwater-related applications. The scattering of acoustic waves from such structures contains information relating to the geometry and composition of the structure, which makes it possible to identify the structure by the remote sensing. Therefore, it is of considerable interest to predict the acoustic fields both radiated and scattered by a submerged vibrating structure.

Analytical approaches to such kind of fluid-structure interaction problems are almost invariably concerned with spherical or infinite circular cylindrical shells subjected to axisymmetric excitations for which the classical method of separation of variables is available. When analyzing the sound radiated or scattered by submerged elastic shells of more complicated shapes, it is almost indispensable to use numerical codes that can handle the complexity of the structure in question.

For a complex structure subjected to known applied forces, the finite element method (FEM) has become an accepted, well-proven, and highly successful analysis tool. Therefore, for the sound/structure interaction model, it is almost consistent to formulate the structural dynamic equations *via* FEM techniques. When applying to the case of interior problems where the fluid is inside the structure, FEM also gives satisfactory solutions. However, in the case of exterior problem in which fluid occupies an unbounded domain, the FEM is inefficient. When FEM has been applied to unbounded exterior problems, the domain has to be truncated and radiating boundary conditions have to be enforced. In addition, they are limited by computer memory and runtime considerations as well. To deal with the unbounded exterior acoustic field problems, the boundary element method based on the utilization of the Helmholtz integral equation has been the most popular numerical tool. The BEM method has several advantages over a FEM treatment of the acoustic problem, including a reduction of dimensionality of the problem by one, and an automatic satisfaction of the radiation condition [2]. The elegance of this method is the mathematical simplicity of the resulting integral expressions.

There are many works on the coupled finite element/boundary element method. For example, Jeans and Mathews presented a unique coupled FEM/BEM method for the elastoacoustic analysis of fluid-filled thin shells [3]. They concluded that except for problems having a significant density difference between the internal and external acoustic fields, for example, air and water, their formulation was suitable.

2. The coupled FEM/BEM model

Our purpose is to investigate the acoustic transmission through thin elastic shells with different fluids on both the inside and outside. There are different fluids on the inside and outside of the thin elastic shell. A thin elastic shell is defined on the closed surface *S*. The shell submerged in an infinite fluid with density ρ^e in the exterior domain *E* and contains a fluid with density ρ^i in the interior domain *D*. The fluids on the inside and outside are assumed to be inviscid and compressible. The surface of the shell is assumed smooth. The normal vector at arbitrary point of the shell surfaces is uniquely determined. Its direction is defined to point into the exterior domain. In the following derivation, assuming there are a point sound source in the interior domain.

According to Hamilton's principle, the finite element governing equation for the dynamic fluid-structure interaction system is given by

$$[-\omega^2 M - i\omega C + K]\{U\} = \{F_I\} + \{F_A\} \tag{1}$$

where *M, K, C* are the mass, stiffness and damping matrices respectively. *U* denotes the displacement and ω is the circular frequency. F_A represents the known applied excitation forces and F_I represents the interaction forces generated by the acoustic fluid acting on the fluid-structure interaction surfaces. The vector of interaction force can be defined through the structure coupled matrix $L_s(L_s^i \text{ and } L_s^e)$ and the nodal acoustic pressures $\{\varphi^i\}$ and $\{\varphi^e\}$, that is

$$\{F_I\} = L_s^i\{\varphi^i\} - L_s^i\{\varphi^e\} \tag{2}$$

Where $\{\varphi^i\}$ and $\{\varphi^e\}$ are the interior and exterior surface acoustic pressures respectively. L_s^i and L_s^e are the structure coupled matrices on the interior surface and exterior surface. The coupled matrix of element is defined as

$$L = \iint_S [N]_f^T \{\bar{n}\}[N]dS \tag{3}$$

Where $[N]_f^T$ is the shape function matrix about displacement in the finite element method[5]. $[N]$ is the shape function matrix about acoustic pressure in the boundary element method and $\{\bar{n}\}$ is the vector of the direction cosines of normal vector.

In operator notations, the surface Helmholtz integral equations for exterior problem can be given as,

$$\left[-\frac{1}{2}I + M_k\right]\varphi^e = L_k \frac{\partial \varphi^e}{\partial n} \tag{4}$$

For interior problem

$$\left[\frac{1}{2}I + M_k\right]\varphi^i = L_k \frac{\partial \varphi^i}{\partial n} + I\varphi^I \tag{5}$$

where the integral operators M_k and L_k are defined as

$$M_k \mu = \iint_S \mu \frac{\partial G}{\partial n} dS \tag{6}$$

$$L_k \mu = \iint_S \mu G dS \tag{7}$$

where $G(p,q)$ is the free-space Green's function for the three-dimensional acoustic wave equation.

To form the coupled FEM/BEM model, the kinetic continuity boundary condition can be expressed as

$$\frac{\partial \varphi^e}{\partial n} = \omega^2 \rho^e Q^e \{U\} \tag{8}$$

$$\frac{\partial \varphi^i}{\partial n} = \omega^2 \rho^i Q^i \{U\} \tag{9}$$

where Q is defined as the kinetic coupled matrix.

Then the final coupled FEM/BEM formulation can be expressed as

$$[-\omega^2 M + K - i\omega C]\{U\} = L_s^i \{\varphi^i\} - L_s^e \{\varphi^e\} \tag{10}$$

$$\left[-\frac{1}{2}I + A^e\right]\{\varphi^e\} = \omega^2 \rho^e B^e Q^e \{U\} \tag{11}$$

$$\left[\frac{1}{2} + A^i\right]\{\varphi^i\} = \omega^2 \rho^i B^i Q^i \{U\} + \{\varphi^I\} \tag{12}$$

where A and B are defined the discretized coefficient matrices of the integral operators M_k and L_k respectively.

To avoid the nonuniqueness problem occurring in the exterior boundary element method, the Burton and Miller formulation [5] is applied.

3. Numerical example

In order to test the correctness of the coupled FEM and BEM method, a fluid-filled submerged elastic spherical shell excited by a point sound source located at the center has been analyzed. In this example, the fluid mediums inside the shell is air and outside the spherical shell is sea water.
The external radius of the spherical shell is 1.01 m and the internal radius is 1.0 m. Therefore, the relative thickness of the spherical shell is 1%. This shell is a thin shell. The density of the spherical shell is $\rho = 7.81 \times 10^3 kg/m^3$. The elasticity modulus is $E = 2.07 \times 10^{11} Pa$ and Poisson's ratio is $\mu = 0.3$. The air density is $\rho^a = 1.21 kg/m^3$ and sound velocity in air is $c^a = 346 m/s$. While, the sea water density is $\rho^w = 1030 kg/m^3$ and sound velocity in sea water is $c^w = 1500 m/s$. The spherical shell is discretized using 96 surface elements.

In Fig. 1 and Fig. 2 the calculated frequency range is from 50 to 2400 Hz. In these figures, the numerical results agree quite well with the corresponding analytical solutions.

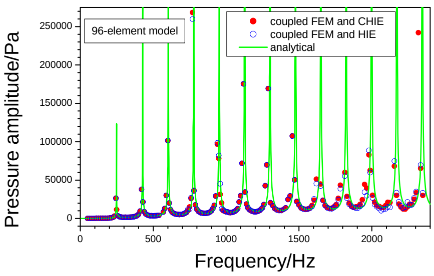

Figure 1. Frequency response of exterior acoustic surface pressure amplitude for a spherical shell

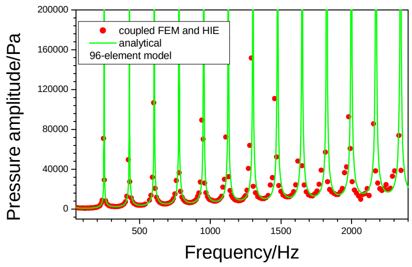

Figure 2. Frequency response of interior acoustic surface pressure amplitude for a spherical shell

4. Conclusions

A coupled finite element/boundary element method is developed for the simulation of sound transmission through thin elastic shell with different fluids on the inside and outside of the shell. This method and the corresponding in-house program is validated by the numerical simulation of sound transmission through a thin spherical shell with air inside and water outside of the shell.

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References

[1] Junger, M.C. and Feit, D. (1986) Sound Structure and Their Interaction, 2nd ed., MIT, Cambridge, MA.
[2] Ciskowski, R.D. and Brebbia, C.A. (1991) Boundary element methods in acoustics, Souththampton Boston: Computational Mechanics Publications, 61-74; 109-129
[3] Jeans, R. A. and Mathews, I. C. (1990) Solution of fluid-structure interaction problems using a coupled finite element and variational boundary element technique, *J. Acoust. Soc. Am.*, **88**, 2459-2466

[4] Yan, Z. Y. (2000) Sound transmission through thin elastic shell with different fluids on both the inside and outside. Ph. D Thesis, Shanghai Jiao Tong University, P.R. China.

[5] Burton, A. J.; Miller, G. F. (1971) The application of integral equation methods to the numerical solution of some exterior boundary value problems. *Proc.R.Soc.London Ser.* **A323**, 201-210.